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# HOMOMORPHISIMS OF L-VAGUE SEMIRINGS OF L-SEMIRING

**R. Panduranga Rao**
*Assistant Professor, Department of Humanities and Basic sciences,*
*Ellenki college of Engineering and Technology, Patelguda village,*
*Ameenpur, Sangareddy-502319, Telangana State, India.*

**Dr. V.S. Kumar**
*Department of Mathematis, JNTUH College of Engineering, JNTU Kukatapally*
*Hyderabad, Telangana State, India.*

**Dr. B. Nagesawara Rao**
*Department of Mathematics, Avanthi Institute of Engineering College,*
*Cherukapalli, Bhogapuram, Vizianagaram Dist. Andhra Pradesh, India.*

**Abstract:** In this paper we introduce L-vague semirings of L-semiring, homomorphisim of L-vague semiring of L-semiring and studied their properties. These concepts are used in the development of some important results and theorems about L-vague semirings of L-semiring. Also some of their important properties have been investigated.

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## INTRODUCTION

The concept of Latice was first defined by Dedekind in 1897 and then doveloped by Birkhoft. G, imposed an operation an open problem “Is there a common abstraction which includes Boolean algebra, Boolean rings and lattice ordered group or L-group is an algebraic structure connecting lattic and group. To answer this problem many common abstractions, namely dually residuated lattice ordered semigroups, commutative lattice ordered groups. Lattice ordered rings, latice ordered near rings and lattice ordered semirings are presented. Amoung them the algebraic structure lattice ordered semiring or L-semiring was introduced by Ranga Rao. P., [13].

Also the concept proposed by Zadeh. L.A. [16] defining a fuzzy subset A of a given universe X characterizing the membership of an element x of X belonging to A by means of a membership function $\mu_A$ defined from $X$ into [0, 1] has revolutionized the theory of Mathematical modeling. Decision making etc., in handling the imprecise real life situations mathematically. Now, several branches of fuzzy mathematics like fuzzy algebra, fuzzy topology, fuzzy control theory, fuzzy measure theory etc., have emerged. But in the decision making, the fuzzy theory takes care of membership of an element x only, that is the evidence against x belonging to A. It is felt by several decision makers and researchers that in proper decision making, the evidence belongs to A and evidence not belongs to A are both necessary and how much X belongs to A or how much x does not belongs to A are necessary.

Several generalizations of Zadeh’s fuzzy set theory have been proposed, such as L-fuzzy sets [4]. Interval valued fuzzysets, Intuitionistic fuzzy sets by Atanassov. K.T. [1], Vague sets [3] are mathematically equivalent. Any such set A of a given Universe X can be charactrierized by means of a pair of function $(t_A, f_A)$ where $t_A$ and $f_A$ are functions from X in to [0 1] such that $0 \leq t_A(x) + f_A(x) \leq 1$ for all x in X. The set $t_A$ is called the truth function and the set $f_A$ is called false function or non membership function and $t_A(x)$ gives the evidence of how much x ∈ A $f_A(x)$ gives the evidence of how much x does not ∈ A. These concepts are being applied in several areas like decisionmaking, fuzzy control, knowledge discovery and fault diagonsis etc. It is believed the vague sets (or equivalently instuitionisticfuzzy sets) will more useful in decision making, and other areas of Mathematical modeling. Through Atanassov's instuitionistic fuzzysets, Gau and Buehrer and some other areas of Mathematical modeling. Since then the theory fuzzy sets developed extensivelyand embraced almost all subjects like engineering science and Technology. But the membership function$\mu_A$ gives only a approximation belong to A. To avid this and obtain a better estimation and analysis of data decision making. Gau. W.L. and Bueher D.J. [3] have initiated the study ofvague sets with the hope that they form a better tool to understand, interpret and solve real life problems which are in general vague, than the theory of vague sets do. Ranjit Biswas [9] initiated the study of vague groups by Ramakrishna. N. [6], [7], [8] and Eswarlal.T. [2] are grate extended the study of vague algebra.

The objective of this paper is to contribute further to the study of vague algebra by introducing the concepts of L-vague semiring of a L-semiring and homomorphism of L-vague semiring of L-vague semiring respectively.

## PRELIMINARIES

In this section we briefly present the necessary material on latices, Boolean latices, Brouwerian latices and illustriate with examples.

**Definition 1. 4** A poset $(L, \leq\ )$ is called a lattice if $sup\{x, y\}$ also denoted by $(x \vee y)$ and $inf\{x, y\}$ also denoted by $(x \wedge y)$ exists for every pair of elements $x, y\ in\ L$.

**Definition 2. 4** A lattice $(L, \leq\ )$ in which every subset of L has g.l.b and l.u.b in it is called a complete lattice.

**Definition 3. 4** A lattice L is said to be distributive if it satisfies $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ and $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ for all $x, y, z$ in $L$.

**Definition 4. 4** A lattice L is said to be bounded if L has least element and greatest element.usually least element of L is denoted by $0_L$ and greatest element is denoted by $1_L$.

**Definition 5. 2** An L-vague set $\mu$ in X is a pair $\mu = (t_\mu, f_\mu)$ where $t_\mu : X \to L$ $f_\mu : X \to L$ are mappings such that $t_\mu \leq 1_L - f_\mu$ and $f_\mu \leq 1_L - t_\mu$ The mapping $t_\mu : X \to L$ is defined the degree of membership function and $f_\mu : X \to L$ defined by the degree of non membership function of the element $x \in X$ to $\mu \subset X$ respectively. The function $t_\mu, f_\mu$ satisfy the condition $t_\mu \leq 1_L - f_\mu$ and $f_\mu \leq 1_L - t_\mu$ ie $t_\mu(x) \leq 1_L - f_\mu(x)$ and $f_\mu(x) \leq 1_L - t_\mu(x)$ for x in X,where $1_L - t_\mu(x)$ and $1_L - f_\mu(x)$ are elements in Brouwerian lattice L.

We recall here that in ordinary vague sets and Boolean vague sets if $(t_A, f_A)$ is a vague set or Boolean vague set then the condition $t_A \leq 1 - f_A$ or $t_A \leq f_A^1$ implies $f_A \leq 1 - t_A$, $f_A \leq t_A^1$. But in our L-vague sets it is not the case, and so in the definition itself we have in included that $t_A \leq 1_L - f_A^1$ and $f_A \leq 1_L - t_A^1$ which is not un natural as this implies that truth is contained in the complement of falsity and falsity is contained in the complement of truth.

**Definition 6. 2** Let $(G, *)$ be a group. An L-vague set $A = (t_A, f_A)$ of G is said to be L-vague group of $G$ if it satisfies the following conditions

(1) $V_A(x * y) \geq \vee\{V_A(x), V_A(y)\}$ for all $x, y \in G$
(2) $V_A(x^{-1}) \geq \{V_A(x)\}$ ie
(1) $t_A(x * y) \geq \wedge\{t_A(x), t_A(y)\}$
(2) $f_A(x * y) \leq \vee\{f_A(x), f_A(y)\}$
(3) $t_A(x^{-1}) \geq t_A(x)$
(4) $f_A(x^{-1}) \leq f_A(x)$ for all $x, y \in G$.

**Definition 7. 9** Let $A$ be a Vague group of a group $G$ then the set $\mathbb{N}(A) = \{a \in G | V_A(axa^{-1}) = V_A(x)$ for all $a, \in G\}$ is called vague normalizer of $A$.

**Definition 8. 7** Let $A$ be a vague group of a group $G$ Then the Set $\mathbb{C}(A) = \{a \in G | V_A([a, x]) = V_A(e)$ for all $x \in G\}$ is called vague centralizer of $A$, where $[a, x] = a^{-1}x^{-1}ax$.

**Defination 9. 7** Let A be a vague group of G, and $GV_A = \{x \in G : V_A(x) = V_A(e)\}$. then the order of A is defined as the order of the crisp sub group $GV_A$ and it is denoted by $O(A)$.

**Definition 10. 11** Let $A$ be a L-vague set of a universe $G$ with true-membership function $t_A$, and false membership function $f_A$. For $\alpha,\ \beta \epsilon[0,1]$ with $\alpha \ \leq\ \beta$, the $(\alpha,\beta)$ cut or L-vague cut set of a L-vague set $A$ is the crisp subset of $G$ is given by $A_{(\alpha,\beta)} = \{x|x\epsilon G,\ V_A(x) \geq [\alpha,\beta]\}$ i.e, $A_{(\alpha,\beta)}$ = $\{x|x\epsilon G,\ t_A(x) \geq \alpha$, and $1 - f_A(x) \geq \beta\}$.

**Definition 11. 8** A non-empty set R is called lattice ordered semi-ring or L-semiring if it has two binary operations + and . in a binary relation $\leq$ defined on it and satisfy the following conditions

(1) $(R,+,.)$ is a semiring
(2) $(R,\leq)$ is a latice
(3) $x \leq y \Rightarrow a + x \leq a + y$, for all $x, yinR$
(4) $x \geq 0, y \geq 0 \Rightarrow xy \geq 0$, for all $x, yinR$

**Examples 12.**

1. $(2Z,+,.,\vee,\wedge)$ is a L-semiring, Where Z is the set of all integers.
2. $(3Z,+,.,\vee,\wedge)$ is a L-semiring, Where Z is the set of all integers and $2, 3 \in Z$.

**Definition 13. 8** Let R be a L-semiring. A fuzzy set A of R is said to be a fuzzy L-semiring of R if it satisfies the following conditions:

1. $\mu_A(x + y) \geq min\{\mu_A(x), \mu_A(y)\}$
2. $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)$
3. $\mu_A(x \vee y) \geq min\{\mu_A(x), \mu_A(y)\}$
4. $\mu_A(x \wedge y) \geq min\{\mu_A(x), \mu_A(y)\}$ for all $x, yinR$.

**Definition 14. 2** A vague set A of semi ring S is said to be of L-vague Ideal if

(1) $t_A(x - y) \geq \wedge\{t_A(x), t_A(y)\}$
(2) $f_A(x - y) \leq \vee\{f_A(x), f_A(y)\}$
(3) $t_A(xy) \geq \wedge\{t_A(x), t_A(y)\}$
(4) $f_A(xy) \leq \vee\{f_A(x), f_A(y)\}$ for all $x, y \in S$.

**Definition 15. 10** The vague set $A$ of set $X$ with $t_A(x) = 0$ and $f_A(x) = 1$, for all $x \in X$ is called zero vague set of $X$. It is denoted by $\bar{0} = (0, 1)$.

**Definition 16. 10** The vague set $A$ of a set $X$ with $t_A(x) = 1$ and $f_A(x) = 0$, for all $x \in X$ is called the unit vague set of $X$. It is denoted by $\bar{1} = (1, 0)$.

## HOMOMORPHISM OF L-VAGUE SEMIRINGS AND L-VAGUE IDEALS

In this section, we introduce and study homomorphism of L-vague semirings and L-Vague Ideals. Now we introduce the following

**Definition 17.** Let S be L-semiring. A L-vague set $A = (t_A, f_A)$ of S is said to be an L-vague semiring of S. if it satisfies the following.

1. $t_A(x + y) \geq \wedge\{t_A(x), t_A(y)\}$ and $f_A(x + y) \leq \vee\{f_A(x), f_A(y)\}$
2. $t_A(xy) \geq \wedge\{t_A(x), t_A(y)\}$ and $f_A(xy) \leq \vee\{f_A(x), f_A(y)\}$
3. $t_A(x \vee y) \geq \wedge\{t_A(x), t_A(y)\}$ and $f_A(x \vee y) \leq \vee\{f_A(x), f_A(y)\}$
4. $t_A(x \wedge y) \geq \wedge\{t_A(x), t_A(y)\}$ and $f_A(x \wedge y) \leq \vee\{f_A(x), f_A(y)\}$

**Example 18.** Let $Z_3 = \{0, 1, 2\}$ be a L-semiring with addition and multiplication modulo 3 on three operations. Take L-vague set $A = \{(0, 0.3, 0.2), (1, 0.4, 0.3), (2, 0.5, 0.4)\}$ and L-vague set $B = \{(0, 0.2, 0.1), (1, 0.3, 0.2), (2, 0.4, 0.3)\}$ are L-vague semirings of a L-semiring $Z_3$.

**Example 19.** Let $Z_3 = \{0, 1, 2, \}$ be a L-semiring with addition and multiplication modulo 3 on three operations. Take L-vague set $A = \{(0, 0.2, 0.1), (1, 0.3, 0.2), (2, 0.4, 0.3)\}$ and L-vague set $B = \{(0, 0.2, 0.3), (1, 0.3, 0.4), (2, 0.4, 0.5)\}$ are L-vague semirings of a L-semiring $Z_3$.

**Definition 20.** Let $S_1$ and $S_2$ be two semirings and $\phi : S_1 \rightarrow S_2$ be a maping. Let A be a vague set of $S_1$.Then the image of L-vague set A can be defined by $(V_{\phi(A)}(x) = (t_{\phi(A)}(x), f_{\phi(A)}(x))$
where $t_{\phi(A)}(x) = t_A(\phi(x)$ and $f_{\phi(A)}(x) = f_A(\phi(x)$.

**Definition 21.** Let $S_1$ and $S_2$ be two semirings and $\phi : S_1 \rightarrow S_2$ be a maping. Let A be a vague set of $S_2$. Then the inverse image of vague set A can be defined by $(V_{\phi^{-1}(A)}(x) = (t_{\phi^{-1}(A)}(x), f_{\phi^{-1}(A)}(x))$
where $t_{\phi^{-1}(A)}(x) = t_A(\phi^{-1}(x)$ and $f_{\phi^{-1}(A)}(x) = f_A(\phi^{-1}(x)$.

**Definition 22.** A vague set A of a semiring S is said to be L-vague left ideal if

(1) $t_A(x - y) \geq \wedge\{t_A(x), t_A(y)\}$
(2) $f_A(x - y) \leq \vee\{f_A(x), f_A(y)\}$

(3) $t_A(xy) \geq t_A(y)$
(4) $f_A(xy) \leq f_A(y)$ for all $x, y \in S$.

**Definition 23.** A vague set A of a semiring S is said to be L-vague right ideal if

(1) $t_A(x-y) \geq \wedge\{t_A(x), t_A(y)\}$
(2) $f_A(x-y) \leq \vee\{f_A(x), f_A(y)\}$
(3) $t_A(xy) \geq t_A(x)$
(4) $f_A(xy) \leq f_A(x)$ for all $x, y \in S$.

**Definition 24.** A vague set A of a semiring S is said to be L-vague ideal if

(1) $t_A(x-y) \geq \wedge\{t_A(x), t_A(y)\}$
(2) $f_A(x-y) \leq \vee\{f_A(x), f_A(y)\}$
(3) $t_A(xy) \geq \vee\{t_A(x), t_A(y)\}$
(4) $f_A(xy) \leq \wedge\{f_A(x), f_A(y)\}$ for all $x, y \in S$.

**Theorem 25.** Let $S_1, S_2$ be any two semirings and $\phi : S_1 \to S_2$ homomorphisim. Then the homomorphic image of L-vague semiring of $S_1$ is a L-vague semiring $S_2$.

*Proof.* Let $S_1, S_2$ be any two L vague semirings and $\phi : S_1 \to S_2$ be a semiring homomorphism.

(1) Let $x, y \in S_1$.
We have $t_A(\phi(x)+(\phi(y)) = \{t_A(\phi(x+y))\} \geq t_A(x+y)$
since $\phi$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$
$\therefore\ t_A(\phi(x+y)) \geq \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$...(2.9.1)
(2) Let $x, y \in S_1$.
We have $f_A(\phi(x)+(\phi(y)) = \{f_A(\phi(x+y))\} \leq f_A(x+y)$
since $\phi$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore\ f_A(\phi(x+y)) \leq \vee\{f_A(\phi(x)), f_A(\phi(y))\}$...(2.9.2)
(3) Let $x, y \in S_1$.
We have $t_A(\phi(x).(\phi(y)) = \{t_A(\phi(xy))\} \geq t_A(xy)$
since $\phi$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$
$\therefore\ t_A(\phi(xy)) \geq \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$...(2.9.3)
(4) Let $x, y \in S_1$.
We have $f_A(\phi(x)\phi(y)) = \{f_A(\phi(xy))\} \leq f_A(xy)$
since $\phi$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore\ f_A(\phi(xy)) \leq \vee\{f_A(\phi(x)), f_A(\phi(y))\}$...(2.9.4)
(5) Let $x, y \in S_1$.
We have $t_A(\phi(x)\vee(\phi(y)) = \{t_A(\phi(x\vee y))\} \geq t_A(x\vee y)$
since $\phi$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$
$\therefore\ t_A(\phi(x \vee y)) \geq \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$...(2.9.5)
(6) Let $x, y \in S_1$.
We have $f_A(\phi(x) \wedge \phi(y)) = \{f_A(\phi(xy))\} \leq f_A(xy)$
since $\phi$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore\ f_A(\phi(x \wedge y)) \leq \vee\{f_A(\phi(x)), f_A(\phi(y))\}$...(2.9.6)
from (2.9.1), (2.9.2), (2.9.3)(2.9.4), (2.9.5), (2.9.6) we have the homomorphic image of a L-vague semiring of $S_1$ is a L-vague semiring of $S_2$. ■

**Theorem 26.** Let $\phi : S_1 \to S_2$ be a semiring homomorphism from $S_1$ *into* $S_2$. Let B be a L-vague semiring of $S_2$. Then $\phi^{-1}(B)$ is L-vague semiring $S_1$.

*Proof.* Let B be a L-vague semiring of $S_2$., Let $x, x \in S_1$. Then Let $S_1, S_2$ be any two L vague semirings and $\phi : S_1 \to S_2$ be a semiring homomorphism.

(1) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) + (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x+y))\}$
$\geq t_A(x+y)$
since $\phi^{-1}$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(x+y)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(2.10.1)
(2) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) + (\phi^{-1}(y)) = \{f_A(\phi^{-1}(x+y))\}$
$\leq f_A(x+y)$
since $\phi^{-1}$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore\ f_A(\phi^{-1}(x+y)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(2.10.2)
(3) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x).(\phi(y)) = \{t_A(\phi^{-1}(xy))\} \geq t_A(xy)$
since $\phi$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(xy)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(2.10.3)
(4) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x)\phi(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$

since $\phi$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore\ f_A(\phi^{-1}(xy)) \leq \vee\{f_A(\phi^{-1}(x)),$
$f_A(\phi^{-1}(y))\}$...(2.10.4)

(5) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) \vee (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x \vee y))\}$
$\geq t_A(x \vee y)$
since $\phi$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(x \vee y)) \geq \wedge\{t_A(\phi^{-1}(x)),$
$t_A(\phi^{-1}(y))\}$...(2.10.5)

(6) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) \wedge \phi^{-1}(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$
since $\phi^{-1}$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore\ f_A(\phi^{-1}(x \wedge y)) \leq \vee\{f_A(\phi^{-1}(x)),$
$f_A(\phi^{-1}(y))\}$...(2.10.6)
from (2.10.1), (2.10.2), (2.10.3)(2.10.4), (2.10.5), (2.10.6) we have the homomorphic image of a L-vague semiring of $S_1$ is a L-vague semiring of $S_2$.

■

**Theorem 27.** Let $\phi : S_1 \to S_2$ be a L-vague semiring homomorphism from $S_1$ in to a $S_2$. Let B be a L-vague left ideal of $S_2$. Then $\phi^{-1}(B)$ is L-vague left ideal of $S_1$.

*Proof.* Let B be a L-vague left ideal of $S_2$. Let $x_1, x_2 \in S_1$ be any element.

(1) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) - (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x - y))\}$
$\geq t_A(x - y)$
since $\phi^{-1}$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(x - y)) \geq \wedge\{t_A(\phi^{-1}(x)),$
$t_A(\phi^{-1}(y))\}$...(2.11.1)

(2) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) - (\phi^{-1}(y)) = \{f_A(\phi^{-1}(x - y))\}$
$\leq f_A(x - y)$
since $\phi^{-1}$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore\ f_A(\phi^{-1}(x - y)) \leq \vee\{f_A(\phi^{-1}(x)),$
$f_A(\phi^{-1}(y))\}$...(2.11.2)

(3) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x).(\phi^{-1}(y)) = \{t_A(\phi^{-1}(xy))\} \geq t_A(y)$
since $\phi$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$\geq t_A(t_A(\phi^{-1}(y))$
$\therefore\ t_A(\phi^{-1}(xy)) \geq \{t_A(\phi^{-1}(y))\}$...(2.11.3)

(4) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x)\phi(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$
since $\phi$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$\leq f_A(\phi^{-1}(y))$
$\therefore\ f_A(\phi^{-1}(xy)) \leq \{f_A(\phi^{-1}(y))\}$...(2.11.4)
From (2.11.1), (2.11.2), (2.11.3) and (2.11.4)$\phi^{-1}(B)$ is L-vague left ideal of $S_1$. ■

**Theorem 28.** Let $\phi : S_1 \to S_2$ be a semiring homomorphism from $S_1$ in to semiring $S_2$. Let B a L-vague right ideal of $S_2$. Then $\phi^{-1}(B)$ is L-vague right ideal of $S_1$.

*Proof.* Let B be a L-vague semiring of $S_2$. Let $x, y \in S_1$. Now,

(1) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) - (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x - y))\}$
$\geq t_A(x - y)$
since $\phi^{-1}$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(x - y)) \geq \wedge\{t_A(\phi^{-1}(x)),$
$t_A(\phi^{-1}(y))\}$...(2.12.1)

(2) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) - (\phi^{-1}(y)) = \{f_A(\phi^{-1}(x - y))\}$
$\leq f_A(x - y)$
since $\phi^{-1}$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore\ f_A(\phi^{-1}(x - y)) \leq \vee\{f_A(\phi^{-1}(x)),$
$f_A(\phi^{-1}(y))\}$...(2.12.2)

(3) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x).(\phi^{-1}(y)) = \{t_A(\phi^{-1}(xy))\} \geq t_A(y)$
since $\phi$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$\geq \{t_A(t_A(\phi^{-1}(x))\}$
$\therefore\ t_A(\phi^{-1}(xy)) \geq \{t_A(\phi^{-1}(x))\}$...(2.12.3)

(4) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x)\phi(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$
since $\phi$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$\leq f_A(\phi^{-1}(x))$
$\therefore\ f_A(\phi^{-1}(xy)) \leq f_A(\phi^{-1}(x))$...(2.12.4)
From (2.12.1), (2.12.2), (2.12.3) and (2.12.1) $\phi^{-1}(B)$ is L-vague right ideal of $S_1$. ■

**Theorem 29.** Let $\phi : S_1 \to S_2$ be a semiring homomorphism from $S_1$ in to a $S_2$. Let B be a L-vague ideal of $S_2$. Then $\phi^{-1}(B)$ is L-vague ideal of $S_1$.

*Proof.* Proof follows from theorem 2.11 and theorem 2.12 ■

**Theorem 30.** Let $\phi : S_1 \to S_2$ be a surjective semiring homomorphism and Let A be a L-vague semiring of $S_1$. Then $\phi(A)$ is L-vague semiring of $S_2$.

*Proof.* Since A is L-vague semiring of $S_1$. Let $x, y. \in S_2$. be any element. Then $\exists x, y \in S_2$. such that $\phi(x) = y$ and $\phi^{-1}(y) = x$

(1) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) + (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x + y))\} \geq t_A(x + y)$
since $\phi^{-1}$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(x + y)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(2.14.1)

(2) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) + (\phi^{-1}(y)) = \{f_A(\phi^{-1}(x + y))\} \leq f_A(x + y)$
since $\phi^{-1}$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore\ f_A(\phi^{-1}(x + y)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(2.14.2)

(3) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x).(\phi(y)) = \{t_A(\phi^{-1}(xy))\} \geq t_A(xy)$
since $\phi$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(xy)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(2.14.3)

(4) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x)\phi(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$
since $\phi$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore\ f_A(\phi^{-1}(xy)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(2.14.4)

(5) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) \vee (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x \vee y))\} \geq t_A(x \vee y)$
since $\phi$ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(x \vee y)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(2.14.5)

(6) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) \wedge \phi^{-1}(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$
since $\phi^{-1}$ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore\ f_A(\phi^{-1}(x \wedge y)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(2.14.6)
from (2.14.1), (2.14.2), (2.14.3), (2.14.4), (2.14.5) and (2.14.6) we have the homomorphic image of a L-vague semiring of $S_1$ is a L-vague semiring of $S_2$. ■

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