International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 12 (2018) pp. 10535-10538

Bipolar Fuzzy Planar Graphs

[1]S. Yahya Mohamed
[1]PG & Research Department of Mathematics, Government Arts College, Tiruchirappalli-620022, India.

[2]N.Subashini
[2]Department of Mathematics, Saranathan College of Engineering, Tiruchirappalli-620012, India.

Abstract

Bipolar fuzzy planar graph is an important subclass of fuzzy graph. Here, Bipolar fuzzy planar graphs and its several properties are presented. The crossing of edges in fuzzy planar graph is not allowed. But, we define bipolar fuzzy planar graph in such a way that the crossing of edges is allowed. Also, we define the bipolar fuzzy planarity value which measures the amount of planarity of a bipolar fuzzy planar graph.

Keywords: Bipolar fuzzy graph, Bipolar fuzzy planar graph, multigraph strong edge, weak edge and strength of bipolar fuzzy graph.

INTRODUCTION

In 1965,L.A.Zadeh[13] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1,1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree

[-1,0) of an element indicates that the element somewhat satisfies the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. In many domains, it is important to be able to deal with bipolar fuzzy information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several direction. Akram [2] introduced the concept of bipolar fuzzy graphs and defined different operations on it. A.Nagoorgani and K.Radha [6,7] introduced the concept of regular fuzzy graphs in 2008 and discussed about the degree of a vertex in some fuzzy graphs. K.Radha and N.Kumarave l[9] introduced the concept of edge degree, total edge degree and discussed about the degree of an edge in some fuzzy graphs. In this paper, fuzzy intersection graphs have been defined from the concept of intersection of fuzzy sets. Samanta and Pal [11] introduced fuzzy tolerance graphs as the generalization of fuzzy intersection graphs. They also defined fuzzy threshold graphs [10]. Fuzzy competition graphs [10] are another kind of fuzzy graphs which are the intersection of the fuzzy neighbourhoods of vertices of a fuzzy graph. Abdul-Jabbar et al. [1] introduced the concept of fuzzy planar graph. In this paper, the crisp planar graph is considered and the membership values are assigned on vertices and edges. Again Nirmala and Dhanabal [8] defined special fuzzy planar graphs. In this papers, the crossing of edges in fuzzy planar graph is not allowed. But, in our work, we define bipolar fuzzy planar graph in such a way that the crossing of edges is allowed. Also, we define the bipolar fuzzy planarity value which measures the amount of planarity of a bipolar fuzzy planar graph. It is also shown that an image can be represented by a bipolar fuzzy planar graph and contraction of such image can be made with the help of bipolar fuzzy planar graph. The bipolar fuzzy multi-graphs and bipolar fuzzy planar graphs are illustrated by examples. These results have certain applications in subway tunnels, routes, oil/gas pipelines representation, etc.

PRELIMINARIES

Definition 2.1 finite graph is a graph $G = (V, E)$ such that V and E are finite sets. An infinite graph is one with an infinite set of vertices or edges or both. Most commonly in graph theory, it is implied that the graphs discussed are finite.

Definition 2.2 A multi-graph is a graph that may contain multiple edges between any two vertices, but it does not contain any self loops. A graph can be drawn in many different ways. A graph may or may not be drawn on a plane without crossing of edges.

Definition 2.3 A drawing of a geometric representation of a graph on any surface such that no edges intersect is called embedding.

Definition 2.4 A graph G is planar if it can be drawn in the plane with its edges only intersecting at vertices of G. So the graph is non-planar it cannot be drawn without crossing.

Definition 2.5 A planar graph with cycles divides the plane into a set of regions, also called faces. The length of a face in a plane graph G is the total length of the closed walk(s) in G bounding the face. The portion of the plane lying outside a graph embedded in a plane is infinite region.

Definition 2.6 For the fuzzy graph $\varepsilon = (V, \sigma, \mu)$, an edge (x,y) is called strong if $\frac{1}{2}\min(\sigma(x), \sigma(y)) \leq \mu(x, y)$ and weak otherwise. The strength of the fuzzy edge (x,y) is represented by the value $\frac{\mu(x,y)}{\min(\sigma(x),\sigma(y))}$.

Definition 2.7 If an edge (x, y) of a fuzzy graph satisfies the condition $\mu(x, y) = \min(\sigma(x), \sigma(y))$, then this edge is called effective edge.Two vertices are said to be effective adjacent if they are the end vertices of the same effective edge. Then the effective incident degree of a fuzzy graph is defined as the number of effective incident edges on a vertex v. If all the edges of a fuzzy graph are effective, then the fuzzy graph becomes complete fuzzy graph.

Definition 2.8 By a bipolar fuzzy graph, we mean a pair G = (A,B) where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation on E such that $\mu_B^P\ (x, y) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N\ (x, y) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all$(x, y) \in E.$ We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E respectively. Note that B is symmetric bipolar fuzzy relation on A. we use the notation xy for an element of E. Thus, G = (A,B) is a bipolar fuzzy graph of $G^* = (V, E)$ if $\mu_B^P\ (xy) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N\ (xy) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all $xy \in E$.

BIPOLAR FUZZY PLANAR GRAPH.

Planarity is important in connecting the wire lines, gas lines, water lines, printed circuit design, etc. But, sometimes little crossing may be accepted to these design of such lines/ circuits. So bipolar fuzzy planar graph is an important topic for these connections.

A crisp graph is called non-planar graph if there is atleast one crossing between the edges for all possible geometrical representations of the graph. Let a crisp graph G has a crossing for a certain geometrical representation between two edges $[\mu_B^P(ab), \mu_B^N(ab)]$ and $[\mu_B^P(cd), \mu_B^N(cd)]$. In bipolar fuzzy concept, we say that this two edges have membership values (-1,1). If we remove the edge $[\mu_B^P(cd), \mu_B^N(cd)]$, the bipolar fuzzy graph becomes planar. In bipolar fuzzy sense, we say that the edges $[\mu_B^P(ab), \mu_B^N(ab)]$ and $[\mu_B^P(cd), \mu_B^N(cd)]$ have membership value

(-1,1) and (0,0) respectively.

Let $\varepsilon = (V, A, B)$ be a bipolar fuzzy graph and for a certain geometric representation, the graph has only one crossing between two bipolar fuzzy edges $\left[[\mu_A^P(w), \mu_A^N(w)], [\mu_A^P(x), \mu_A^N(x)], [\mu_B^P(wx), \mu_B^N(wx)]\right]$and $\left[[\mu_A^P(y), \mu_A^N(y)], [\mu_A^P(z), \mu_A^N(z)], [\mu_B^P(yz), \mu_B^N(yz)]\right]$.
If $[\mu_B^P(wx), \mu_B^N(wx)] = (-1,1)$ and $[\mu_B^P(yz), \mu_B^N(yz)] =$ (0,0), then we say that the bipolar fuzzy graph has no crossing. Similarly, if $[\mu_B^P(wx), \mu_B^N(wx)]$ has value near to (-1,1) and $[\mu_B^P(wx), \mu_B^N(wx)]$ has value near to (0,0), the crossing will not be important for the planarity. If $[\mu_B^P(wx), \mu_B^N(wx)]$ has value near to (-1,1) and $[\mu_B^P(wx), \mu_B^N(wx)]$ has value near to (-1,1), then the crossing very important for the planarity.

Before going to the main definition, some co-related terms are discussed below.

INTERSECTING VALUE IN BIPOLAR FUZZY MULTIGRAPH.

In Bipolar Fuzzy multigraph when two edges intersect at a point, a value is assigned to that point in the following way. Let in a Bipolar Fuzzy multigraph $\varepsilon = (V, A, B)$, B contains two edges $[\mu_B^P(ab), \mu_B^N(ab)]$ and $[\mu_B^P(cd), \mu_B^N(cd)]$ which are intersected at a point P.

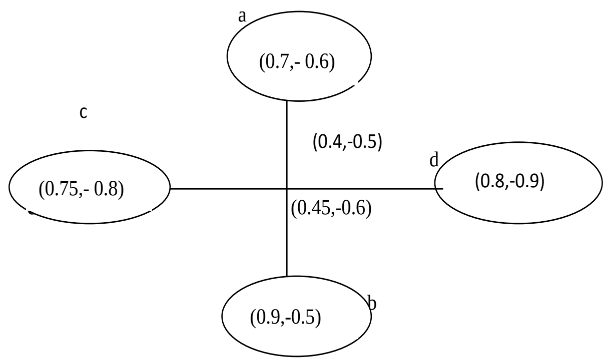

Figure 1. Intersecting value between two intersecting edges.

Strength of the Bipolar fuzzy edge $[\mu_B^P(ab), \mu_B^N(ab)]$ can be measured by the value $S_{[\mu_B^P(ab),\mu_B^N(ab)]} = \frac{[\mu_B^P(ab),\mu_B^N(ab)]}{[min[\mu_A^P(a),\mu_A^P(b)],max[\mu_A^N(a),\mu_A^N(b)]]}$ =(0.5714,1) $\geq$ 0.5 and

$S_{[\mu_B^P(cd),\mu_B^N(cd)]}) \geq$ 0.5, then the bipolar fuzzy edge is called strong otherwise weak.

We define the intersecting value at the point P by

$$I_{[\mu_A^P(P),\mu_A^N(P)]} = \left(\frac{S_{[\mu_B^P(ab)+\mu_B^P(cd)]}}{2}, \frac{S_{[\mu_B^N(ab)+\mu_B^N(cd)]}}{2}\right)$$

If the number of point of intersections in a bipolar fuzzy multigraph increases, planarity decreases. So for bipolar fuzzy multigraph $I_{[\mu_A^P(P),\mu_A^N(P)]}$ is inversely proportional to the planarity. Based on this concept, a new terminology is introduced below for a bipolar fuzzy planar graph.

Definition.4.1: Let ε be a bipolar fuzzy multigraph and for a certain geometrical representation $[\mu_A^P(P_1), \mu_A^N(P_1)], [\mu_A^P(P_2), \mu_A^N(P_2)], \ldots \ldots [\mu_A^P(P_n), \mu_A^N(P_n)]$ be the points of intersections between the edges. ε is said to be bipolar fuzzy planar graph with fuzzy planarity value f, where

$$f = \frac{1}{1+I_{[\mu_A^P(P_1),\mu_A^N(P_1)]}+I_{[\mu_A^P(P_2),\mu_A^N(P_2)]}+\cdots+I_{[\mu_A^P(P_n),\mu_A^N(P_n)]}}$$

It is obvious that f is bounded and the range of f is $-1 \leq f \leq 1$.

If there is no point of intersection for a certain geometrical representation of a bipolar fuzzy planar graph, then its bipolar fuzzy planarity value is $(-1,1)$. In this case, the underlying crisp graph of this bipolar fuzzy graph is the crisp planar graph. If f decreases, then the number of points of intersection between the edges increases and obviously the nature of planarity decreases. From this analogy, one can say that every bipolar fuzzy graph is a bipolar fuzzy planar graph with certain bipolar fuzzy planarity value.

Example. 4.2: Here an example is given to calculate the intersecting value at the intersecting point between two edges. Two edges $[\mu_B^P(ab), \mu_B^N(ab)]$ and $[\mu_B^P(cd), \mu_B^N(cd)]$ are intersected where $[\mu_A^P(a), \mu_A^N(a)] = (0.7, -0.6)$, $[\mu_A^P(b), \mu_A^N(b)] = (0.9, -0.5)$,

$[\mu_A^P(c), \mu_A^N(c)] = (0.75, -0.8)$, $[\mu_A^P(d), \mu_A^N(d)] = (0.8, -0.9)$, $[\mu_B^P(ab), \mu_B^N(ab)] = (0.4, -0.5)$ & $[\mu_B^P(cd), \mu_B^N(cd)] = (0.45, -0.6)$

Strength of the edge $[\mu_B^P(ab), \mu_B^N(ab)] = (0.5714,1)$ and that of $[\mu_B^P(cd), \mu_B^N(cd)] = (0.6,0.75)$.

Thus the intersecting value at the point is (0.5857,0.875)

Bipolar fuzzy planarity value for a bipolar fuzzy multigraph is calculated from the following theorem.

Theorem 4.3: Let ε be a bipolar fuzzy multigraph such that edge membership value of each intersecting edge is equal to the minimum of positive membership values and maximum of negative membership values of its end vertices. The bipolar fuzzy planarity value f of ε is given by $f = \frac{1}{1+N_p}$ where N_p is the number of point of intersections between the edges in ε.

Proof.

Let $\varepsilon = (V, A, B)$ be a bipolar fuzzy multigraph such that edge membership values of each intersecting edge is equal to minimum of its vertex positive membership values and maximum of its vertex negative membership values.For the bipolar fuzzy multigraph

$[\mu_B^P(xy)] = min\ [\mu_A^P(x), \mu_A^P(y)]$ & $[\mu_B^N(xy)] = max\ [\mu_A^N(x), \mu_A^N(y)]$ for each intersecting edge $[\mu_B^P(xy), \mu_B^N(xy)]$.

Let $[\mu_A^P(P_1), \mu_A^N(P_1)], [\mu_A^P(P_2), \mu_A^N(P_2)], \dots \dots [\mu_A^P(P_k), \mu_A^N(P_k)]$ be the point of intersection between the edges in ε, k being an integer. For any intersecting edge $[\mu_B^P(ab), \mu_B^N(ab)]$ in ε,

$$I_{[\mu_B^P(ab), \mu_B^N(ab)]} = \frac{[\mu_B^P(ab), \mu_B^N(ab)]}{\{min\ [\mu_A^P(a), \mu_A^P(b)]\ , max\ [\mu_A^N(a), \mu_A^N(b)]\}} = (1,1)$$

Therefore, for $[\mu_A^P(P_1), \mu_A^N(P_1)]$, the point of intersection between the edges $[\mu_B^P(ab), \mu_B^N(ab)]$ and $[\mu_B^P(cd), \mu_B^N(cd)]$, $I_{[\mu_A^P(P_1), \mu_A^N(P_1)]} = (1,1)$.

Hence $I_{[\mu_A^P(P_i), \mu_A^N(P_i)]} = (1,1)$ for i = 1,2,….k.

$$\text{Now } f = \frac{1}{1+\left\{I_{[\mu_A^P(P_1), \mu_A^N(P_1)]} + I_{[\mu_A^P(P_2), \mu_A^N(P_2)]} + \dots + I_{[\mu_A^P(P_n), \mu_A^N(P_n)]}\right\}} = \frac{1}{1+N_p}$$

Where N_P is the number of point of intersection between the edges in ε.

Definition.4.4: A bipolar fuzzy planar graph ε is called strong bipolar fuzzy planar graph if the bipolar fuzzy planarity value of the graph is greater than 0.5.

Theorem.4.5: Let ε be a strong bipolar fuzzy planar graph. The number of point of intersection between strong edges in ε is atmost one.

Proof.

Let $\varepsilon = (V, A, B)$ be a strong bipolar fuzzy planar graph. Let, if possible, ε has atleast two point of intersections Let $[\mu_A^P(P_1), \mu_A^N(P_1)], [\mu_A^P(P_2), \mu_A^N(P_2)]$ between two strong edges in ε.

For any strong edge $[\mu_B^P(ab), \mu_B^N(ab)]$,

$[\mu_B^P(ab)] \geq \frac{1}{2} min\ [\mu_A^P(a), \mu_A^P(b)]$ & $[\mu_B^N(ab)] \leq \frac{1}{2} max\ [\mu_A^N(a), \mu_A^N(b)]$

So $I_{[\mu_B^P(ab), \mu_B^N(ab)]} \geq 0.5$

Thus for two intersecting strong edges $[\mu_B^P(ab), \mu_B^N(ab)]$ and $[\mu_B^P(cd), \mu_B^N(cd)]$,

$$I_{[\mu_A^P(P_1), \mu_A^N(P_1)]} \geq 0.5 \text{ and } I_{[\mu_A^P(P_2), \mu_A^N(P_2)]} \geq 0.5$$

Then $1+I_{[\mu_A^P(P_1), \mu_A^N(P_1)]} + I_{[\mu_A^P(P_2), \mu_A^N(P_2)]} \geq 2$

Therefore $f \leq 0.5$

It contradicts the fact that the bipolar fuzzy graph is a strong bipolar fuzzy planar graph.

So number of point of intersections between strong edges can not be two. It is clear that if the number of point of intersection of strong edges increases, the bipolar fuzzy planarity value decreases. Similarly, if the number of point of intersection of strong edges is one, then the bipolar fuzzy planarity value $f > 0.5$. Any bipolar fuzzy planar graph without any crossing between edges is a strong bipolar fuzzy planar graph. Thus, we conclude that the maximum number of point of intersections between the strong edges in ε is one.

Definition. 4.6: Let $\varepsilon = (V, A, B)$ be a bipolar fuzzy planar graph. A bipolar fuzzy face of ε is a region, bounded by the set of bipolar fuzzy edges $E' \subseteq E$, of a geometric representation of ε. The membership value of the bipolar face is

$min\left\{\left\{\frac{[\mu_B^P(xy)]}{min\ [\mu_A^P(x), \mu_A^P(y)]}\right\}, \left\{\frac{[\mu_B^N(xy)]}{max[\mu_A^N(x), \mu_A^N(y)]}\right\}\right\}$ A bipolar fuzzy face is called strong bipolar fuzzy face if its membership value is greater than 0.5, and weak face otherwise. Every bipolar fuzzy planar graph has an infinite region which is called outer bipolar fuzzy face. Other faces are called inner bipolar fuzzy faces.

Example.4.7: In fig.2, F_1, F_2 & F_3 are three bipolar fuzzy faces. F_1 is bounded by the edges $[\mu_B^P(v_1v_2), \mu_B^N(v_1v_2)]$, $[\mu_B^P(v_2v_3), \mu_B^N(v_2v_3)]$ & $[\mu_B^P(v_3v_1), \mu_B^N(v_3v_1)]$ with membership value(0.8333,0.5714). Similarly, F_2 is a bipolar fuzzy bounded face. F_3 is the outer bipolar fuzzy face with membership value (0.333,0.375). So F_1 is a strong bipolar fuzzy face and F_2, F_3 are weak bipolar fuzzy faces.

Every strong bipolar fuzzy face has membership value greater than 0.5. So every edge of a strong bipolar fuzzy face is a strong bipolar fuzzy edge.

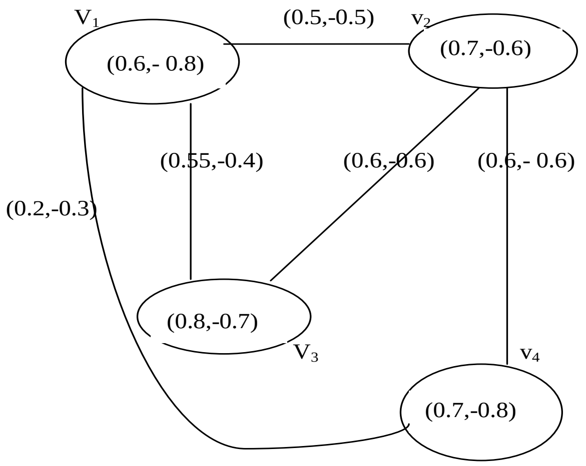

Figure 2. Example of faces in bipolar fuzzy planar graph

CONCLUSION

This study describes the bipolar fuzzy multigraphs, bipolar fuzzy planar graphs, and a very important consequence of bipolar fuzzy planar graph known as bipolar fuzzy dual graphs. In crisp planar graph, no edge intersects other.In bipolar fuzzy graph an edge may be weak or strong. We define bipolar fuzzy planar graph in such a way that an edge can intersect other edge. But, this facility violates the definition of of planarity of graph.This is to be investigated in near future.

REFERENCES

[1] N.Abdul-jabbar,J.H.Naoom and, E.H.Ouda, Fuzzy dual graph, Journal of Al.Nahrain University, 12 (4),168-171,2009.

[2] M.Akram, Bipolar fuzzy graphs, Information sciences,DOI 10.1016/j.ins 2011.07.037,2011.

[3] S.Arumugam and S.Velammal, edge domination in graphs, Taiwanese journal of Mathematics, Vol.2,PP.173-179 June 1998.

[4] V.K.Balakrishnan, Graph theory,McGraw-Hill,1997.

[5] Ganesh Ghorai and Madhumangal Pal, Faces and dual of *m*-polar fuzzy planar graphs, Journal of Intelligent & Fuzzy Systems 31 (2016) 2043–2049 DOI:10.3233/JIFS-16433 IOS Press 2043

[6] A.Nagoorgani and R.J.Hussain, Fuzzy effective distance k-dominating sets and their applications, International journal of Algorothms, computing and Mathematics,2(3),25-36,2009.

[7] A.Nagoorgani, K.Radha, On regular fuzzy graphs, journal of physical sciences,Vol.12,33-44,2008.

[8] A.Nagoorgani, and K.Radha The degree of a vertex in some fuzzy graphs, International journal of algorithms, computing and Mathematics, Vol 2,No.3,Aug 2009.

[9] G.Nirmala and K.Dhanabal, special planar fuzzy graph configurations, International Journal of scientific and Research publications,2(7),1-4,2012.

[10] K.Radha and N.Kumaravel, The degree of an edge in Cartesian product and composition of two fuzzy graphs, International journal of applied mathematics and statistical sciences (IJAMSS) IASET, Vol.2,Issue2 65-78 may 2013.

[11] S.Samanta and M.Pal, Fuzzy K-competition graphs and p-competition fuzzy graphs,Fuzzy Engineering and Information,5(2),191-204,2013.

[12] S.Samanta and M.Pal, Fuzzy tolerance graphs, International Journal of Latest trends in Mathematics,1(2), 57-67,2011.

[13] S.Samanta and M.Pal, Irregular bipolar Fuzzy graphs, International Journal Applications of Fuzzy sets,2,91-102,2012.

[14] Sovan samanta and Madhumangal pal, Fuzzy planar graphs, DOI 10.1109 / TFUZZ.2014.2387875, IEEE Transactions of fuzzy systems.

[15] L.A.Zadeh,Fuzzy sets, Information and control 8(1965) 338-353.