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Teaching and Learning of Mathematics through Technology at Senior Secondary School

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Abstract

There is a general Mathematics phobia among the young school students. In this paper, we made an effort to find ways to motivate and create interest among students. We organized a training programme and introduced open source software, namely, GeoGebra to a group of senior secondary students of Synod Higher Secondary School, Aizawl, Mizoram, India. We discussed some of the problems from their text book. We analyzed the impact of the training programme through the pre and post questionnaires. The analysis showed that the training programme was successful in motivating and generating more interest among the students. We conclude that in classroom teaching, the teacher should put more emphasis in motivating the students and use software based methods as far as possible.

Keywords: Teaching and learning mathematics, Technology based learning, Mathematics education

INTRODUCTION

Today, in many countries around the world, there is a significant gap between the knowledge and skills students learn in school and the knowledge and skills workers need in workplaces and communities. Employers report that they need students who are better prepared in skills such as professionalism and work ethic, oral and written communication, teamwork and collaboration, critical thinking and problem solving, application of information technology, and leadership. So the emphasis in schools is increasingly on learning how to learn, rather than just acquiring specific technical skills that keep changing anyway. New fields in Mathematics such as Operations Research, Control theory, Signal Processing and cryptography have been generated which need technology. Students of today need an understanding of mathematics that allows them to produce and interpret technology-generated results, to develop and to evaluate alternative solution paths, and to recognize and understand the mathematical limitation of particular technological tools (Balacheff and Kaput, 1996). To satisfy our demand mathematics teaching practices have been changing all over the world especially in developed countries. Research shows that teachers are stepping away from the chalkboard or white board and move forward toward the use of calculator, computers and interactive whiteboards. By using Dynamic Geometry software, Lavy and Shriki, (2010) explored the changes in prospective mathematics teachers with regard to their mathematical knowledge in problem posing activities in geometry. They found that the students improved their knowledge of geometrical concepts and shapes involved. And, in the process of creating the problem and working out its solution, they improved their understanding of the interconnections between the concept and the shape involved.

TECHNOLOGY BASED LEARNING IN MIZORAM

Although many countries use technology for teaching and learning of mathematics, in India especially in Mizoram the remote part of India the teaching method still we practice for learning mathematics is a traditional method which is a lecture type classroom teaching. In this paper we study how technology based learning of mathematics can influence students in learning mathematics. We conducted a training programme at Synod Higher Secondary School in Aizawl, Mizoram. The aim of the workshop is to demonstrate to the student that through technology one can explore variables and relationship among variables. And to illustrate that technology is a significant tool for promoting their mathematical problem solving ability, reasoning and exploration. We also demonstrated how technology can improve their understanding. Jimoyiannis, (2010) argues that true learning in the 21st century requires students being able to use Information and Communication Technology, not only for enhancing the memorization of facts, but also for problem solving in real world settings. Güven *et al*. (2010) show that Dynamic Geometry Software can be used in supporting deductive proof by showing empirical evidence as the source of insight for the proof.

Zhang and Jiao, (2011) categorized technology-based mathematics learning approaches into two areas: (1) online learning environment using already developed math contents; and (2) dynamic mathematics software in which the teaching contents have not been developed. The first category is a traditional courseware based on online learning environment, in which teachers and students can use passively the instructional materials. On the other hand, dynamic math software enables a learner to benefit from learning-by-doing (Nilsson and Pareto, 2010) which is difficult to be achieved by traditional tools or technologies (Zhang and Jiao, 2011). Dynamic math software (Stahl *et al*. 2010) supports learners' self explorations. However, dynamic software-based instructions might have negative effect to the high performance students (Zhang and Jiao, 2011), and it still needs more engaging and collaborative activities. Despite the effectiveness of technology and its potential for mathematics learning, teachers still need to leverage technology resources in ways that extend and increase their effectiveness as meaningful pedagogical tools (Ertmer and Ottenbreit-Leftwich, 2010).

DYNAMIC SOFTWARE GEOGEBRA

GeoGebra was designed by Markus Hohenwater as an open-source dynamic mathematics software that incorporates geometry, algebra and calculus into a single, open-source, user-friendly package (Hohenwarter *et al.* 2009). This software has combined features of older software programs such as Maple, Derive, Cabri and Geometer's Sketchpad (Sahaa *et al.* 2010). GeoGebra is free and user-friendly software that connects geometry and algebra (White, 2012). GeoGebra's support materials are rather impressive (especially for a free program), where it provides wide-ranging online help feature, 42-page help manual in pdf format, downloadable tutorials, and a variety of detailed lessons using video-based step-by-step examples. These materials are very concise, easily accessible, and professionally done, with supplementary suggestions contributed by users. This concerted assisted environment is described as focusing on "quality versus quantity" in the GeoGebra website (Grandgenett, 2007).

A review of literature also shows that use of GeoGebra has a positive impact on students' understanding of geometry. Erhan and Andreasen, (2013) suggested that students improved their mathematics understanding after using the dynamic geometry software. Students were able to explore and form conjectures and therefore had better scores as well. GeoGebra was described as raising the enthusiasm for the effective and wise application of technology to the teaching/learning enterprise (Fahlberg-Stojanovska and Stojanovski, 2009; Hewson, 2009). Also, the teacher should strongly encourage geometric visualizations through the lesson plan (Dikovic, 2007). Leong, (2013) conducted a study to determine the effects of the dynamic software, Geometer's Sketchpad (GSP) in the teaching and learning of graph functions. This study was conducted among six students in a Malaysian secondary school. A quasi-experimental design using intact sampling was employed. A significant difference was observed in the achievement of the experimental group as compared to the control. This indicates that the dynamic software (GSP) had a positive effect on student achievement and attitude towards learning mathematical concepts.

THE TRAINING PROGRAMME

The programme was divided into two parts; theory and practical. In the theory class, we introduced Geogebra through power point presentation followed by demonstration. The problems below are picked from their current school textbook (Aggarwal, 2016) to demonstrate how ICT (Information and Communication Technology) can help students in understanding the concepts better.

Picture 1: Senior Secondary students in the Training Programme.

Problem 1: Determine the domain for the following real functions:

(1) $\mathrm{F}(x)=\sqrt{9-x^2}$ (2) $\mathrm{G}(x)=\dfrac{1}{1-x^2}$ (3) $\mathrm{H}(x)=\dfrac{x^2}{1+x^2}$

Solution: (1).The given function $\mathrm{F}(x)=\sqrt{9-x^2}$ is undefined when

$$9-x^2<0 \quad i.e. \quad x^2>9 \Rightarrow x<-3 \quad and \quad x>3$$

Hence, the domain of the function F is given by [-3, 3].

(2). The given function $\mathrm{G}(x)=\dfrac{1}{1-x^2}$ is undefined when $x=\pm 1$

Thus, the domain of the function G is given by $R-\{-1,1\}$

(3). The function $\mathrm{H}(x)=\dfrac{x^2}{1+x^2}$ is defined for all real numbers. Thus, the domain of the function H is R.

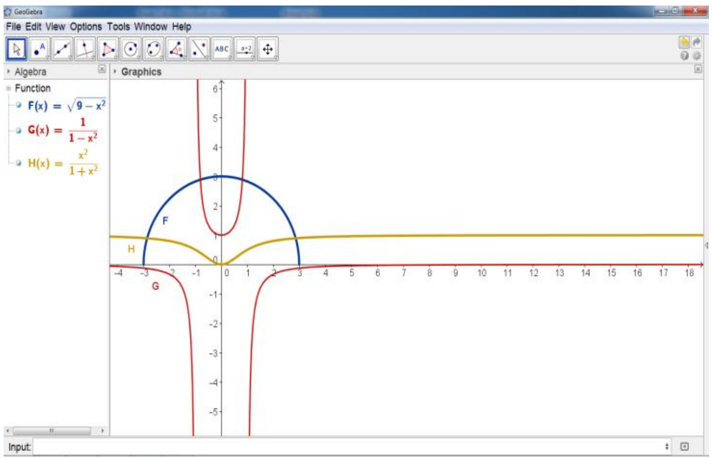


Figure 1

Problem 2: Find the limit $\lim_{x \to 0} \frac{\sin x}{x}$.

Solution: Generally students at school level had a serious problem on understanding limits and limits of a function. The explanation given in the textbook is as follows:

(1) $$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\left[\frac{x-\frac{x^3}{3!}+\frac{x^5}{5!}-.......}{x}\right]$$

$$= \lim_{x\to 0} x\left[\frac{1-\frac{x^2}{3!}+\frac{x^4}{5!}-.......}{x}\right]$$

$$= \lim_{x\to 0}\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-......\right)=1$$

A better way to understand how $\lim_{x\to 0}\frac{\sin x}{x}$ = 1 is through the following graphs from GeoGebra.

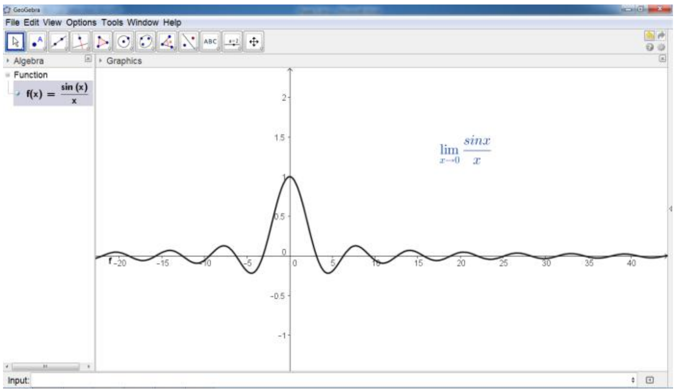


Figure 2(a)

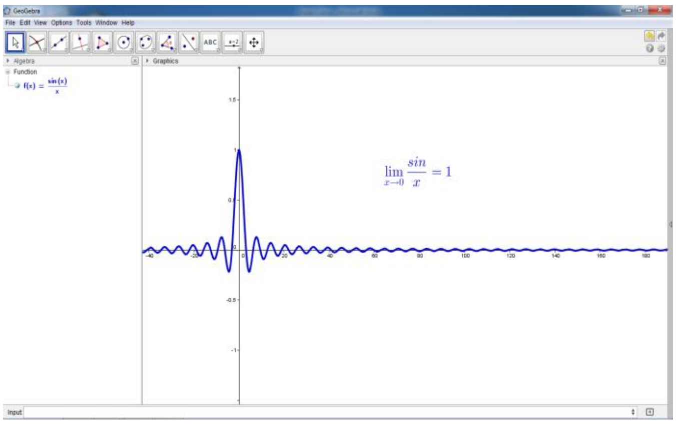


Figure 2(b)

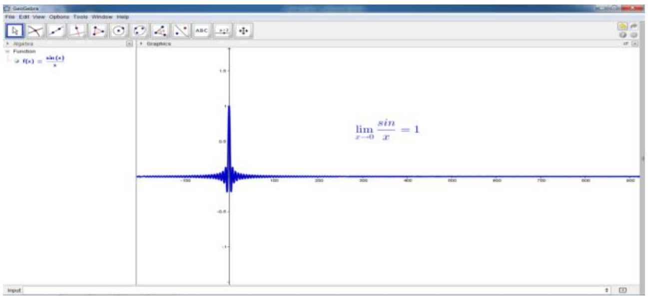


Figure 2(c)

Problem 3: The classic box problem

An open box is to be made out of a piece of cardboard measuring 24cm x 24 cm by cutting off equal squares from the corners and turning up the sides. Find the height of the box when it has maximum volume.

Solution: Let the length of the side of the square cut off from the corners be x cm.

Then the height of the box = x cm as shown in the figure.

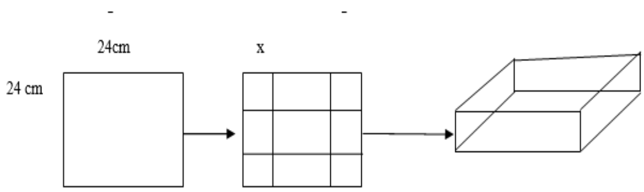


Figure 3(a): Biggest box problem

$$\therefore V = lenght \times breadth \times height$$

$$= (24-x)\times(24-x)\times x \ = x(24-x)^2$$

$$= 4x^3 - 96x^2 + 576x$$

$$\Rightarrow \frac{dV}{dx} = 12(x^2 - 16x + 48) \text{, And } \frac{d^2V}{dx^2} = 24(x-8)$$

Now, $\frac{dV}{dx} = 0 \Rightarrow x^2 - 16x + 48 = 0$. i.e.

$$(x-12)(x-4)=0 \ \Rightarrow x = 4(\because x \neq 12)$$

$$\left[\frac{d^2V}{dx^2}\right]_{x=4} = -96 < 0. \quad \therefore \text{ V is maximum at x = 4.}$$

Hence, the volume of the box is maximum when its height is 4 cm. Its volume is 1024 cubic cm.

The classical Box problem can be demonstrated using GeoGebra as shown below:

a) Type the equation $V = 4x^3 - 96x^2 + 576x$, in the input bar and press Enter. Figure 3(b) show the graph of the function. To change the axis click button and the scroll the axis to get figure 3(c).

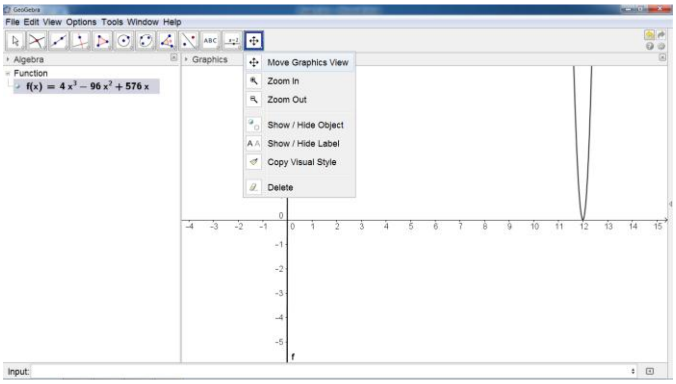


Figure 3(b)

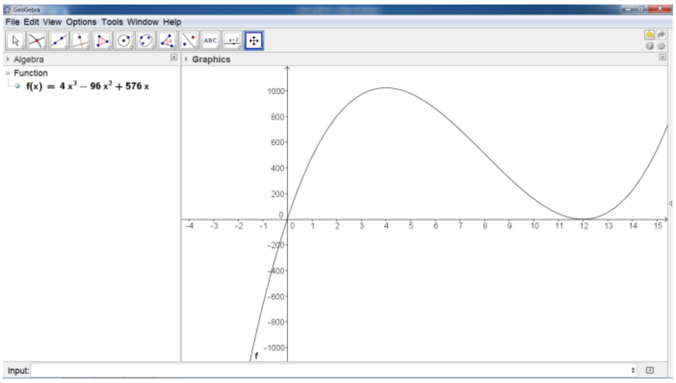


Figure 3(c)

b) Find the derivative of $V(x)$. Using properties change the color of $V(x)$ and $V'(x)$ with red and blue as shown in figure 3(d).

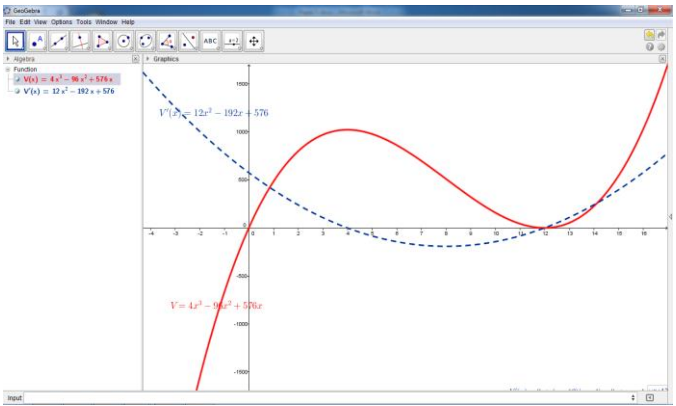


Figure 3(d)

c) Select intersection button and find the intersection of $V'(x)$ with x-axis. Which gives A (4, 0) and B (12, 0). This shows that x = 4 or x = 12 are the extreme points for the function $V(x)$ as shown in figure 3(e).

d)

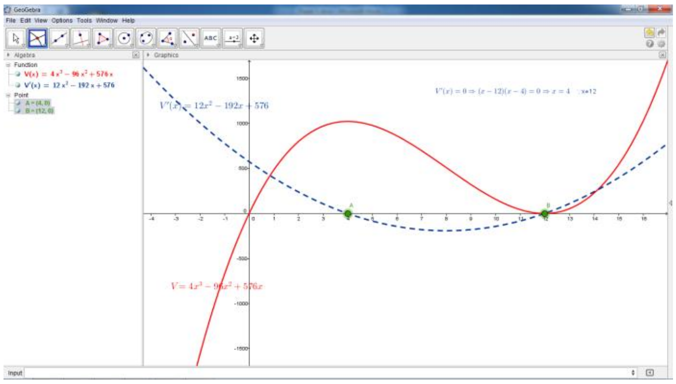


Figure 3(e)

e) Draw perpendicular through A by selecting perpendicular line in the tool bar. Click A and then click x-axis to get 3(f).

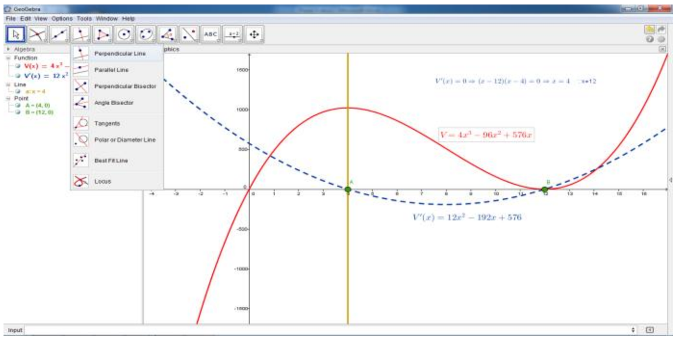


Figure 3(f)

f) In the next figure 3(f) we find the intersection of $V(x)$ with the perpendicular by selecting intersection button and then click $V(x)$ curve and x = 4 line. We get the point C (4, 1024). Which give the required height of the Box and the greatest volume of the Box.

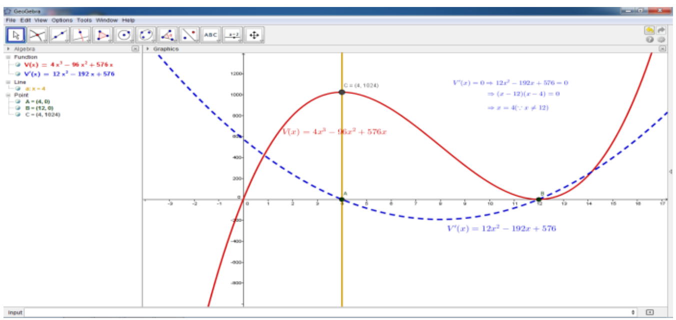


Figure 3(g)

Problem 4: A small manufacturer has employed 5 skilled men and 10 semiskilled men and makes an article in two qualities, a deluxe model and an ordinary model. The making of a deluxe model requires 2 hours work by a skilled man and 2 hours work by a semiskilled man. The ordinary model requires 1 hour by a skilled man and 3 hours by a semiskilled man. By union rules, no man can work more than 8 hours per day. The manufacturer gains Rs 15 on the deluxe model and Rs 10 on the ordinary model. How many of each type should be made in order to maximize his total daily profit? Also find the maximum daily profit.

Solution: Let us assume that x deluxe models and y ordinary models of articles are made each day.

Labors	Time needed (in hrs) Deluxe Models	Time needed (in hrs) Ordinary Models	Total Time (in hrs)
Skilled	2	1	8
Semiskilled	2	3	8

x = number of deluxe model produced per day

y = number of ordinary model produced per day

Then, 2x + y = 8x5
and 2x + 3y = 8x10

Then the objective function for the problem is

Maximize Z = 15x + 10y

Subject to the constrains

$$x \geq 0, y \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$2x + y \leq 40 \ldots\ldots\ldots\ldots\ldots (2)$$

$$2x + 3y \leq 80 \ldots\ldots\ldots\ldots\ldots (3)$$

Now, our aim is to solve the given maximum problem using GeoGebra. The step of drawing to find the solution of the problem is as follows:

1) The value $x \geq 0, y \geq 0$ means that we need only first quadrant in the graph sheet.

2) We draw the first constrain by taking $2x + y = 40$ and then use polygon tool to mark the corner of the polygon.

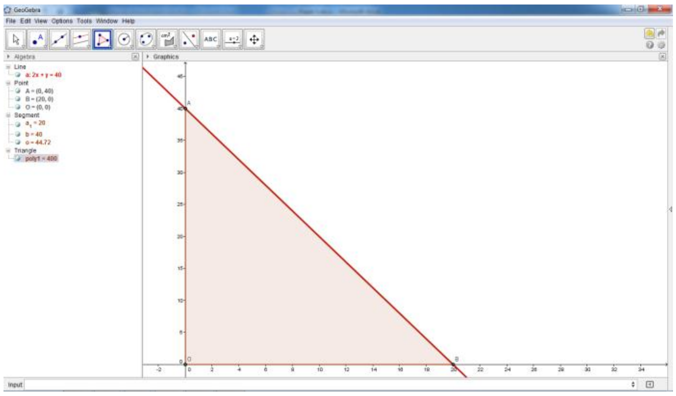


Figure 4(a)

3) We draw the second constrain by taking $2x + 3y = 80$ and then use polygon tool to mark the corner of the second polygon. Then the point O (0, 0), C (0, 26.67), G (10, 20) and B (20, 0) are the feasible solution.

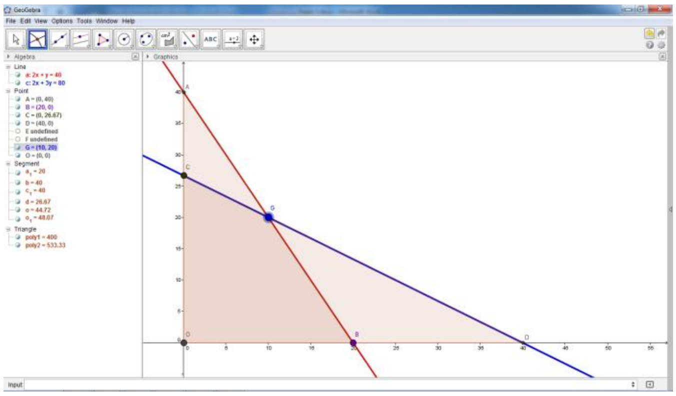


Figure 4(b)

4) We check the corner points to find the required solution. Use ZC= 15*x(C) +10*y(C) and so on we get. We find that Z will be maximum at x = 10 and y = 20. Hence, the manufacturer should produce 10 deluxe models and 20 ordinary models per day to get a maximum profit of Rs 350.

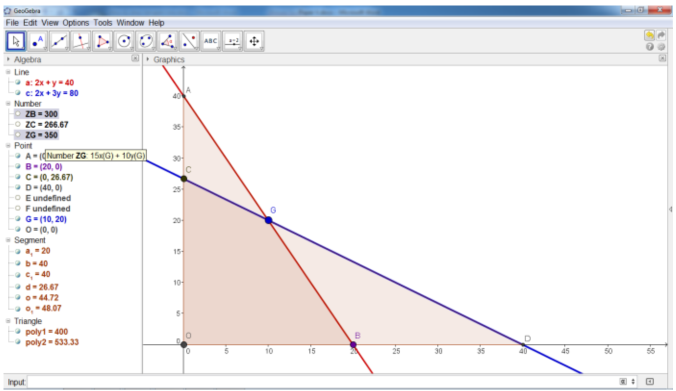


Figure 4(c)

Practical session

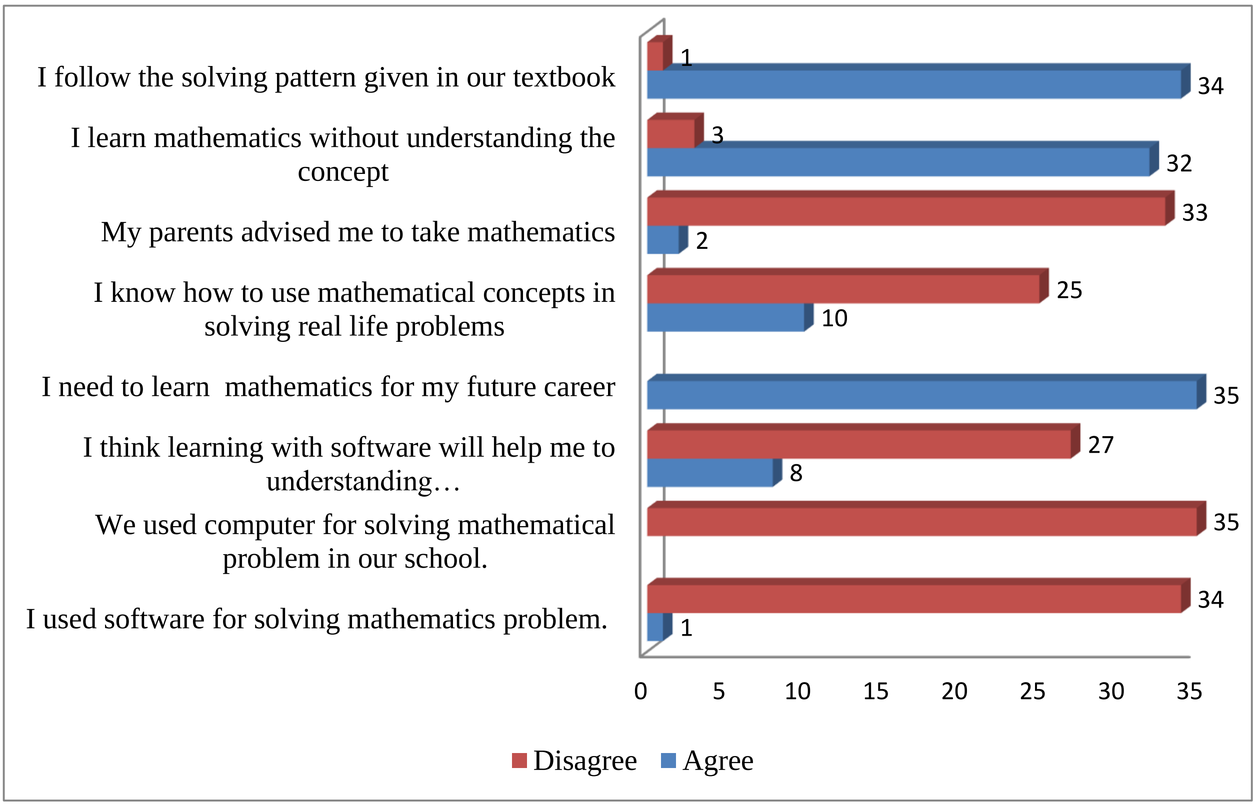
The second part of the programme was practical session in the computer Laboratory. Presentation was made demonstrating how to operate the GeoGebra software. After indicating the notation used in the display windows students are given similar problem used in the theory class to work on their own. It was observed that all the participants were happy about the programme. Their basic knowledge about the subject increased. They want to attend more such training programmes.

Picture 2: Students performing in practical class

Analysis of student's Response

Pre-questionnaire containing eight items was set up to find out the students' attitude towards Mathematics before attending the programme and their level of understanding of Mathematics. From the table below we find that students at Synod Higher Secondary School never used software based Methods in the class. Majority of students thought that learning through software may not help them understand the Mathematical concepts

Table 1. Pre-questionnaire report.



On the last part of the programme, students were asked to fill up post-questionnaire. The post questionnaire was designed to find out the impact of the training programme. There are eight items in the questionnaire which focus mainly on whether the use of software (GeoGebra) for learning mathematics had influenced students' attitude or not. The table below highlights students' response about GeoGebra software for learning mathematics. The following points were assigned:

Strongly Agree – 4, Agree – 3, Disagree a little – 2 and Disagree completely – 1.

Students' level of satisfaction is categorized as High, Medium and Low as follows:

If 3<Mean < 4, High satisfaction

If 2<Mean < 3, Medium satisfaction

If 1<Mean<2, Low satisfaction

From the table below one can see that students gave the highest score on 8th item which has mean 3.54 and Standard deviation 0.657. The lowest response was given on 4th item which has mean 2.97 and standard deviation 0.453 which is almost high response in the categorized measurement. On all items except on 4th item their mean response are higher than three. This proves that the majority of students attending the workshop gain not only knowledge but also acquired better concepts of what they had learnt in the classroom.

Table 2. Post-questionnaire report

Sl. No	Items	Number	Means	SD
1	The training programme helped me to understand the importance of software for learning mathematics	35	3.11	0.404
2	I feel that in this training programme, ICT helped me to visualize and explore the concepts of what we had learnt in classroom	35	3.17	0.514
3	I feel that ICT should be used to solve problems which cannot be done by hand	35	3.31	0.471
4	I feel a lot more confident about learning mathematics now than I did before	35	2.97	0.453
5	Learning through ICT must be included in the curricula	35	3.17	0.568
6	The training programme helped me to understand the application of mathematics in the real world problems	35	3.11	0.631
7	ICT gave a better explanation of calculus concepts	35	3.23	0.598
8	I want to attend such kind of training programmes again in future	35	3.54	0.657

CONCLUSION

In this paper, we introduced GeoGebra to a group of 35 students of senior secondary school level. The main objective of the training programme was to motivate and generate more interest among the students. After analysis of the pre and post questionnaires, it was observed that students were excited about the methodology we introduced in the training programmed and they wanted to attend more such training in their future. We were able to clarify many mathematical concepts that they did not know earlier. We conclude that teachers should try to motivate the students at the beginning and use Information and Communication Technology based methods as far as possible.

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