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Shortest Path Problems Using Complete Partitions

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Abstract

The minimum spanning tree of a graph G consists of all the vertices. Constructing minimum spanning tree of a weighted graph G consists of selected edges. The procedure to find a minimum spanning tree may be given by two familiar algorithms namely Prim's algorithm and Kruskal's algorithm. Here an attempt has been made to find the minimum spanning tree of a weighted graph using partitioning of its weights. Here depending on the minimum weight two procedures have been adopted.

Keywords: Partitions, Complete partitions, Kruskal's algorithm.

Classification MSC: Primary 11A67, Secondary 05C05

INTRODUCTION

The theory of partitions [1], [2], [3], [4], [5], [6] is one of very few branches of mathematics that can be appreciated by anyone who is endowed with little more than a lively interest in the subject. In the theory of numbers [7], [8], [9] or combinatorial problems from all sources wherever discrete objects are counted or classifies its applications are found. The theory of partitions has an interesting history. Certain special problems in partitions certainly date back to the middle ages. Euler indeed laid the foundations of the theory of partitions. Many of the other great mathematicians – Cayley, Gauss, Hardy, Jacobi, Lagrange, Legendre, Littlewood, Rademacher, Ramanujan, Schur and Sylvester have contributed to the development of number theory. The representation of positive integers by sums of other positive integers is known as the fundamental additive decomposition process. For studying partitions, the graphical representation is an effective elementary device. Representation of a positive integer as a sum two or more squares is also a partition, where each part is a square or a square number. There are many kinds of partitions depending on the parts represented. The partition of a positive integer n can be defined as : A finite non – decreasing sequence $\lambda = (\lambda_1, \lambda_2, ...\lambda_k)$ such that $\sum_{i=1}^{k} \lambda_i = n$ and $\lambda_i > 0$ for all $i = 1, 2, ...k$. The λ_i are called the parts of the partition and k is called the length of the partition.

Number theory, an interesting branch of mathematics that deals with integers and their properties plays an important role in discrete mathematics. Graphs [10], [11], [12], [13], [14] are discrete structures consisting of vertices and edges that connect these vertices. Depending on the type and number of edges that can connect a pair of vertices, there are many kinds of different graphs. The graph models can be used to represent almost every problem involving discrete arrangement of objects, where we are not concerned with their internal properties but with their inter – relationship. Eventhough Graph theory is an old subject, one of the reasons for the recent interest in it is its applicability in many diverse fields such as computer science, physical sciences, electrical and communication engineering and economics. A subgraph of a graph G which consists of all vertices is called a spanning subgraph. A graph in which each edge $'e'$ is assigned a non – negative real number, $w(e)$ is called a weighted graph. $w(e)$, called the weight of the edge $'e'$ may represent distance, time, cost etc., in some units.

A shortest path between two vertices in a weighted graph is a path of least weight. In an unweighted graph, a shortest path means one with the least number of edges. If the subgraph T of a connected graph G is a tree containing all the vertices of G, then T is called a spanning tree of G. If G is a connected weighted graph, the spanning tree of G with the smallest total weight is called the minimum spanning tree of G. There exists two popular algorithms for constructing minimum spanning trees. The weight of a minimum spanning tree is unique, whereas different minimum spanning trees are possible, as two or more edges can have the same weight. In Prim's algorithm edges of minimum weight that are incident on a vertex already in the spanning tree and not forming a circuit are selected, whereas in Kruskal's algorithm edges of minimum weight that are not necessarily incident on a vertex already in the spanning tree and not forming a circuit are selected.

First we give the well known definition of complete partition.

Definition :

A partition of a positive interger n is a finite non- decreasing sequence $\lambda = (\lambda_1, \lambda_2, ...\lambda_k)$ such that $\sum_{i=1}^{k} \lambda_i = n$ and $\lambda_i > 0$ for all $i = 1, 2, ...k$. The λ_i are called the parts of the partition and k is called the length of the partition. We sometimes write $\lambda = (1^{m_1} 2^{m_2} ....),$ which means there are exactly m_i parts equal to i in the partition λ. For example, there are five partitions of $4:(1^4)$, $(1^2\ 2), (2^2), (13)$ and (4). A complete partition of an integer n is a partition $\lambda = (\lambda_1, \lambda_2, ...\lambda_k)$ of n, with $\lambda_1 = 1$, such that each integer $i, 1 \le i \le n$, can be represented as a sum of elements of $\lambda_1, \lambda_2, ...\lambda_k$. In other words, each i can be expressed as $\sum_{j=1}^{k} \alpha_j \lambda_j$, where α_j is either 0 or 1.

We now provide an algorithm to find the shortest spanning tree of a graph G using complete partition of the number of vertices of G.

PARTITIONS ALGORITHM

Step 1 :- For a given graph G the edges are arranged in the order of increasing weights.

Step 2 :- Choose an edge with minimum weight to be an Edge of the required spanning tree.

Step 3 :- For an edge with weight of $w(e)$, $w(e)$ greater than 1 find the complete partition $C(l, k)$ where l and k are given by $1 \le k \le \left\lfloor \frac{n+1}{2} \right\rfloor$ and $\lceil \log_2(n+1) \rceil \le l \le n.$

Step 4 :- Divide the weights of each edge by l.

Step 5 :- Edges with minimum ratios that do not form a circuit with the already selected edges are successively added.

Step 6 :- The procedure is stopped after $(n-1)$ edges have been selected.

Step 7 :- To find the minimum weight:

Case (i) : If $w(e) = 1$ for any edge find the sum of ratios of the weights of the spanning tree multiply by l

and add 1.

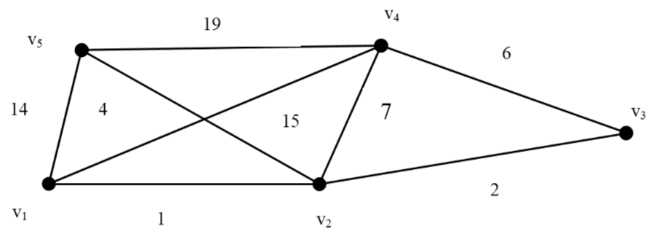
Case(ii) : The same procedure can be adopted even if $w(e_i) = 1$ for more than one i.

Case (iii) : If $w(e) > 1$ for all edges simply multiply by l .

Step 8 :- The rounded of integer is taken as weight of the shortest spanning tree.

ILLUSTRATION 1:

Graph with one of its edges is of unit weight



(i). Spanning tree by Kruskal's algorithm :

Edge	Weight
Let $(v_1, v_2) = e_1$	1
$(v_2, v_3) = e_2$	2
$(v_1, v_4) = e_7$	4
$(v_3, v_4) = e_3$	6
$(v_2, v_4) = e_8$	7
$(v_1, v_5) = e_5$	14
$(v_2, v_5) = e_6$	15
$(v_4, v_5) = e_4$	19

Since there are 5 vertices in the graph, we should stop the procedure for finding the edges of the minimum spanning tree, when 4 edges have been found out.

First we choose an edge e_1 whose minimum weight is 1. Next we choose e_2 whose minimum weight is 2. Now, we continue from node v_1 choose the edge e_7 which does not form a circuit whose minimum weight is 4. Continuing this process by Kruskal's we get a minimum spanning tree having minimum weight. Finally we choose the edge e_6.

Now the required minimum spanning tree consists of the 4 edges e_1, e_2, e_7 and e_6.

The total length of the minimum spanning tree

= 1 + 2 + 4 + 14 = 21.

(ii). Spanning tree by Partitions Algorithm:

Now, we apply complete partitions. The minimum weight greater than unity is 2 for the edge e_2 (i.e). $w(e_2) = 2.$

The partitions of 2 are 2 = 2

= 1 + 1.

Complete partitions of 2 is (1^2).

(i.e). 2 = 1 + 1 = (1^2)

Here the length of the complete partition is $l = 2$ with the largest part $k = 1$.

∴Divide the weights by 2. Now the weights of the corresponging edges are tabulated to get the minimum spanning tree.

Edge	Weight
$(v_1, v_2) = e_1$	1
$(v_2, v_3) = e_2$	1
$(v_1, v_4) = e_7$	2
$(v_3, v_4) = e_3$	3
$(v_2, v_4) = e_8$	3.5
$(v_1, v_5) = e_5$	7
$(v_2, v_5) = e_6$	7.5
$(v_4, v_5) = e_4$	9.5

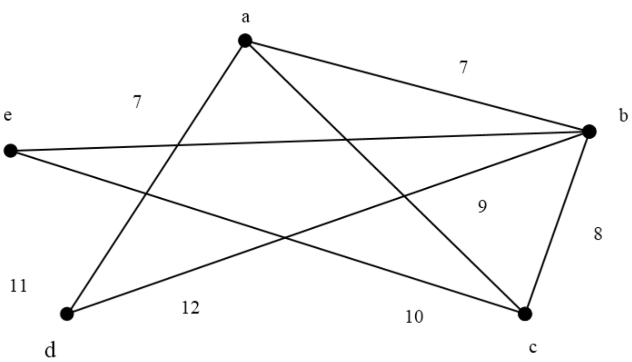
Now we apply the algorithm to find the minimum spanning tree. The minimum spanning tree consists of the 4 edges e_1, e_2, e_7 and e_6.

It is observed that the spanning tree is the same. By this algorithm the total length of the minimum spanning tree

$= (1 + 2 + 7)l + 1 = 10 \times l + 1 = 20 + 1 = 21.$

ILLUSTRATION 2:

Graph whose edges are of weight greater than one



(i). Spanning tree by Kruskal's algorithm :

Edge	Weight
Let (a, b) = e_1	7
(e, b) = e_4	7
(b, c) = e_2	8
(c, a) = e_7	9
(c, e) = e_3	10
(a, d) = e_6	11
(d, b) = e_5	12

First we choose an edge e_1 whose minimum weight is 7. Next we choose e_4 which does not form a circuit whose minimum weight is 7. Now we continue from node b, choose the edge e_2 whose weight is 8. Continuing this process by Kruskal's we get a minimum spanning tree having minimum weight. Finally we choose the edge e_6 whose weight is 11.

The required minimum spanning tree consists of the 4 edges e_1, e_4, e_2 and e_6.

The total length of the minimum spanning tree

$$= 7 + 7 + 8 + 11 = 33.$$

(ii) Spanning tree by Partitions Algorithm:

Now we apply complete partitions. In this problem the minimum weight is 7. Before applying complete partitions first we have to find out the partitions of 7.

The complete partitions of 7 are (1^7), $(1^5\ 2)$, $(1^3\ 2^2)$, $(1^4\ 3)$, $(1^2\ 2\ 3)$.

Now we choose the complete partition of 7 as $(1^2 23)$ with $l = 4$ and $k = 3$ where l is the length and k is the largest part of the minimum weight. Divide each of the weights by 4. The weights of the corresponding edges are tabulated to get the minimum spanning tree.

Edge	Weight
Let (a, b) = e_1	1.75
(e, b) = e_4	1.75
(b, c) = e_2	2
(c, a) = e_7	2.25
(c, e) = e_3	2.5
(a, d) = e_6	2.75
(d, b) = e_5	3

Now we apply the algorithm to find the minimum spanning tree. The minimum spanning tree consists of the 4 edges e_1, e_4, e_2 and e_6.

It is observed that the spanning tree is the same.

∴ By this algorithm the total length of the minimum spanning tree $= 1.75 + 1.75 + 2 + 2.75 \times l$

$= 8.25 \times l = 33.$

CONCLUSION

The algorithm appears to be the same as Kruskal's algorithm, but on dividing by the minimum weight the calculation to find the total weight is made easier. But to get the minimum weight we adopt a slightly different procedure. This algorithm may be extended to any other problem which can be represented as network.

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