International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 12 (2018) pp. 10389-10393

Operations in Fuzzy Labeling Graph through Matching and Complete Matching

S. Yahya Mohamad[1] and S.Suganthi[2]

[1] *PG & Research Department of Mathematics, Government Arts College, Trichy– 620 022, Tamilnadu, India.*

[2]*Department of Mathematics, Bharathiyar Arts & Science College for Women, Deviyakurichi, Tamilnadu, India.*

Abstract

In this paper, the operations in fuzzy labeling graph through matching and complete matching are introduced and also some properties based on the operations in matching and complete matching are studied.

Keywords: Complete matching, union, intersection, symmetric difference.

2010 Mathematics Subject Classification: 05C72, 54E50, 03F55

INTRODUCTION

Many real world systems can be modeled using graphs. Graph represents the connections between the entities in these systems. The foundation for graph theory was laid in 1735 by Euler when he solved the Konigsberg bridge problem. A mathematical frame work to describe the phenomena of uncertainity in real life situation has been suggested by Zadeh in 1965. Research on the theory of fuzzy sets has been witnessing an exponential growth; both within mathematics and in its applications. Fuzzy graphs are generalization of graphs. Fuzzy graphs are countered in fuzzy set theory. A fuzzy set was defined by L. A. Zadeh in 1965.Every element in the universal set is assigned a grade of membership ,a value in [0,1].The elements in the universal set along with their grades of membership form a fuzzy set. In 1965 Fuzzy relations on a set was first defined by Zadeh. There are several operations on G_1 and G_2. Operations on (crisp) graphs such as union, intersection, symmetric difference were extended to fuzzy graphs through matching. A matching is a set of edges which are non-adjacent. Here our results consider for simple connected fuzzy labeling graph.

PRELIMINARIES

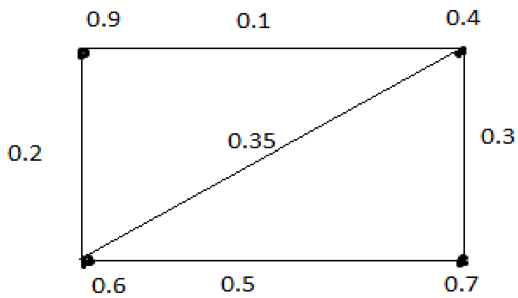
Definition 2.1

Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of U × V. A **fuzzy graph** G = (α, β) is a pair of functions α : V → [0, 1] and β: V ×V → [0,1] where for all u,v ∈V, we have β (u ,v) ≤ min {α(u) , α (v)}.

Definition 2.2

A graph G = (α, β) is said to be a ***fuzzy labeling graph*** if α: V → [0, 1] and β: V ×V → [0, 1] is a bijective such that the membership value of edges and vertices are distinct and β (u, v) < min {α (u), α (v)} for all u, v ∈ V.

Example: 2.3



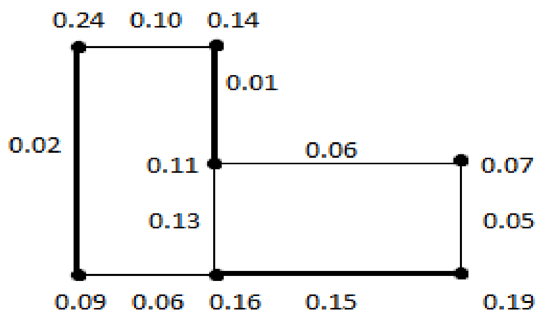
Definition: 2.4

An edge is said to be a **link** if it is neither a loop nor multiple edges.

Definition: 2.5

A subset M of $\beta\ (v_i, v_{i+1}),\ 1 \le i \le\ n$ is called a ***matching*** in fuzzy graph if its elements are links and no two are adjacent in G. The two ends of an edge in M are said to be matched under M.

Example: 2.6



Definition 2.7

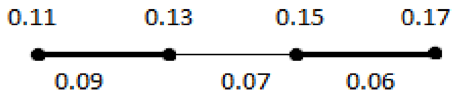
The two ends of an edge in a matching M are said to be matched under M.

A matching M saturates a vertex v then v is said to be *M-saturated*.

Definition 2.8

If every vertex of fuzzy graph is M-saturated then the matching is said to be complete or perfect. It is denoted by C_M.

Example: 2.9



Definition 2.10

Let G_1 and G_2 be two graphs then the symmetric difference between two graphs is defined by $(G_1 \; \Delta \; G_2) = (G_1 - G_2) \cup (G_2 - G_1)$.

Definition 2.11

The difference of two sets $(\mu_A - \mu_B)$ is defined to be the set of all elements which are in μ_A *but not in* μ_B.

Definition 2.12

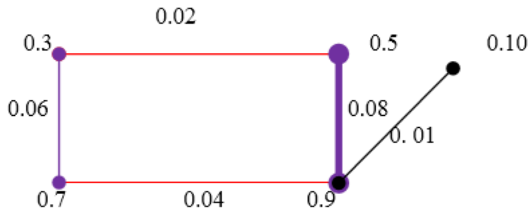
Let μ_A and μ_B be two fuzzy sets. The symmetric difference between two fuzzy sets is defined by $(\mu_A \Delta \mu_B) = (\mu_A - \mu_B) \cup (\mu_B - \mu_A)$.

MAIN RESULTS

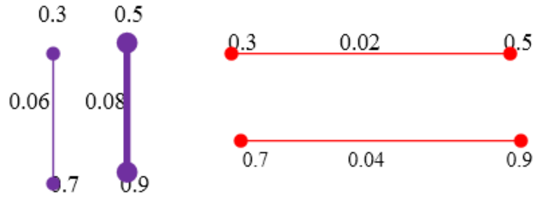
Definition: 3.1

Let M_1 ad M_2 be two matchings in a fuzzy labeling graph with vertex set $\propto_1$ (v) ,$\propto_2$ (v) and edge set β_1 and β_2 . *Then* **Union of M_1 and M_2** $(M_1 \cup M_2)$ consists of a vertex set($\alpha_1 \cup \alpha_2$)which is the subset of $\propto$(v) and edge set $(\beta_1 \cup \beta_2)$ which is the subset of β .

Example: 3.2

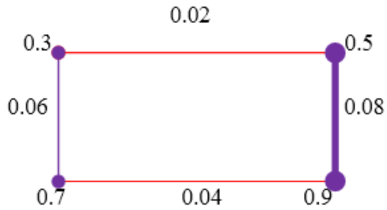


(FLG)



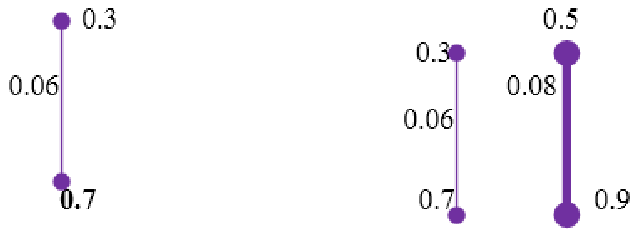
M_1 M_2

Here $M_1 = \{0.06, 0.08\} \subset \beta 1$ and $M_2 = \{0.02, 0.04\} \subset \beta 2$ have no edges in common



$M_1 \cup M_2$

Here $M_1 = \{0.06\} \subset \beta 1$ and $M_2 = \{0.06, 0.08\} \subset \beta 2$ have edges in common



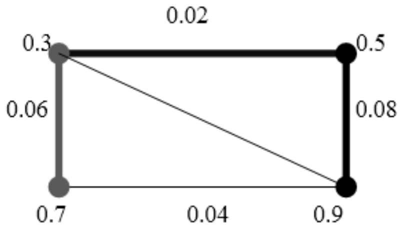
M_1 **M_2**

$M_1 \cup M_2$

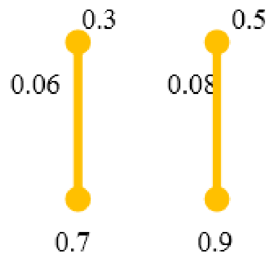
Definition: 3.3

Let M_1 ad M_2 be two matchings in a fuzzy labeling graph with vertex set $\propto_1$ (v) ,$\propto_2$ (v) and edge set β_1 and β_2 . Then **intersection of M_1 and M_2** ($M_1 \cap M_2$) consists of a vertex set($\alpha_1 \cap \alpha_2$) which is the subset of $\propto$(v) and edge set ($\beta_1 \cap \beta_2$) which is the subset of β .

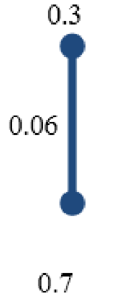
Example: 3.4



Here M_1= {0.06}$\subset \beta 1$ and M_2 = {0.06, 0.08}$\subset \beta 2$



M₁ **M2**

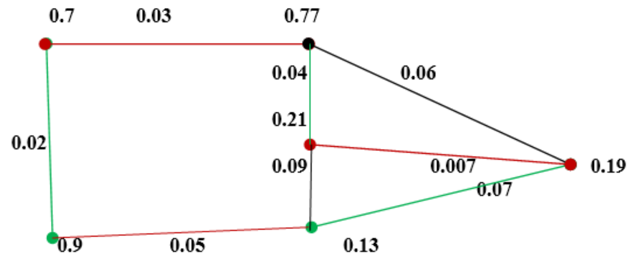


$M_1 \cap M_2$

Definition: 3.5

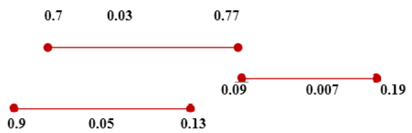
Let M_1 ad M_2 be two matchings in a fuzzy labeling graph with vertex set $\propto_1$ (v) ,$\propto_2$ (v) and edge set β_1 and β_2 . Then **symmetric difference between M_1 and M_2** ($M_1 \Delta M_2$) is defined by (M_1 –M_2)U (M_2 –M_1) .

Example: 3.6

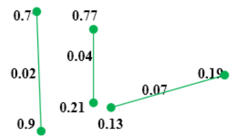


(FLG)

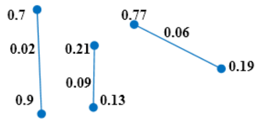
Here M1 = {0.03, 0.05, 0.007} $\subset \beta 1$, M2 = {0.02, 0.04, 0.07}$\subset \beta 2$



M_1

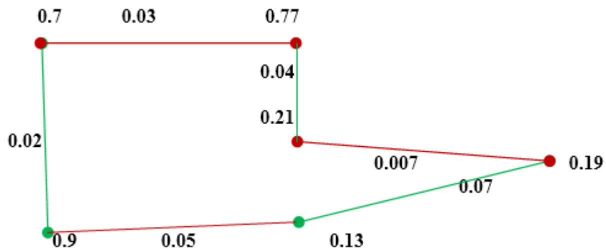


M_2



M_3

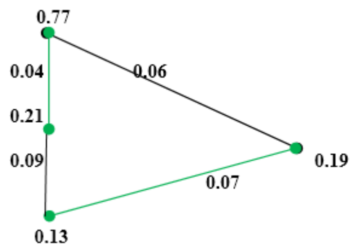
Now, $M_1 \Delta M_2$ = {0.03, 0.05, 0.007}∪ {0.02, 0.04, 0.07}



$M_1 \Delta M_2$

Now M_2 = {0.02, 0.04, 0.07}} $\subset \beta 2$ and M3 = {0.02, 0.09, 0.06} $\subset \beta 3$ then

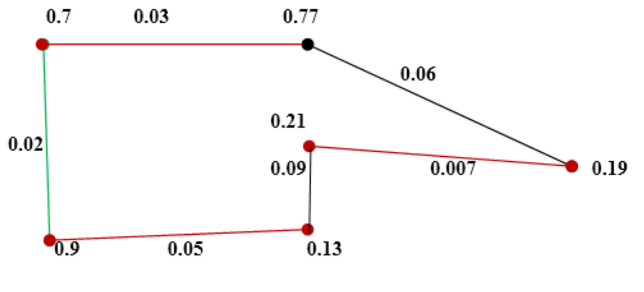
$M_2 \Delta M_3$ = {0.04, 0.07} ∪ {0.09, 0.06}



$M_2 \Delta M_3$

Here M_1 = {0.03, 0.05, 0.007} ⊂ $\beta 1$ and M_3 = {0.02, 0.09, 0.06} ⊂ $\beta 3$ then

$M_1 \Delta M_3$ = {0.03, 0.05, 0.007} ∪{0.02, 0.09, 0.06}



$M_1 \Delta M_3$

Theorem: 3.7

Let M_1 and M_2 be two matching in a fuzzy labeling graph. Then union of M_1 and M_2 is also contains a matching.

Proof:

Let M_1 and M_2 be two matching in a fuzzy labeling graph (FLG).Then M_1 and M_2 have the edges either distinct or repeated edges.

Suppose M_1 and M_2 have distinct edges. Then union of M1 and M_2 form a connected graph which contains a cycle with even number of vertices.

Otherwise, M_1 and M_2 have the edges in common. Then union of M_1 and M_2 form a disconnected graph with exactly two components.

One of these two components is a common edge in both M_1 and M_2 and another one is a path .This path contains the edges alternatively in M_1 and M_2.

But in two cases, there exists a matching. Hence union of M_1 and M_2 is also contains a matching.

Note:

The example 3.2 proved above result.

Theorem: 3.8

Let M_1 and M_2 be two perfect matching in a fuzzy labeling graph. Then union of M_1 and M_2 is also contains a perfect matching.

Proof:

Let M_1 and M_2 be two perfect matching in a fuzzy labeling graph(FLG).Then M_1 and M_2 have the edges either distinct or repeated edges.

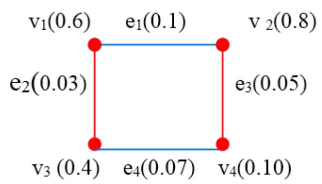
Suppose M_1 and M_2 have distinct edges. Then union of M1 and M_2 form a connected graph which contains a cycle with even number of vertices.

Otherwise, M_1 and M_2 have the edges in common. Then union of M_1 and M_2 form a disconnected graph with exactly two components.

One of these two components is a common edge in both M_1 and M_2 and another one is a even cycle.

In both cases, the number of vertices is equal to the number of vertices in a fuzzy labeling graph. Hence union of M_1 and M_2 is also contains a perfect matching.

Example:



Theorem: 3.9

Let M_1 and M_2 be two matching in a fuzzy labeling graph. Then intersection of M_1 and M_2 is also contains a matching.

Proof:

Let M_1 and M_2 be two matching in a fuzzy labeling graph (FLG).Then M_1 and M_2 have the edges either distinct or repeated edges.

But intersection applied only for the repeated edges. So we take the matchings M_1 and M_2 have the edges in common.

Then intersection of M_1 and M_2 form a complete graph with two vertices or disconnected graph. One of these components is K_2 and components are combination of K_2 .

There is also exists matching. Hence intersection of M_1 and M_2 is also contains a matching.

Note:

The example 3.4 proved above result.

Theorem: 3.10

Let M_1 and M_2 be two perfect matching in a fuzzy labeling graph. Then intersection of M_1 and M_2 is also contains a perfect matching.

Proof:

Let M_1 and M_2 be two perfect matching in a fuzzy labeling graph(FLG).Then M_1 and M_2 have the edges either distinct or repeated edges.

But intersection applied only for the repeated edges. So we take the perfect matchings M_1 and M_2 have the edges in common.

Here two cases will arise. (i) M_1 and M_2 have only one edge in common. (ii)) M_1 and M_2 have more than one edges in common.

In case (i) the intersection is a K_2 which is the edge common in both M_1 and M_2 and in case (ii) the intersection is a disconnected graph with each components be K_2.

Here the number of vertices is equal to twice the number of edges common in both perfect matchings in a fuzzy labeling graph.

Hence intersection of M_1 and M_2 is also contains a perfect matching.

Theorem: 3.11

Let M_1 and M_2 be two perfect matching in a fuzzy labeling graph. Then symmetric difference between M1 and M_2 ($M_1 \Delta M_2$) is also contains a perfect matching.

Proof:

Let M_1 and M_2 be two perfect matching in a fuzzy labeling graph(FLG).Then M_1 and M_2 have the edges either distinct or repeated edges.

Suppose M_1 and M_2 have distinct edges. Then symmetric difference between M1 and M_2 ($M_1 \Delta M_2$) form a connected graph which contains a cycle with even number of vertices.

Otherwise, M_1 and M_2 have the edges in common. . Then symmetric difference between M1 and M_2 ($M_1 \Delta M_2$) form a connected graph which contains a cycle with even number of vertices.

Hence in both cases, the number of vertices is equal to the number of vertices in a fuzzy labeling graph. Hence symmetric difference between M1 and M_2 ($M_1 \Delta M_2$) is also contains a perfect matching.

Note

The example 3.6 proved above result.

CONCLUSION

In this paper, the operations on matching like union, intersection and symmetric difference are defined and found. Some theorems related to the concept are discussed and verify the results through examples. In future we extend this concept to some special graph like fuzzy regular graph, fuzzy strong regular graph, fuzzy bipartite graph etc.

REFERENCES

[1] Frank Harary, Graph Theory, Indian Student Edition, Narosa/Addison Wesley, 1988.

[2] Frank Harary and Gordon W. Wilcox, Boolean operations on graphs, Math. Scand. 20 (1967), 41-51.

[3] A. Nagoorgani and S. R. Latha, On irregular fuzzy graphs, Appl. Math. Sci. 6(11) (2012), 517-523.

[4] A. Nagoorgani and S. R. Latha, Isomorphism on irregular fuzzy graphs, Inter. J. Math. Sci. Engg. Appls. (IJMSEA) 6(III) (2012), 193-208.

[5] A. Nagoorgani and J. Malarvizhi, Isomorphism on fuzzy graphs, Inter. J. Comput. Math. Sci. 2 (2008), 200-206.

[6] Narsingh Deo, Graph Theory with applications to Engineering and Computer Science.

[7] K .Ranganathan, R.Balakrishnan, A Text Book of Graph Theory, Springer.

[8] A. Rosenfeld, Fuzzy graphs, Fuzzy Sets and their Applications to Cognitive and Decision Process, L. A. Zadeh, K. S. Fu, K. Tanaka and M. Shimura, eds., Academic Press, New York, 1975, pp. 75-95.

[9] M. S. Sunitha and A. Vijayakumar, Complement of fuzzy graph, Indian J. Pure Appl. Math. 33(9) (2002), 1451-1464.

[10] L. A. Zadeh, Fuzzy sets Information and Control 8 (1965), 338-353.