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Reliability and Availability Analysis for a Three-Unit Gas Turbine Power Generating System with Seasonal Effect and FCFS Repair Pattern

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Abstract

The present paper develops a reliability model for a gas turbine power generating system which comprises two gas turbines and one steam turbine. Two different seasons, i.e., summer and the winter have been taken into consideration while developing the model. Initially, both the gas turbines and the steam turbine are operative and this working is called working in full capacity. However, the situations of working of 'two gas turbines without steam turbine,' 'one gas turbine along with steam turbine', and 'one gas turbine without steam turbine' also occur and the same have been considered. The system is analysed by making use of semi-Markov processes and regenerative point technique. Reliability and Mean time to system failure (MTSF) is obtained. The availability analysis is done under various capacities and different seasons. Graphical study is also done for particular cases.

Keywords: Two gas turbines, One steam turbine, Different seasons, FCFS repair pattern, Reliability, Availabilities.

INTRODUCTION

Literature of reliability contains a good number of research papers on one or two or more units systems [1-3, 5-8]. Parashar and Taneja [9] and Taneja et al. [14] discussed systems with two dissimilar units of different nature. However, situations may be there where the two units may be dissimilar but the nature of the work done by them is same. Such a situation was discussed by Singh and Taneja [12-13] for a gas turbine power plant. However, they did not consider the parameter 'Temperature' which also affects the working and efficiency of a gas turbine system. One such situation was discussed by Rajesh et al. [10] where effects of temperature on production of a system comprising one gas turbine and one steam turbine have been taken into account. Such a system necessarily goes to down mode on failure of gas turbine irrespective of operability of steam turbine, as steam turbine cannot work without working of gas turbine. However, this problem can be overcomed to some extent if number of gas turbine is increased, i.e., redundancy is introduced.

Keeping the above observations in view, the present paper carries out the reliability and availability analysis of a gas turbine system consisting of two gas turbines and one steam turbine considering the working of the system in two different seasons, i.e., summer and the winter. Initially, both the gas turbines and the steam turbine are operative and this period of working is called working in full capacity. On failure of the steam turbine, both or one of the gas turbines may be kept in upstate depending upon the requirement and readiness of paying higher amount by the buyer of the power, otherwise the system is put to down state. When the system works with failure of at least one of the turbines, its working is termed as working under reduced capacity. On failure of both gas turbines, the system goes to down state as steam turbine cannot work without working of a gas turbine. System is analysed by making use of semi-Markov processes and regenerative point technique. Expressions for reliability, MTSF and availability under various capacities have been obtained for summer as well as winter. Computational work along with graphical study has been done using R by making use of the contributions made by R Core Team (2017) [4, 11]. The information gathered on maintenance and production of a power station comprising two gas and one steam turbines has been used for estimating various parameters involved in the study.

Main assumptions of the model are:

i. Failure times are assumed to follow exponential distribution whereas the repair times have arbitrary distributions.

ii. After every repair, unit becomes as good as new.

iii. All the random variables are independent.

iv. System fails completely on failure of all the three units.

v. First come first served (FCFS) pattern is applicable among the repair for gas and steam turbines. However, there is no compulsion of FCFS pattern as far as repair of the two gas turbines are concerned as we are assuming them identical.

Other assumptions are as usual.

NOTATION, TRANSITION PROBABILITIES AND MEAN SOJOURM TIMES

Notation

Symbol	Meaning
O_{gt}^{wi} :	Gas turbine operative in winter
O_{st}^{wi} :	Steam turbine operative in winter
O_{gt}^{su} :	Gas turbine operative in summer
O_{st}^{su} :	Steam turbine operative in summer
U_{rgt}^{wi} :	Gas turbine under repair in winter
U_{rst}^{wi} :	Steam turbine under repair in winter
U_{rgt}^{su} :	Gas turbine under repair in summer
U_{rst}^{su} :	Steam turbine under repair in summer
d_{gt}^{wi} :	Gas turbine put to down mode in winter
d_{st}^{wi} :	Steam turbine put to down mode in winter
d_{gt}^{su} :	Gas turbine put to down mode in summer
d_{st}^{su} :	Steam turbine put to down mode in summer
W_{rgt}^{wi} :	Gas turbine waiting for repair in winter
W_{rgt}^{su} :	Gas turbine waiting for repair in summer
W_{rst}^{wi} :	Steam turbine waiting for repair in winter
W_{rst}^{su} :	Steam turbine waiting for repair in summer
U_{Rst}^{wi} :	Repair of steam turbine continuing from previous state in winter
U_{Rst}^{su} :	Repair of steam turbine continuing from previous state in summer
$h_1(t)$:	Probability density function (p.d.f.) of time of changing the season from winter to summer
$h_2(t)$:	p.d.f. of time of changing the season from summer to winter
λ_1 :	Failure rate of gas turbine in winter
λ_2 :	Failure rate of gas turbine in summer
α_1 :	Failure rate of steam turbine in winter
α_2 :	Failure rate of steam turbine in summer
$g_1(t)$:	p.d.f. of repair time of gas turbine
$g_2(t)$:	p.d.f. of repair time of steam turbine
p_{11} :	Probability that there is demand for higher rate but this demand is not greater than production of one gas turbine in winter
p_{12} :	Probability that there is demand for higher rate and this demand is greater than production of one gas turbine in winter
$p_1 = p_{11} + p_{12}$:	Probability that there is demand for higher rate and this demand is either greater than or not greater than production of one gas turbine in winter
$q_1 = 1 - p_1$:	Probability that no customer is ready to pay higher amount in winter
p_{21} :	Probability that there is demand for higher rate but this demand is not greater than production of one gas turbine in summer
p_{22} :	Probability that there is demand for higher rate and this demand is greater than production of one gas turbine in summer
$p_2 = p_{21} + p_{22}$:	Probability that there is demand for higher rate and this demand is either greater than or not greater than production of one gas turbine in summer
$q_2 = 1 - p_2$:	Probability that no customer is ready to pay higher amount in winter
$\phi_i(t)$:	Cumulative distribution function (c.d.f.) of the first passage time from regenerative state i to failed state
$AW_i^f(t)$:	Probability that system is up in full capacity during winter at instant t given that it entered state i at t = 0
$AW_i^I(t)$:	Probability that system is up in reduced capacity of type I during winter at instant t given that it entered state i at t = 0
$AW_i^{II}(t)$:	Probability that system is up in reduced capacity of type II during winter at instant t given that it entered state i at t = 0
$AW_i^{III}(t)$:	Probability that system is up in reduced capacity of type III during winter at instant t given that it entered state i at t = 0
$AS_i^f(t)$:	Probability that system is up in full capacity during summer at instant t given that it entered state i at t = 0
$AS_i^I(t)$:	Probability that system is up in reduced capacity of type I during summer at instant t given that it entered state i at t = 0
$AS_i^{II}(t)$:	Probability that system is up in reduced capacity of type II during summer at instant t given that it entered state i at t = 0
$AS_i^{III}(t)$:	Probability that system is up in reduced capacity of type III during summer at instant t given that it entered state i at t = 0
Ⓢ :	Laplace Stieltjes Convolution
©:	Laplace Convolution

Transition Probabilities

Various states of the system with season wise (winter and summer) status are shown in **Table 1**. The epochs of entry into states 0, 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 17, 24 and 25 are regeneration points and thus 0, 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 17, 24 and 25 are regenerative states. States 9, 11, 21 and 23 are failed states. States 0 and 1 are states where the system works in full capacities (both gas turbines and steam turbine are operative) in winter and summer respectively. States 4 and 16 are the states where the system works at reduced capacity when only two gas turbines are operative [Type I capacity, say] in winter and summer respectively. States 5 and 17 are the states where the system works at reduced capacity when one gas turbine and one steam turbine are operative [Type II capacity, say] in winter and summer respectively. States 3, 6, 8 and 12 are the states where the system works at reduced capacity when only one gas turbine is operative [Type III capacity, say] in winter. States 15, 18, 20 and 24 are the states where the system works at reduced capacity when only one gas turbine is operative [Type III capacity, say] in summer. States 10 and 13 are the states where the steam turbine is compulsorily put to down mode on failure of both gas turbines as the former cannot work without the operability of at least one gas turbine in winter. States 22 and 25 are the states where the steam turbine is compulsorily put to down mode on failure of both gas turbines as the former cannot work without the operability of at least one gas turbine in summer. States 2 and 7 are the states the gas turbine(s) is/are put to down mode when the steam turbine is failed and no buyer is ready to pay higher amount so as to make the system to work at reduced capacities in winter. States 14 and 19 are the states the gas turbine(s) is/are put to down mode when the steam turbine is failed and no buyer is ready to pay higher amount so as to make the system to work at reduced capacities in summer.

Table 1. Possible states with status

State No	Status	State No	Status	State No	Status
0	$O_{gt}^{wi}, O_{gt}^{wi}, O_{st}^{wi}$	9	$W_{rgt}^{wi}, W_{rgt}^{wi}, U_{Rst}^{wi}$	18	$W_{rgt}^{su}, O_{gt}^{su}, U_{Rst}^{su}$
1	$O_{gt}^{su}, O_{gt}^{su}, O_{st}^{su}$	10	$U_{Rgt}^{wi}, W_{rgt}^{wi}, d_{st}^{wi}$	19	$U_{Rgt}^{su}, d_{gt}^{su}, U_{rst}^{su}$
2	$d_{gt}^{wi}, d_{gt}^{wi}, U_{rst}^{wi}$	11	$U_{Rgt}^{wi}, W_{rgt}^{wi}, W_{rst}^{wi}$	20	$U_{Rgt}^{su}, O_{gt}^{su}, W_{rst}^{su}$
3	$O_{gt}^{wi}, d_{gt}^{wi}, U_{rst}^{wi}$	12	$O_{gt}^{wi}, w_{rgt}^{wi}, U_{rst}^{wi}$	21	$W_{rgt}^{su}, W_{rgt}^{su}, U_{Rst}^{su}$
4	$O_{gt}^{wi}, O_{gt}^{wi}, U_{rst}^{wi}$	13	$U_{rgt}^{wi}, W_{rgt}^{wi}, d_{st}^{wi}$	22	$U_{Rgt}^{su}, W_{rgt}^{su}, d_{st}^{su}$
5	$U_{rgt}^{wi}, O_{gt}^{wi}, O_{st}^{wi}$	14	$d_{gt}^{su}, d_{gt}^{su}, U_{rst}^{su}$	23	$U_{Rgt}^{su}, W_{rgt}^{su}, W_{rst}^{su}$
6	$W_{rgt}^{wi}, O_{gt}^{wi}, U_{Rst}^{wi}$	15	$O_{gt}^{su}, d_{gt}^{su}, U_{rst}^{su}$	24	$O_{gt}^{su}, W_{rgt}^{su}, U_{rst}^{su}$
7	$U_{Rgt}^{wi}, d_{gt}^{wi}, W_{rst}^{wi}$	16	$O_{gt}^{su}, O_{gt}^{su}, U_{rst}^{su}$	25	$U_{rgt}^{su}, W_{rgt}^{su}, d_{st}^{su}$
8	$U_{Rgt}^{wi}, O_{gt}^{wi}, W_{rst}^{wi}$	17	$U_{rgt}^{su}, O_{gt}^{su}, O_{st}^{su}$		

Let $q_{ij}(t)$ be the probability density function (p.d.f.) of the first passage time from regenerative state i to regenerative state j or failed state j, then we have:

$$q_{01}(t) = e^{-(2\lambda_1+\alpha_1)t}h_1(t) \quad q_{02}(t) = q_1\alpha_1 e^{-(2\lambda_1+\alpha_1)t}\overline{H}_1(t) \quad q_{03}(t) = p_{11}\alpha_1 e^{-(2\lambda_1+\alpha_1)t}\overline{H}_1(t)$$

$$q_{04}(t) = p_{12}\alpha_1 e^{-(2\lambda_1+\alpha_1)t}\overline{H}_1(t) \quad q_{05}(t) = 2\lambda_1 e^{-(2\lambda_1+\alpha_1)t}\overline{H}_1(t) \quad q_{10}(t) = e^{-(2\lambda_2+\alpha_2)t}h_2(t)$$

$$q_{1,14}(t) = q_2\alpha_2 e^{-(2\lambda_2+\alpha_2)t}\overline{H}_2(t) \quad q_{1,15}(t) = p_{21}\alpha_2 e^{-(2\lambda_2+\alpha_2)t}\overline{H}_2(t) \quad q_{1,16}(t) = p_{22}\alpha_2 e^{-(2\lambda_2+\alpha_2)t}\overline{H}_2(t)$$

$$q_{1,17}(t) = 2\lambda_2 e^{-(2\lambda_2+\alpha_2)t}\overline{H}_2(t) \quad q_{20}(t) = g_2(t) \quad q_{30}(t) = g_2(t).e^{-\lambda_1 t}$$

$$q_{35}^{(6)}(t) = \lambda_1 t e^{-\lambda_1 t} g_2(t) \quad q_{39}^{(6)}(t) = \lambda_1^2 t e^{-\lambda_1 t}\overline{G}_2(t)$$

$q_{3,13}^{(6,9)}(t)=\lfloor 1-e^{-\lambda_1 t}-\lambda_1 te^{-\lambda_1 t}\rfloor g_2(t)$ $\quad q_{40}(t)=g_2(t).e^{-2\lambda_1 t}$

$q_{45}^{(6)}(t)=2\lfloor e^{-\lambda_1 t}-e^{-2\lambda_1 t}\rfloor g_2(t)$ $\quad q_{49}^{(6)}(t)=2\lambda_1\left[e^{-\lambda_1 t}-e^{-2\lambda_1 t}\right]\overline{G}_2(t)$ $\quad q_{4,13}^{(6,9)}(t)=\lfloor 1-2e^{-\lambda_1 t}+e^{-2\lambda_1 t}\rfloor g_2(t)$

$q_{50}(t)=g_1(t).e^{-(\alpha_1+\lambda_1)t}$ $\quad q_{52}^{(7)}(t)=\frac{q_1\alpha_1}{\alpha_1+\lambda_1}\left[1-e^{-(\alpha_1+\lambda_1)t}\right]g_1(t)$

$q_{54}^{(8)}(t)=p_1\lfloor e^{-\lambda_1 t}-e^{-(\alpha_1+\lambda_1)t}\rfloor g_1(t)$ $\quad q_{55}^{(10)}(t)=\frac{\lambda_1}{\alpha_1+\lambda_1}\left[1-e^{-(\alpha_1+\lambda_1)t}\right]g_1(t)$

$q_{5,11}^{(8)}(t)=p_1\lambda_1\left[e^{-\lambda_1 t}-e^{-(\alpha_1+\lambda_1)t}\right]\overline{G}_1(t)$ $\quad q_{5,12}^{(8,11)}(t)=\frac{p_1}{\alpha_1+\lambda_1}\left[\alpha_1\left(1-e^{-\lambda_1 t}\right)-\lambda_1\left(e^{-\lambda_1 t}-e^{-(\alpha_1+\lambda_1)t}\right)\right]g_1(t)$

$q_{12,5}(t)=e^{-\lambda_1 t}g_2(t)$ $\quad q_{12,9}(t)=\lambda_1 e^{-\lambda_1 t}\overline{G}_2(t)$ $\quad q_{13,5}(t)=g_1(t)$

$q_{14,1}(t)=g_2(t)$ $\quad q_{15,1}(t)=g_2(t).e^{-\lambda_2 t}$ $\quad q_{15,17}^{(18)}(t)=\lambda_2 te^{-\lambda_2 t}g_2(t)$

$q_{15,21}^{(18)}(t)=\lambda_2^2 te^{-\lambda_2 t}\overline{G}_2(t)$

$q_{15,25}^{(18,21)}(t)=\lfloor 1-e^{-\lambda_2 t}-\lambda_2 te^{-\lambda_2 t}\rfloor g_2(t)$ $\quad q_{16,1}(t)=g_2(t).e^{-2\lambda_2 t}$

$q_{16,17}^{(18)}(t)=2\lfloor e^{-\lambda_2 t}-e^{-2\lambda_2 t}\rfloor g_2(t)$ $\quad q_{16,21}^{(18)}(t)=2\lambda_2\left[e^{-\lambda_2 t}-e^{-2\lambda_2 t}\right]\overline{G}_2(t)$

$q_{16,25}^{(18,21)}(t)=\lfloor 1-2e^{-\lambda_2 t}+e^{-2\lambda_2 t}\rfloor g_2(t)$ $\quad q_{17,1}(t)=g_1(t).e^{-(\alpha_2+\lambda_2)t}$

$q_{17,14}^{(19)}(t)=\frac{q_2\alpha_2}{\alpha_2+\lambda_2}\left[1-e^{-(\alpha_2+\lambda_2)t}\right]g_1(t)$ $\quad q_{17,16}^{(20)}(t)=p_2\lfloor e^{-\lambda_2 t}-e^{-(\alpha_2+\lambda_2)t}\rfloor g_1(t)$

$q_{17,17}^{(22)}(t)=\frac{\lambda_2}{\alpha_2+\lambda_2}\left[1-e^{-(\alpha_2+\lambda_2)t}\right]g_1(t)$ $\quad q_{17,23}^{(20)}(t)=p_2\lambda_2\left[e^{-\lambda_2 t}-e^{-(\alpha_2+\lambda_2)t}\right]\overline{G}_1(t)$

$q_{17,24}^{(20,23)}(t)=\frac{p_2}{\alpha_2+\lambda_2}\left[\alpha_2\left(1-e^{-\lambda_2 t}\right)-\lambda_2\left(e^{-\lambda_2 t}-e^{-(\alpha_2+\lambda_2)t}\right)\right]g_1(t)$

$q_{24,17}(t)=e^{-\lambda_2 t}g_2(t)$ $\quad q_{24,21}(t)=\lambda_2 e^{-\lambda_2 t}\overline{G}_2(t)$

$q_{24,25}^{(21)}(t)=\lfloor 1-e^{-\lambda_2 t}\rfloor g_2(t)$ $\quad q_{25,17}(t)=g_1(t)$

The transition probabilities p_{ij} are then be obtained using the relation $p_{ij}=\lim_{s\to 0} q_{ij}^*(s)$.

Mean Sojourm Times

Mean Sojourn time (μ_i) in state i, i.e., the expected time of stay in state i are

$\mu_0=\frac{1}{\alpha_1+2\lambda_1}\left[1-h_1^*(\alpha_1+2\lambda_1)\right]$ $\quad \mu_1=\frac{1}{\alpha_2+2\lambda_2}\left[1-h_2^*(\alpha_2+2\lambda_2)\right]$ $\quad \mu_2=\int_0^\infty \overline{G}_2(t)\,dt$

$\mu_3=\frac{1}{\lambda_1}\left[1-g_2^*(\lambda_1)\right]$ $\quad \mu_4=\frac{1}{2\lambda_1}\left[1-g_2^*(2\lambda_1)\right]$

$$\mu_5 = \frac{1}{\alpha_1+\lambda_1}\left[1-g_1^*(\alpha_1+\lambda_1)\right] \qquad \mu_{12} = \frac{1}{\lambda_1}\left[1-g_2^*(\lambda_1)\right] \qquad \mu_{13} = \int_0^\infty \overline{G}_1(t)\,dt$$

$$\mu_{14} = \int_0^\infty \overline{G}_2(t)\,dt = \mu_2 \qquad \mu_{15} = \frac{1}{\lambda_2}\left[1-g_2^*(\lambda_2)\right] \qquad \mu_{16} = \frac{1}{2\lambda_2}\left[1-g_2^*(2\lambda_2)\right]$$

$$\mu_{17} = \frac{1}{\alpha_2+\lambda_2}\left[1-g_1^*(\alpha_2+\lambda_2)\right] \qquad \mu_{24} = \frac{1}{\lambda_2}\left[1-g_2^*(\lambda_2)\right] \qquad \mu_{25} = \int_0^\infty \overline{G}_1(t)\,dt = \mu_{13}$$

The unconditional mean time taken by the system to transit for any state j when it is counted from epoch of entrance into state i is mathematical stated as

$$m_{ij} = \int_0^\infty t\,q_{ij}(t)dt = -q_{ij}^{*'}(0)$$

$$m_{01}+m_{02}+m_{03}+m_{04}+m_{05} = \mu_0 \qquad m_{10}+m_{1,14}+m_{1,15}+m_{1,16}+m_{1,17} = \mu_1$$

$$m_{20} = \mu_2 \qquad m_{30}+m_{35}^{(6)}+m_{39}^{(6)} = 2\mu_3 + g_2^{*'}(\lambda_1)$$

$$m_{40}+m_{45}^{(6)}+m_{49}^{(6)} = 2\mu_3 - \mu_4$$

$$m_{50}+m_{52}^{(7)}+m_{54}^{(8)}+m_{55}^{(10)}+m_{5,11}^{(8)} = \frac{p_1}{\lambda_1}\left[1-g_1^*(\lambda_1)\right] - \frac{p_1\lambda_1}{\alpha_1+\lambda_1}\mu_5 - \left|1-\frac{p_1\alpha_1}{\alpha_1+\lambda_1}\right| g_1^{*'}(0)$$

$$m_{12,5}+m_{12,9} = \mu_{12} \qquad m_{13,5} = \mu_{13} \qquad m_{14,1} = \mu_{14} = \mu_2$$

$$m_{15,1}+m_{15,17}^{(18)}+m_{15,21}^{(18)} = 2\mu_{15} + g_2^{*'}(\lambda_2) \qquad m_{16,1}+m_{16,17}^{(18)}+m_{16,21}^{(18)} = 2\mu_{15} - \mu_{16}$$

$$m_{17,1}+m_{17,14}^{(19)}+m_{17,16}^{(20)}+m_{17,17}^{(22)}+m_{17,23}^{(20)} = \frac{p_2}{\lambda_2}\left[1-g_1^*(\lambda_2)\right] - \frac{p_2\lambda_2}{\alpha_2+\lambda_2}\mu_{17} - \left|1-\frac{p_2\alpha_2}{\alpha_2+\lambda_2}\right| g_1^{*'}(0)$$

$$m_{24,17}+m_{24,21} = \mu_{24} \qquad m_{25,17} = \mu_{25} = \mu_{13}$$

RELIABILITY AND MEAN TIME TO SYSTEM FAILURE

To determine the reliability and mean time to system failure (MTSF) of the system, we regard the failed states as absorbing states. Let $\phi_i(t)$ be the c.d.f. of the first passage time from regenerative state i to failed state, then the recursive relations for $\phi_i(t)$ are:

$$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t) + Q_{02}(t)Ⓢ\phi_2(t) + Q_{03}(t)Ⓢ\phi_3(t) + Q_{04}(t)Ⓢ\phi_4(t) + Q_{05}(t)Ⓢ\phi_5(t)$$

$$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0(t) + Q_{1,14}(t)Ⓢ\phi_{14}(t) + Q_{1,15}(t)Ⓢ\phi_{15}(t) + Q_{1,16}(t)Ⓢ\phi_{16}(t) + Q_{1,17}(t)Ⓢ\phi_{17}(t)$$

$$\phi_2(t) = Q_{20}(t)Ⓢ\phi_0(t)$$

$$\phi_3(t) = Q_{30}(t)Ⓢ\phi_0(t) + Q_{35}^{(6)}(t)Ⓢ\phi_5(t) + Q_{39}^{(6)}(t)$$

$$\phi_4(t) = Q_{40}(t)Ⓢ\phi_0(t) + Q_{45}^{(6)}(t)Ⓢ\phi_5(t) + Q_{49}^{(6)}(t)$$

$$\phi_5(t) = Q_{50}(t)Ⓢ\phi_0(t) + Q_{52}^{(7)}(t)Ⓢ\phi_2(t) + Q_{54}^{(8)}(t)Ⓢ\phi_4(t) + Q_{55}^{(10)}(t)Ⓢ\phi_5(t) + Q_{5,11}^{(8)}(t)$$

$$\phi_{12}(t) = Q_{12,5}(t)ⓢ\phi_5(t) + Q_{12,9}(t)$$

$$\phi_{13}(t) = Q_{13,5}(t)ⓢ\phi_5(t)$$

$$\phi_{14}(t) = Q_{14,1}(t)ⓢ\phi_1(t)$$

$$\phi_{15}(t) = Q_{15,1}(t)ⓢ\phi_1(t) + Q_{15,17}^{(18)}(t)ⓢ\phi_{17}(t) + Q_{15,21}^{(18)}(t)$$

$$\phi_{16}(t) = Q_{16,1}(t)ⓢ\phi_1(t) + Q_{16,17}^{(18)}(t)ⓢ\phi_{17}(t) + Q_{16,21}^{(18)}(t)$$

$$\phi_{17}(t) = Q_{17,1}(t)ⓢ\phi_1(t) + Q_{17,14}^{(19)}(t)ⓢ\phi_{14}(t) + Q_{17,16}^{(20)}(t)ⓢ\phi_{16}(t) + Q_{17,17}^{(22)}(t)ⓢ\phi_{17}(t) + Q_{17,23}^{(20)}(t)$$

$$\phi_{24}(t) = Q_{24,17}(t)ⓢ\phi_{17}(t) + Q_{24,21}(t)$$

$$\phi_{25}(t) = Q_{25,17}(t)ⓢ\phi_{17}(t)$$

Taking Laplace Stieltjes Transform (L.S.T.) of these relations and solving them for $\phi_0^{**}(t)$, the reliability of the system 'R(t)' is given as

R(t) = Inverse Laplace Transform of $\dfrac{1-\phi_0^{**}(s)}{s}$

where

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)},$$

$$D(s) = \begin{vmatrix}
1 & -Q_{01}^{*}(s) & -Q_{02}^{*}(s) & -Q_{03}^{*}(s) & -Q_{04}^{*}(s) & -Q_{05}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-Q_{10}^{*}(s) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_{1,14}^{*}(s) & -Q_{1,15}^{*}(s) & -Q_{1,16}^{*}(s) & -Q_{1,17}^{*}(s) & 0 & 0 \\
-Q_{20}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-Q_{30}^{*}(s) & 0 & 0 & 1 & 0 & -Q_{35}^{(6)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-Q_{40}^{*}(s) & 0 & 0 & 0 & 1 & -Q_{45}^{(6)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-Q_{50}^{*}(s) & 0 & -Q_{52}^{(7)*}(s) & 0 & -Q_{54}^{(8)*}(s) & 1-Q_{55}^{(10)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -Q_{12,5}^{*}(s) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -Q_{13,5}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{14,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{15,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -Q_{15,17}^{(18)*}(s) & 0 & 0 \\
0 & -q_{16,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -Q_{16,17}^{(18)*}(s) & 0 & 0 \\
0 & -q_{17,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & -Q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-Q_{17,17}^{(20)*}(s) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_{24,17}^{*}(s) & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_{25,17}^{*}(s) & 0 & 1
\end{vmatrix}$$

$$N(s) = \begin{vmatrix}
0 & -Q_{01}^{*}(s) & -Q_{02}^{*}(s) & -Q_{03}^{*}(s) & -Q_{04}^{*}(s) & -Q_{05}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_{1,14}^{*}(s) & -Q_{1,15}^{*}(s) & -Q_{1,16}^{*}(s) & -Q_{1,17}^{*}(s) & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{39}^{(6)*}(s) & 0 & 0 & 1 & 0 & -Q_{35}^{(6)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{49}^{(6)*}(s) & 0 & 0 & 0 & 1 & -Q_{45}^{(6)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{5,11}^{(8)*}(s) & 0 & -Q_{52}^{(7)*}(s) & 0 & -Q_{54}^{(8)*}(s) & 1-Q_{55}^{(10)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{12,9}^{*}(s) & 0 & 0 & 0 & 0 & -Q_{12,5}^{*}(s) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -Q_{13,5}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{14,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
Q_{15,21}^{(18)*}(s) & -q_{15,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -Q_{15,17}^{(18)*}(s) & 0 & 0 \\
Q_{16,21}^{(18)*}(s) & -q_{16,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -Q_{16,17}^{(18)*}(s) & 0 & 0 \\
Q_{17,23}^{(20)*}(s) & -q_{17,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & -Q_{17,14}^{(19)*}(s) & 0 & -Q_{17,16}^{(20)*}(s) & 1-Q_{17,17}^{(20)*}(s) & 0 & 0 \\
Q_{24,21}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_{24,17}^{*}(s) & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_{25,17}^{*}(s) & 0 & 1
\end{vmatrix}$$

The mean time to system failure (MTSF) when the system starts from the state '0' is given by

$$T_0 = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} = \frac{D'(0)-N'(0)}{D(0)}$$

where $D'(0)$, $N'(0)$ and $D(0)$ can be obtained from $D'(s)$, $N'(s)$ and $D(s)$ by replacing s by zero

AVAILABILITY IN DIFFERENT SEASONS AND CAPACITIES/TYPES

Availability in Full Capacity during Winter

Letting $AW_i^f(t)$ as the probability that system is up in full capacity during winter at instant t given that it entered state i at t = 0, we have using the arguments of the theory of regenerative process, the availability $AW_i^f(t)$ is seen to be satisfying the following recursive relations:

$$AW_0^f(t) = MW_0^f(t) + q_{01}(t)©AW_1^f(t) + q_{02}(t)©AW_2^f(t) + q_{03}(t)©AW_3^f(t) + q_{04}(t)©AW_4^f(t) + q_{05}(t)©AW_5^f(t)$$

$$AW_1^f(t) = q_{10}(t)©AW_0^f(t) + q_{1,14}(t)©AW_{14}^f(t) + q_{1,15}(t)©AW_{15}^f(t) + q_{1,16}(t)©AW_{16}^f(t) + q_{1,17}(t)©AW_{17}^f(t)$$

$$AW_2^f(t) = q_{20}(t)©AW_0^f(t)$$

$$AW_3^f(t) = q_{30}(t)©AW_0^f(t) + q_{35}^{(6)}(t)©AW_5^f(t) + q_{3,13}^{(6,9)}(t)©AW_{13}^f(t)$$

$$AW_4^f(t) = q_{40}(t)©AW_0^f(t) + q_{45}^{(6)}(t)©AW_5^f(t) + q_{4,13}^{(6,9)}(t)©AW_{13}^f(t)$$

$$AW_5^f(t) = q_{50}(t)©AW_0^f(t) + q_{52}^{(7)}(t)©AW_2^f(t) + q_{54}^{(8)}(t)©AW_4^f(t) + q_{55}^{(10)}(t)©AW_5^f(t) + q_{5,12}^{(8,11)}(t)©AW_{12}^f(t)$$

$$AW_{12}^f(t) = q_{12,5}(t)©AW_5^f(t) + q_{12,13}^{(9)}(t)©AW_{13}^f(t)$$

$$AW_{13}^f(t) = q_{13,5}(t)©AW_5^f(t)$$

$$AW_{14}^f(t) = q_{14,1}(t)©AW_1^f(t)$$

$$AW_{15}^f(t) = q_{15,1}(t)©AW_1^f(t) + q_{15,17}^{(18)}(t)©AW_{17}^f(t) + q_{15,25}^{(18,21)}(t)©AW_{25}^f(t)$$

$$AW_{16}^f(t) = q_{16,1}(t)©AW_1^f(t) + q_{16,17}^{(18)}(t)©AW_{17}^f(t) + q_{16,25}^{(18,21)}(t)©AW_{25}^f(t)$$

$$AW_{17}^f(t) = q_{17,1}(t)©AW_1^f(t) + q_{17,14}^{(19)}(t)©AW_{14}^f(t) + q_{17,16}^{(20)}(t)©AW_{16}^f(t) + q_{17,17}^{(22)}(t)©AW_{17}^f(t) + q_{17,24}^{(20,23)}(t)©AW_{24}^f(t)$$

$$AW_{24}^f(t) = q_{24,17}(t)©AW_{17}^f(t) + q_{24,25}^{(21)}(t)©AW_{25}^f(t)$$

$$AW_{25}^f(t) = q_{25,17}(t)©AW_{17}^f(t)$$

where $MW_0^f(t) = e^{-(2\lambda_1+\alpha_1)t}\,\overline{H}_1(t)$

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Taking Laplace Transform (L.T.) of the above equations and solving them for $AW_0^{f*}(s)$, the steady state availability of the system is given by

$$AW_0^f = \lim_{s\to 0} sAW_0^{f*}(s) = \frac{N_1(0)}{D_1^{'}(0)}$$

where

$$D_1(s)=\begin{vmatrix}
1 & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & -q_{04}^*(s) & -q_{05}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-q_{10}^*(s) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{1,14}^*(s) & -q_{1,15}^*(s) & -q_{1,16}^*(s) & -q_{1,17}^*(s) & 0 & 0 \\
-q_{20}^*(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-q_{30}^*(s) & 0 & 0 & 1 & 0 & -q_{35}^{(6)*}(s) & 0 & -q_{3,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
-q_{40}^*(s) & 0 & 0 & 0 & 1 & -q_{45}^{(6)*}(s) & 0 & -q_{4,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
-q_{50}^*(s) & 0 & -q_{52}^{(7)*}(s) & 0 & -q_{54}^{(8)*}(s) & 1-q_{55}^{(10)*}(s) & -q_{5,12}^{(8,11)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -q_{12,5}^*(s) & 1 & -q_{12,13}^{(9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -q_{13,5}^*(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{14,1}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{15,1}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -q_{15,17}^{(18)*}(s) & 0 & -q_{15,25}^{(18,21)*}(s) \\
0 & -q_{16,1}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{16,17}^{(18)*}(s) & 0 & -q_{16,25}^{(18,21)*}(s) \\
0 & -q_{17,1}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-q_{17,17}^{(22)*}(s) & -q_{17,24}^{(20,23)*}(s) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{24,17}^*(s) & 1 & -q_{24,25}^{(21)*}(s) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{25,17}^*(s) & 0 & 1
\end{vmatrix}$$

$$N_1(s)=\begin{vmatrix}
MW_0^f(t) & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & -q_{04}^*(s) & -q_{05}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{1,14}^*(s) & -q_{1,15}^*(s) & -q_{1,16}^*(s) & -q_{1,17}^*(s) & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -q_{35}^{(6)*}(s) & 0 & -q_{3,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -q_{45}^{(6)*}(s) & 0 & -q_{4,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -q_{52}^{(7)*}(s) & 0 & -q_{54}^{(8)*}(s) & 1-q_{55}^{(10)*}(s) & -q_{5,12}^{(8,11)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -q_{12,5}^*(s) & 1 & -q_{12,13}^{(9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -q_{13,5}^*(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{14,1}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{15,1}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -q_{15,17}^{(18)*}(s) & 0 & -q_{15,25}^{(18,21)*}(s) \\
0 & -q_{16,1}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{16,17}^{(18)*}(s) & 0 & -q_{16,25}^{(18,21)*}(s) \\
0 & -q_{17,1}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-q_{17,17}^{(22)*}(s) & -q_{17,24}^{(20,23)*}(s) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{24,17}^*(s) & 1 & -q_{24,25}^{(21)*}(s) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{25,17}^*(s) & 0 & 1
\end{vmatrix}$$

Availability in Reduced Capacity of Type I during Winter

Letting $AW_i^I(t)$ as the probability that system is up in reduced capacity of type I during winter at instant t given that it entered state i at t = 0, we have using the arguments of the theory of regenerative process, the availability $AW_i^I(t)$ is seen to be satisfying the following recursive relations:

$$AW_0^I(t) = q_{01}(t)©AW_1^I(t) + q_{02}(t)©AW_2^I(t) + q_{03}(t)©AW_3^I(t) + q_{04}(t)©AW_4^I(t) + q_{05}(t)©AW_5^I(t)$$

$$AW_1^I(t) = q_{10}(t)©AW_0^I(t) + q_{1,14}(t)©AW_{14}^I(t) + q_{1,15}(t)©AW_{15}^I(t) + q_{1,16}(t)©AW_{16}^I(t)$$
$$+ q_{1,17}(t)©AW_{17}^I(t)$$

$$AW_2^I(t) = q_{20}(t)©AW_0^I(t)$$

$$AW_3^I(t) = q_{30}(t)©AW_0^I(t) + q_{35}^{(6)}(t)©AW_5^I(t) + q_{3,13}^{(6,9)}(t)©AW_{13}^I(t)$$

$$AW_4^I(t) = MW_4^I(t) + q_{40}(t)©AW_0^I(t) + q_{45}^{(6)}(t)©AW_5^I(t) + q_{4,13}^{(6,9)}(t)©AW_{13}^I(t)$$

$$AW_5^I(t) = q_{50}(t)©AW_0^I(t) + q_{52}^{(7)}(t)©AW_2^I(t) + q_{54}^{(8)}(t)©AW_4^I(t)$$
$$+ q_{55}^{(10)}(t)©AW_5^I(t) + q_{5,12}^{(8,11)}(t)©AW_{12}^I(t)$$

$$AW_{12}^I(t) = q_{12,5}(t)©AW_5^I(t) + q_{12,13}^{(9)}(t)©AW_{13}^I(t)$$

$$AW_{13}^I(t) = q_{13,5}(t)©AW_5^I(t)$$

$$AW_{14}^I(t) = q_{14,1}(t)©AW_1^I(t)$$

$$AW_{15}^I(t) = q_{15,1}(t)©AW_1^I(t) + q_{15,17}^{(18)}(t)©AW_{17}^I(t) + q_{15,25}^{(18,21)}(t)©AW_{25}^I(t)$$

$$AW_{16}^I(t) = q_{16,1}(t)©AW_1^I(t) + q_{16,17}^{(18)}(t)©AW_{17}^I(t) + q_{16,25}^{(18,21)}(t)©AW_{25}^I(t)$$

$$AW_{17}^I(t) = q_{17,1}(t)©AW_1^I(t) + q_{17,14}^{(19)}(t)©AW_{14}^I(t) + q_{17,16}^{(20)}(t)©AW_{16}^I(t)$$
$$+ q_{17,17}^{(22)}(t)©AW_{17}^I(t) + q_{17,24}^{(20,23)}(t)©AW_{24}^I(t)$$

$$AW_{24}^I(t) = q_{24,17}(t)©AW_{17}^I(t) + q_{24,25}^{(21)}(t)©AW_{25}^I(t)$$

$$AW_{25}^I(t) = q_{25,17}(t)©AW_{17}^I(t)$$

where $MW_4^I(t) = e^{-2\lambda_1 t}\,\overline{G}_2(t)$

Taking Laplace Transform (L.T.) of the above equations and solving them for $AW_0^{I*}(s)$, the steady state availability of the system is given by

$$AW_0^I = \lim_{s\to 0} sAW_0^{*I}(s) = \frac{N_2(0)}{D_1^{'}(0)}$$

where $D_1(s)$ already defined, and

$$N_2(s) = \begin{vmatrix}
0 & -q_{01}^{*}(s) & -q_{02}^{*}(s) & -q_{03}^{*}(s) & -q_{04}^{*}(s) & -q_{05}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{1,14}^{*}(s) & -q_{1,15}^{*}(s) & -q_{1,16}^{*}(s) & -q_{1,17}^{*}(s) & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -q_{35}^{(6)*}(s) & 0 & -q_{3,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
MW_4^I(t) & 0 & 0 & 0 & 1 & -q_{45}^{(6)*}(s) & 0 & -q_{4,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -q_{52}^{(7)*}(s) & 0 & -q_{54}^{(8)*}(s) & 1-q_{55}^{(10)*}(s) & -q_{5,12}^{(8,11)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -q_{12,5}^{*}(s) & 1 & -q_{12,13}^{(9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -q_{13,5}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{14,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{15,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -q_{15,17}^{(18)*}(s) & 0 & -q_{15,25}^{(18,21)*}(s) \\
0 & -q_{16,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{16,17}^{(18)*}(s) & 0 & -q_{16,25}^{(18,21)*}(s) \\
0 & -q_{17,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-q_{17,17}^{(22)*}(s) & -q_{17,24}^{(20,23)*}(s) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{24,17}^{*}(s) & 1 & -q_{24,25}^{(21)*}(s) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{25,17}^{*}(s) & 0 & 1
\end{vmatrix}$$

Availability in Reduced Capacity of Type II during Winter

Letting $AW_i^{II}(t)$ as the probability that system is up in reduced capacity of type II during winter at instant t given that it entered state i at t = 0, we have using the arguments of the theory of regenerative process, the availability $AW_i^{II}(t)$ is seen to be satisfying the following recursive relations:

$$AW_0^{II}(t) = q_{01}(t)©AW_1^{II}(t) + q_{02}(t)©AW_2^{II}(t) + q_{03}(t)©AW_3^{II}(t) + q_{04}(t)©AW_4^{II}(t) + q_{05}(t)©AW_5^{II}(t)$$

$$AW_1^{II}(t) = q_{10}(t)©AW_0^{II}(t) + q_{1,14}(t)©AW_{14}^{II}(t) + q_{1,15}(t)©AW_{15}^{II}(t) + q_{1,16}(t)©AW_{16}^{II}(t) + q_{1,17}(t)©AW_{17}^{II}(t)$$

$$AW_2^{II}(t) = q_{20}(t)©AW_0^{II}(t)$$

$$AW_3^{II}(t) = q_{30}(t)©AW_0^{II}(t) + q_{35}^{(6)}(t)©AW_5^{II}(t) + q_{3,13}^{(6,9)}(t)©AW_{13}^{II}(t)$$

$$AW_4^{II}(t) = q_{40}(t)©AW_0^{II}(t) + q_{45}^{(6)}(t)©AW_5^{II}(t) + q_{4,13}^{(6,9)}(t)©AW_{13}^{II}(t)$$

$$AW_5^{II}(t) = MW_5^{II}(t) + q_{50}(t)©AW_0^{II}(t) + q_{52}^{(7)}(t)©AW_2^{II}(t) + q_{54}^{(8)}(t)©AW_4^{II}(t) + q_{55}^{(10)}(t)©AW_5^{II}(t) + q_{5,12}^{(20,23)}(t)©AW_{12}^{II}(t)$$

$$AW_{12}^{II}(t) = q_{12,5}(t)©AW_5^{II}(t) + q_{12,13}^{(9)}(t)©AW_{13}^{II}(t)$$

$$AW_{13}^{II}(t) = q_{13,5}(t)©AW_5^{II}(t)$$

$$AW_{14}^{II}(t) = q_{14,1}(t)©AW_1^{II}(t)$$

$$AW_{15}^{II}(t) = q_{15,1}(t)©AW_1^{II}(t) + q_{15,17}^{(18)}(t)©AW_{17}^{II}(t) + q_{15,25}^{(18,21)}(t)©AW_{25}^{II}(t)$$

$$AW_{16}^{II}(t) = q_{16,1}(t)©AW_1^{II}(t) + q_{16,17}^{(18)}(t)©AW_{17}^{II}(t) + q_{16,25}^{(18,21)}(t)©AW_{25}^{II}(t)$$

$$AW_{17}^{II}(t) = q_{17,1}(t)©AW_1^{II}(t) + q_{17,14}^{(19)}(t)©AW_{14}^{II}(t) + q_{17,16}^{(20)}(t)©AW_{16}^{II}(t) + q_{17,17}^{(22)}(t)©AW_{17}^{II}(t) + q_{17,24}^{(20,23)}(t)©AW_{24}^{II}(t)$$

$$AW_{24}^{II}(t) = q_{24,17}(t)©AW_{17}^{II}(t) + q_{24,25}^{(21)}(t)©AW_{25}^{II}(t)$$

$$AW_{25}^{II}(t) = q_{25,17}(t)©AW_{17}^{II}(t)$$

where $MW_5^{II}(t) = e^{-(\alpha_1+\lambda_1)t}\,\overline{G}_1(t)$

Taking Laplace Transform (L.T.) of the above equations and solving them for $AW_0^{II*}(s)$, the steady state availability of the system is given by

$$AW_0^{II} = \lim_{s\to 0} sAS_0^{II*}(s) = \frac{N_3(0)}{D_1^{'}(0)}$$

where $D_1(s)$ already defined, and

$$N_3(s)=\begin{vmatrix} 0 & -q_{01}^{*}(s) & -q_{02}^{*}(s) & -q_{03}^{*}(s) & -q_{04}^{*}(s) & -q_{05}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{1,14}^{*}(s) & -q_{1,15}^{*}(s) & -q_{1,16}^{*}(s) & -q_{1,17}^{*}(s) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -q_{35}^{(6)*}(s) & 0 & -q_{3,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{45}^{(6)*}(s) & 0 & -q_{4,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ MW_5^{II}(t) & 0 & -q_{52}^{(7)*}(s) & 0 & -q_{54}^{(8)*}(s) & 1-q_{55}^{(10)*}(s) & -q_{5,12}^{(8,11)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -q_{12,5}^{*}(s) & 1 & -q_{12,13}^{(9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -q_{13,5}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{14,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{15,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -q_{15,17}^{(18)*}(s) & 0 & -q_{15,25}^{(18,21)*}(s) \\ 0 & -q_{16,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{16,17}^{(18)*}(s) & 0 & -q_{16,25}^{(18,21)*}(s) \\ 0 & -q_{17,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-q_{17,17}^{(22)*}(s) & -q_{17,24}^{(20,23)*}(s) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{24,17}^{*}(s) & 1 & -q_{24,25}^{(21)*}(s) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{25,17}^{*}(s) & 0 & 1 \end{vmatrix}$$

Availability in Reduced Capacity of Type III during Winter

Letting $AW_i^{III}(t)$ as the probability that system is up in reduced capacity of type III during winter at instant t given that it entered state i at t = 0, we have using the arguments of the theory of regenerative process, the availability $AW_i^{III}(t)$ is seen to be satisfying the following recursive relations:

$$AW_0^{III}(t)=q_{01}(t)©AW_1^{III}(t)+q_{02}(t)©AW_2^{III}(t)+q_{03}(t)©AW_3^{III}(t)+q_{04}(t)©AW_4^{III}(t)+q_{05}(t)©AW_5^{III}(t)$$

$$AW_1^{III}(t)=q_{10}(t)©AW_0^{III}(t)+q_{1,14}(t)©AW_{14}^{III}(t)+q_{1,15}(t)©AW_{15}^{III}(t)+q_{1,16}(t)©AW_{16}^{III}(t)$$
$$+q_{1,17}(t)©AW_{17}^{III}(t)$$

$$AW_2^{III}(t)=q_{20}(t)©AW_0^{III}(t)$$

$$AW_3^{III}(t)=MW_3^{III}(t)+q_{30}(t)©AW_0^{III}(t)+q_{35}^{(6)}(t)©AW_5^{III}(t)+q_{3,13}^{(6,9)}(t)©AW_{13}^{III}(t)$$

$$AW_4^{III}(t)=q_{40}(t)©AW_0^{III}(t)+q_{45}^{(6)}(t)©AW_5^{III}(t)+q_{4,13}^{(6,9)}(t)©AW_{13}^{III}(t)$$

$$AW_5^{III}(t)=q_{50}(t)©AW_0^{III}(t)+q_{52}^{(7)}(t)©AW_2^{III}(t)+q_{54}^{(8)}(t)©AW_4^{III}(t)$$
$$+q_{55}^{(10)}(t)©AW_5^{III}(t)+q_{5,12}^{(8,11)}(t)©AW_{12}^{III}(t)$$

$$AW_{12}^{III}(t)=MW_{12}^{III}(t)+q_{12,5}(t)©AW_5^{III}(t)+q_{12,13}^{(9)}(t)©AW_{13}^{III}(t)$$

$$AW_{13}^{III}(t)=q_{13,5}(t)©AW_5^{III}(t)$$

$$AW_{14}^{III}(t)=q_{14,1}(t)©AW_1^{III}(t)$$

$$AW_{15}^{III}(t)=q_{15,1}(t)©AW_1^{III}(t)+q_{15,17}^{(18)}(t)©AW_{17}^{III}(t)+q_{15,25}^{(18,21)}(t)©AW_{25}^{III}(t)$$

$$AW_{16}^{III}(t)=q_{16,1}(t)©AW_1^{III}(t)+q_{16,17}^{(18)}(t)©AW_{17}^{III}(t)+q_{16,25}^{(18,21)}(t)©AW_{25}^{III}(t)$$

$$AW_{17}^{III}(t)=q_{17,1}(t)©AW_1^{III}(t)+q_{17,14}^{(19)}(t)©AW_{14}^{III}(t)+q_{17,16}^{(20)}(t)©AW_{16}^{III}(t)$$
$$+q_{17,17}^{(22)}(t)©AW_{17}^{III}(t)+q_{17,24}^{(20,23)}(t)©AW_{24}^{III}(t)$$

$$AW_{24}^{III}(t)=q_{24,17}(t)©AW_{17}^{III}(t)+q_{24,25}^{(21)}(t)©AW_{25}^{III}(t)$$

$$AW_{25}^{III}(t)=q_{25,17}(t)©AW_{17}^{III}(t)$$

where $MW_3^{III}(t) = e^{-\lambda_1 t}\left(1+\lambda_1 t\right)\overline{G}_2(t),\ MW_{12}^{III}(t) = e^{-\lambda_1 t}\,\overline{G}_2(t)$

Taking Laplace Transform (L.T.) of the above equations and solving them for $AW_0^{III*}(s)$, the steady state availability of the system is given by

$$AW_0^{III} = \lim_{s\to 0} sAW_0^{III*}(s) = \frac{N_4(0)}{D_1^{'}(0)}$$

where $D_1(s)$ already defined, and

$$N_4(s) = \begin{vmatrix}
0 & -q_{01}^{*}(s) & -q_{02}^{*}(s) & -q_{03}^{*}(s) & -q_{04}^{*}(s) & -q_{05}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{1,14}^{*}(s) & -q_{1,15}^{*}(s) & -q_{1,16}^{*}(s) & -q_{1,17}^{*}(s) & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
MW_3^{III}(t) & 0 & 0 & 1 & 0 & -q_{35}^{(6)*}(s) & 0 & -q_{3,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -q_{45}^{(6)*}(s) & 0 & -q_{4,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -q_{52}^{(7)*}(s) & 0 & -q_{54}^{(8)*}(s) & 1-q_{55}^{(10)*}(s) & -q_{5,12}^{(18,21)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
MW_{12}^{III}(t) & 0 & 0 & 0 & 0 & -q_{12,5}^{*}(s) & 1 & -q_{12,13}^{(9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -q_{13,5}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{14,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{15,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -q_{15,17}^{(18)*}(s) & 0 & -q_{15,25}^{(18,21)*}(s) \\
0 & -q_{16,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{16,17}^{(18)*}(s) & 0 & -q_{16,25}^{(18,21)*}(s) \\
0 & -q_{17,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-q_{17,17}^{(22)*}(s) & -q_{17,24}^{(20,23)*}(s) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{24,17}^{*}(s) & 1 & -q_{24,25}^{(21)*}(s) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{25,17}^{*}(s) & 0 & 1
\end{vmatrix}$$

Availabilities during Summer

Proceeding in the same fashion as done in the case of obtaining expressions for various types of availabilities during winter, the availabilities under different capacities may be obtained and the same are:

The steady state availability of the system in full capacity = $AS_0^{f} = \lim_{s\to 0} sAS_0^{f*}(s)$

The steady state availability of the system in reduced capacity of Type I = $AS_0^{I} = \lim_{s\to 0} sAS_0^{I*}(s)$

The steady state availability of the system in reduced capacity of Type II = $AS_0^{II} = \lim_{s\to 0} sAS_0^{II*}(s)$

The steady state availability of the system in reduced capacity of Type III = $AS_0^{III} = \lim_{s\to 0} sAS_0^{III*}(s)$

where

$$AS_0^{f*}(s) = \frac{N_5(s)}{D_1(s)},\ AS_0^{I*}(s) = \frac{N_6(s)}{D_1(s)},\ AS_0^{II*}(s) = \frac{N_7(s)}{D_1(s)},\ AS_0^{III*}(s) = \frac{N_8(s)}{D_1(s)}$$

where $D_1(s)$ already defined, and

$$N_5(s)=\begin{vmatrix}
0 & -q_{01}^{*}(s) & -q_{02}^{*}(s) & -q_{03}^{*}(s) & -q_{04}^{*}(s) & -q_{05}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
MS_1^{f}(t) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{1,14}^{*}(s) & -q_{1,15}^{*}(s) & -q_{1,16}^{*}(s) & -q_{1,17}^{*}(s) & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & -q_{35}^{(6)*}(s) & 0 & -q_{3,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & -q_{45}^{(6)*}(s) & 0 & -q_{4,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -q_{52}^{(7)*}(s) & 0 & -q_{54}^{(8)*}(s) & 1-q_{55}^{(10)*}(s) & -q_{5,12}^{(8,11)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -q_{12,5}^{*}(s) & 1 & -q_{12,13}^{(9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -q_{13,5}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -q_{14,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & -q_{15,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -q_{15,17}^{(18)*}(s) & 0 & -q_{15,25}^{(18,21)*}(s)\\
0 & -q_{16,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{16,17}^{(18)*}(s) & 0 & -q_{16,25}^{(18,21)*}(s)\\
0 & -q_{17,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-q_{17,17}^{(22)*}(s) & -q_{17,24}^{(20,23)*}(s) & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{24,17}^{*}(s) & 1 & -q_{24,25}^{(21)*}(s)\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{25,17}^{*}(s) & 0 & 1
\end{vmatrix}$$

$$N_6(s)=\begin{vmatrix}
0 & -q_{01}^{*}(s) & -q_{02}^{*}(s) & -q_{03}^{*}(s) & -q_{04}^{*}(s) & -q_{05}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{1,14}^{*}(s) & -q_{1,15}^{*}(s) & -q_{1,16}^{*}(s) & -q_{1,17}^{*}(s) & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & -q_{35}^{(6)*}(s) & 0 & -q_{3,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & -q_{45}^{(6)*}(s) & 0 & -q_{4,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -q_{52}^{(7)*}(s) & 0 & -q_{54}^{(8)*}(s) & 1-q_{55}^{(10)*}(s) & -q_{5,12}^{(8,11)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -q_{12,5}^{*}(s) & 1 & -q_{12,13}^{(9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -q_{13,5}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -q_{14,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & -q_{15,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -q_{15,17}^{(18)*}(s) & 0 & -q_{15,25}^{(18,21)*}(s)\\
MS_{16}^{I}(t) & -q_{16,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{16,17}^{(18)*}(s) & 0 & -q_{16,25}^{(18,21)*}(s)\\
0 & -q_{17,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-q_{17,17}^{(22)*}(s) & -q_{17,24}^{(20,23)*}(s) & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{24,17}^{*}(s) & 1 & -q_{24,25}^{(21)*}(s)\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{25,17}^{*}(s) & 0 & 1
\end{vmatrix}$$

$$N_7(s)=\begin{vmatrix}
0 & -q_{01}^{*}(s) & -q_{02}^{*}(s) & -q_{03}^{*}(s) & -q_{04}^{*}(s) & -q_{05}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{1,14}^{*}(s) & -q_{1,15}^{*}(s) & -q_{1,16}^{*}(s) & -q_{1,17}^{*}(s) & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & -q_{35}^{(6)*}(s) & 0 & -q_{3,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & -q_{45}^{(6)*}(s) & 0 & -q_{4,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -q_{52}^{(7)*}(s) & 0 & -q_{54}^{(8)*}(s) & 1-q_{55}^{(10)*}(s) & -q_{5,12}^{(8,11)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -q_{12,5}^{*}(s) & 1 & -q_{12,13}^{(9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -q_{13,5}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -q_{14,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & -q_{15,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -q_{15,17}^{(18)*}(s) & 0 & -q_{15,25}^{(18,21)*}(s)\\
0 & -q_{16,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{16,17}^{(18)*}(s) & 0 & -q_{16,25}^{(18,21)*}(s)\\
MS_{17}^{II}(t) & -q_{17,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-q_{17,17}^{(22)*}(s) & -q_{17,24}^{(20,23)*}(s) & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{24,17}^{*}(s) & 1 & -q_{24,25}^{(21)*}(s)\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{25,17}^{*}(s) & 0 & 1
\end{vmatrix}$$

$$N_8(s)=\begin{vmatrix}
0 & -q_{01}^{*}(s) & -q_{02}^{*}(s) & -q_{03}^{*}(s) & -q_{04}^{*}(s) & -q_{05}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{1,14}^{*}(s) & -q_{1,15}^{*}(s) & -q_{1,16}^{*}(s) & -q_{1,17}^{*}(s) & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & -q_{35}^{(6)*}(s) & 0 & -q_{3,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & -q_{45}^{(6)*}(s) & 0 & -q_{4,13}^{(6,9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -q_{52}^{(7)*}(s) & 0 & -q_{54}^{(8)*}(s) & 1-q_{55}^{(10)*}(s) & -q_{5,12}^{(8,11)*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -q_{12,5}^{*}(s) & 1 & -q_{12,13}^{(9)*}(s) & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -q_{13,5}^{*}(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -q_{14,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
MS_{15}^{III}(t) & -q_{15,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -q_{15,17}^{(18)*}(s) & 0 & -q_{15,25}^{(18,21)*}(s)\\
0 & -q_{16,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -q_{16,17}^{(18)*}(s) & 0 & -q_{16,25}^{(18,21)*}(s)\\
0 & -q_{17,1}^{*}(s) & 0 & 0 & 0 & 0 & 0 & 0 & -q_{17,14}^{(19)*}(s) & 0 & -q_{17,16}^{(20)*}(s) & 1-q_{17,17}^{(22)*}(s) & -q_{17,24}^{(20,23)*}(s) & 0\\
MS_{24}^{III}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{24,17}^{*}(s) & 1 & -q_{24,25}^{(21)*}(s)\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_{25,17}^{*}(s) & 0 & 1
\end{vmatrix}$$

RESULTS AND DISCUSSION

To estimate the parameters involved in our study, we gathered information/data from the "Pragati Power Station, Combined Cycle Gas Power Station, IP Estate, Ring Road, New Delhi", India. Assuming

$$g_1(t) = \beta_1 e^{-\beta_1 t},\ \ g_2(t) = \beta_2 e^{-\beta_2 t},$$
$$h_1(t) = \gamma_1 e^{-\gamma_1 t},\ \ h_2(t) = \gamma_2 e^{-\gamma_2 t}$$

and using the gathered information, the maximum likelihood estimates of various parameters are obtained as:

$$\beta_1 = \beta_2 = 0.041667,\ \ \gamma_1 = 0.014840,\ \ \gamma_2 = 0.014840,$$
$$\lambda_1 = \alpha_1 = 0.000417,\ \ \lambda_2 = \alpha_2 = 0.000757$$

The data collected on maintenance and production of a power station comprising two gas and one steam turbines revealed change in power generation with respect to temperature (ambient temperature). The threshold value 28^0C of the parameter Temperature (TEMP) is used to partition the yearly production into two subsets, one subset for TEMP $\leq 28^0$C (considered as winter), and the other subset for TEMP > 28^0C (considered as summer). Various factors in case of winter as well as summer are obtained separately using R programming language and are shown in **Table 2**:

Table 2. Production and revenue for different capacities/types in winter as well as summer

Obtained factor	Statistical technique used in R	Winter	Summer
Temperature (TEMP)	mean	19.71043^0C	32.88819^0C
Production in full capacity	Linear model	299.3257MW	291.6628MW
Production in type I	Linear model	185.9201MW	178.5221MW
Production in type II	Linear model	159.9739MW	150.4371MW
Production in type III	Linear model	94.8562MW	89.0027MW

Now, let us study the behaviour of mean time to system failure and different availabilities on the basis of above values.

Behaviour of the mean time to system failure (MTSF) with respect to failure rates of steam turbines (α_1/α_2) during winter/summer as well as failure rates of gas turbines (λ_1/λ_2) during winter/summer is shown in Figures 1.

From **Figure 1** we observe that MTSF decreases with increase in failure rates of steam turbines in winter (α_1) as well as summer (α_2). It also decreases with increase in failure rates of gas turbines in winter (λ_1) as well as summer (λ_2).

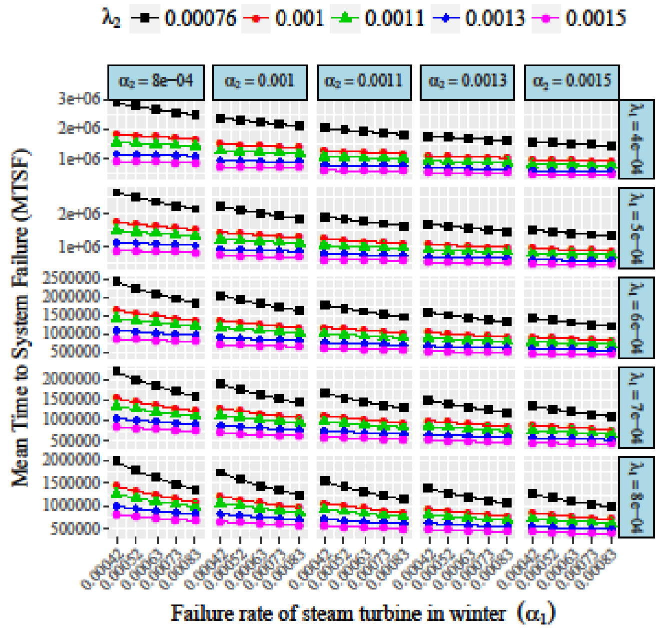

Figure 1: MTSF with respect to failure rates of steam turbines (α_1 / α_2) and gas turbines (λ_1 / λ_2) during winter/summer.

Figures 2 and 3 reveal that:

i) the availability in full capacity during winter/summer decreases with increase in failure rates of steam turbines (for both winter/summer) as well as gas turbines (for both winter/summer); however,

ii) availability in full capacity during winter and summer decreases fast as increase in failure rates of gas turbines compare to increase in failure rates of steam turbines.

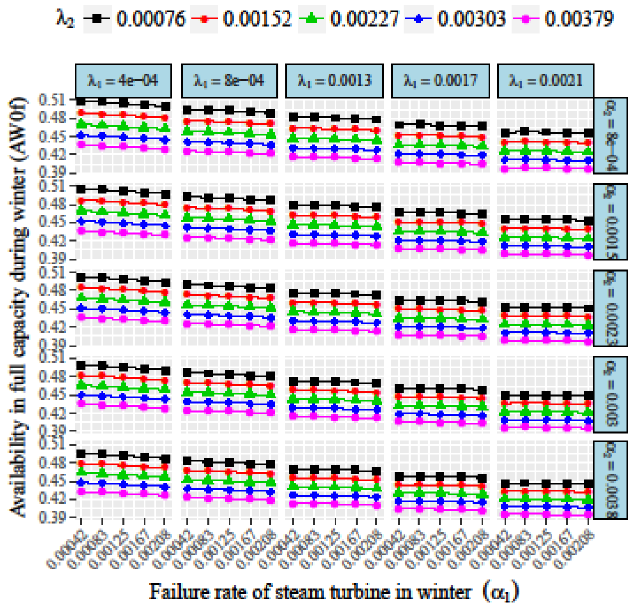

Figure 2: Availability in full capacity during winter (AW0f) with respect to failure rates of steam turbines (α_1/α_2) and gas turbines (λ_1/λ_2) during winter/summer.

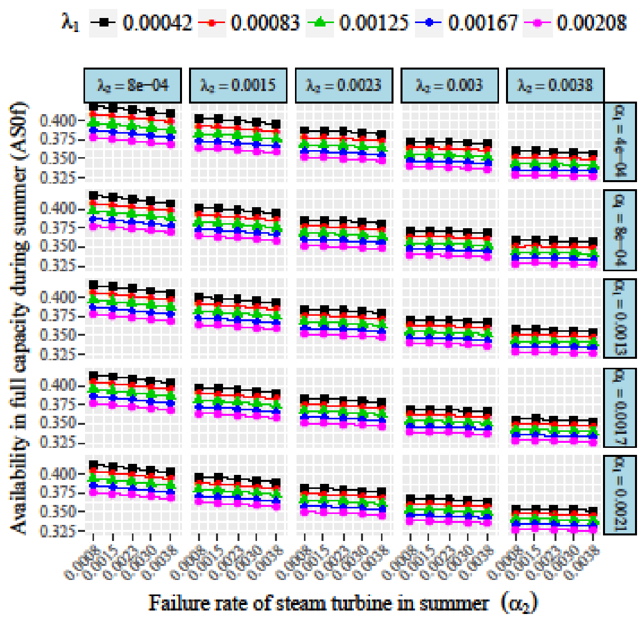

Figure 3: Availability in full capacity during summer (AS0f) with respect to failure rates of steam turbines (α_1/α_2) and gas turbines (λ_1/λ_2) during winter/summer.

Figure 4 shows simultaneous behaviour of availability in full capacity during winter and summer with respect to failure rates of steam turbines (α_1/α_2) and gas turbines (λ_1/λ_2) during winter/summer. Their behaviours are same as explained above from i) to ii). In Figure 4 for better clarity only two values of failure rate of gas turbine (λ_2) during summer are taken instead of five values that we took in Figures 2 and 3.

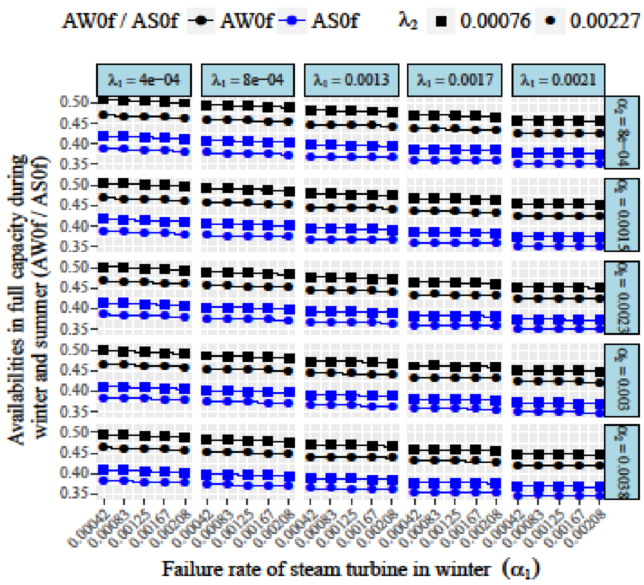

Figure 4: Availability in full capacity during winter/summer (AW0f/AS0f) simultaneously with respect to failure rates of steam turbines (α_1/α_2) and gas turbines (λ_1/λ_2) during winter/summer.

In our model relation between p_{11}, p_{12}, p_1 and q_1 is $p_1 = p_{11} + p_{12}$, $p_1 + q_1 = 1$. In Figure 5 values of these are taken as:

p_{11} takes values 0.8, 0.6, 0.4, 0.2, 0.0, and p_{12} takes values 0.1, 0.3, 0.5, 0.7, 0.9.

Figure 5 confirms many expected and interesting behaviours of the model explained as follows:

Availability in reduced capacity of type I during winter (AW0I):

From the model we note that from full capacity system goes to AW0I when steam turbine fails and there is demand of more than one gas turbine. It means that AW0I should increase with increase in failure rate of steam turbine during winter as well as with increase in the values of the probabilities p_{12}. This same behaviour can be seen from the red line in each panel which is corresponds to AW0I.

Availability in reduced capacity of type III during winter (AW0III):

From the model we note that from full capacity system goes to AW0III when steam turbine fails and demand is less than equal to production of one gas turbine. It means that AW0III should increase with increase in failure rate of steam turbine during winter as well as with increase in the values of the probabilities p_{11} (or decrease in the values of p_{12}). This same behaviour can be seen from the blue line in each panel which is corresponds to AW0III.

Availability in reduced capacity of type II during winter (AW0II):

From the model we note that from full capacity system goes to AW0II when one of the gas turbines fails, and it is independent of the demand as steam turbine is working. It means that AW0II should increase with increase in failure rate of gas turbine during winter and should be almost independent of the demand probabilities p_{11} and p_{12}. The same behaviour can be seen from the green line in each panel which is corresponds to AW0II.

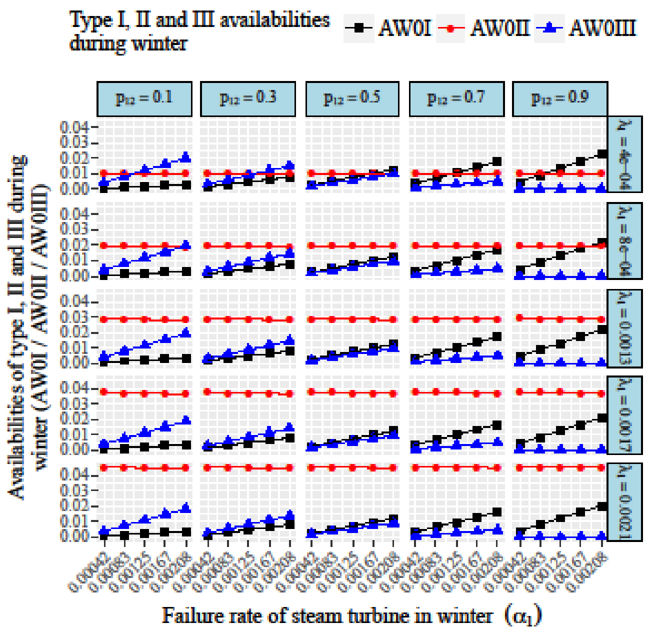

Figure 5: Availabilities of type I, II and III during winter (AW0I/AW0II/AW0III) simultaneously with respect to failure rates of steam and gas turbines (α_1/λ_1) during winter as well as for different probabilities of paying higher amount in reduced capacities.

In our model relation between p_{21}, p_{22}, p_2 and q_2 is $p_2 = p_{21} + p_{22},\ p_2 + q_2 = 1.$ In Figure 6 values of these are taken as:

p_{21} takes values 0.8, 0.6, 0.4, 0.2, 0.0, and p_{22} takes values 0.1, 0.3, 0.5, 0.7, 0.9.

Figure 6 confirms many expected and interesting behaviours of the model explained as follows:

Availability in reduced capacity of type I during summer (AS0I):

From the model we note that from full capacity system goes to AS0I when steam turbine fails and there is demand of more than one gas turbine. It means that AS0I should increase with increase in failure rate of steam turbine during summer as well as with increase in the values of the probabilities p_{22}. The same behaviour can be seen from the red line in each panel which is corresponds to AS0I.

Availability in reduced capacity of type III during summer (AS0III):

From the model we note that from full capacity system goes to AS0III when steam turbine fails and demand is less than equal to production of one gas turbine. It means that AS0III should increase with increase in failure rate of steam turbine during summer as well as with increase in the values of the probabilities p_{21}. The same behaviour can be seen from the blue line in each panel which is corresponds to AS0III.

Availability in reduced capacity of type II during summer (AS0II):

From the model we note that from full capacity system goes to AS0II when one of the gas turbines fails, and it is independent of the demand as steam turbine is working. It means that AS0II should increase with increase in failure rate of gas turbine during summer and should be almost independent of the demand probabilities p_{21} and p_{22}. The same behaviour can be seen from the green line in each panel which is corresponds to AS0II.

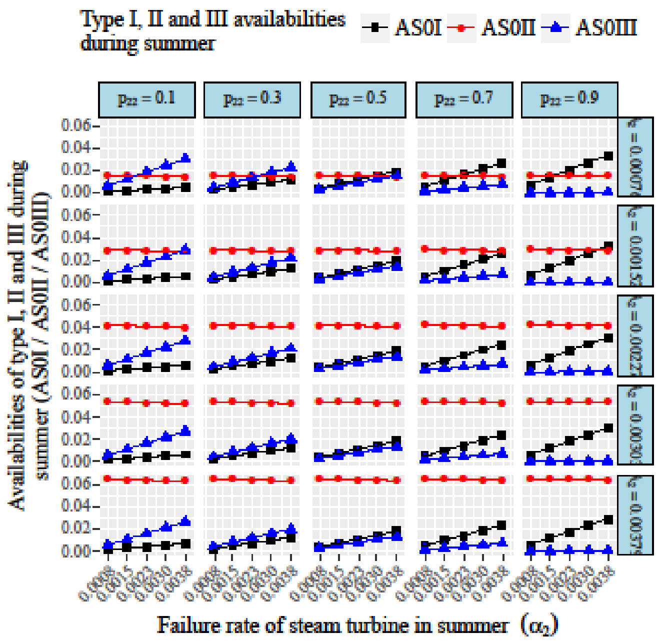

Figure 6: Availabilities of type I, II and III during summer (AS0I/AS0II/AS0III) simultaneously with respect to failure rates of steam and gas turbines (α_2/λ_2) during summer as well as for different probabilities of paying higher amount in reduced capacities.

CONCLUSION

For a power station comprising two gas and one steam turbine, expression for mean time to system failure have been obtained. Simultaneous effects of failure rates of gas and steam turbines in winter as well as summer on mean time to system failure have been graphically analysis. Interesting results have been obtained with regard to availability. Linear model fitted between the production of the power station and the ambient temperature. Behaviour of the availability in different capacities/types and different seasons i.e. winter as well as summer has been depicted with respect to failure rate of steam turbine and gas turbines in summer as well as winter. This will be helpful for users of the such systems to take various important decisions regarding quality of the turbines and use of gas turbine during the failure time of steam turbine.

ACKNOWLEDGEMENTS

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