International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 12 (2018) pp. 10202-10204

Characterization of e-homomorphism of Semi Graphs

Paras D Uchat[1], Maitri Sutaria[2]

[1] *Department of Mathematics, Indian Institute of Teacher Education, Gandhinagar, Gujarat, India.*

[2] *Department of Mathematics, Samarpan Science and Commerce College, Gandhinagar, Gujarat, India.*

Abstract

Semi graph was introduced by E-Sampatkumar[6] which is closely associated with Theory of Graphs. Study of Homomorphism [3] of graphs play vital role in Algebraic Graph Theory. In this Paper we have introduced e-homomorphism of semi graphs and derives its color characteristic[5]. We have also investigated nature of some important parameters under e-homomorphism.

Keywords: e-homomorphism, e-independent set, e-clique set, e-chromatic number.

Mathematics subject classification AMS (2010): 05C60, 05C75, 05E99

INTRODUCTION

Homomorphism[3]can be viewed as a generalization of graph colorings. In Semi graphs edge having three kind of vertices on it, which are end vertex, middle end vertex and middle vertex. This leads to various types of adjacencies between vertices.e-homomorphism of graphs is associate to End adjacency of vertices. Color characteristic [5] of homomorphism between graphs motivate us to establish the color characteristic of e-homomorphism of semi graph. We had also investigated the effect of various parameter like Clique number, Independence number under e-homomorphism.

PRELIMINARIES

Definition 1: Semigraph

A semi graph G is a pair (V,X) where V is a nonempty set whose elements are called vertices of G, and X is a set of n-tuples, called edges of G, of distinct vertices, for various $n \geq 2$. satisfying the following conditions.

(I): Any two edges have at most one vertex in common.

(II): Two edges $E_1 = (u_1, u_2, u_{3,} \dots\dots., u_n)$ and $E_2 = (v_1, v_2, v_3 \dots\dots v_m)$ are considered to be equal if and only if

1. m = n and
2. Either $u_i = v_i$ for $1 \leq i \leq n$ or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

Note : Semi graph allow edges with more than two vertices, however, the vertices in any edge of a semi graph follow a particular order .Vertices in a semigraph are divided into three types Namely end vertices, middle vertices, middle-end vertices .

According to these vertices various types of degrees can be defined:

(1) Degree: deg v is the number of edges having v is an end vertex.

(2) Edge Degree: deg_e v is the number of edges containing vertex v.

(3) Adjacent Degree: $deg_a v$ is the number of vertices adjacent to v

(4) Consecutive Adjacent Degree: $deg_{ca} v$ is the number of vertices which are consecutively adjacent to v.

Definition 2: Coloring of semi graph[6]: colors assign to vertices such that not all vertices in an edge are colored the same. Minimum number of colors required for coloring the semi graph known as chromatic number $\chi(G)$.

Definition 3: e-coloring [6]: a coloring of vertices of semi graph G such that no two end vertices of an edge are colored the same. Minimum number of colors required for e-coloring the semi graph known as e- chromatic number $\chi_e(G)$.

Definition 4: Complete graph [6]:

A semi graph G is said to be complete if any two vertices in G are adjacent. Complete graph on n vertices is denoted by K_n.

Definition 5: Clique number [7] :

A set S of vertices in a semigraph is said to form a clique if any two vertices in S are Pair wise Adjacent. The maximum cardinality of the set S is clique number .It is denoted by $\omega(G)$.

Definition 6: e-independencenumber [6]:

A set S of vertices is e-Independent if no two end vertices of an edge belong to S. Maximum cardinality of an e-independent set is e-independence number.

Now we are introducing similar concepts of external graph theory in semi graph.

According to end vertex adjacency we have introduced e-homomorphism.

Definition 1 :e- homomorphism of semi graphs: A mapping from vertex set of semi graphs G to H is said to be e-homomorphism if it preserve the end vertex adjacency of any two end vertices of G to end vertices of graph H.

Example: e-homomorphism between two graphs is shown below.

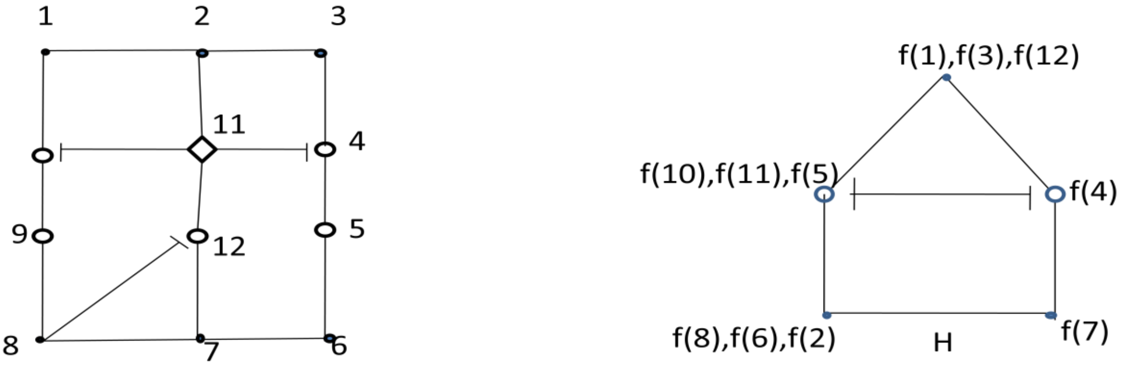

Figure 1:e-homomorphism

Definition 2: e- clique in semi graphs:

We have introduced e–clique in a semi graph is a subset S of V(G) which is a collection of e-vertices if any two vertices in set S are Pair wise adjacent. Maximum cardinality of the set S is **e-clique number**.ω_e(G)

MAIN RESULTS

Theorem 1: For any semi graph G $\chi_e(G) = n$iff there exist e-homomorphism $f: G \to K_n$, for minimum n.

Proof:

If $\chi_e(G) = n$then there exist e-homomorphism $f: G \to K_n$ for minimum n. K_n, is a complete Graph.

Let $n_1 = \{a_1, a_2, \dots \dots, a_n\}$, $n_2 = \{b_1, b_2, \dots \dots \dots, b_n\}$

.

:

:

$n_n = \{t_1, t_2, \dots \dots \dots, t_n\}$ be the e-chromatic partition of n-vertices of graph G.

Clearly, each partition is e-independent and semi edge joining two end vertices Lies in different partition. Construct a map $f: G \to K_n$ Such that each partition is fiber of different vertices of K_n.

Clearly, f is e-homomorphism from G to K_n for minimum n .

Let $f: G \to K_n$ be e-homomorphism where K_n. is a complete graph with n-vertices .

Suppose $(a_1, a_2, \dots \dots, a_n)$ are n-end vertices which are adjacent in K_n as there is e-

In e-Homomorphism from $f: G \to K_n$ there fibers are adjacent n G. So We have n-color class In G. Hence , $\chi_e(G) = n$.

Corollary: Let G and H be any two semi graphs if there is e- homomorphism from G to H then$\chi_e(G) \leq \chi_e(H)$.

Proof: We know well known result that if $\chi_e(G) = n$ then there exist e-homomorphism, $f: G \to k_n$for minimum n. Let$g: G \to H$ be e-homomorphism and let $\chi_e(H) = n$.Then there exist e-homomorphism $h: H \to K_n$ but $hg: G \to k_n$ which is e- homomorphism which implies $\chi_e(G) = n$.

Hence $\chi_e(G) \leq \chi_e(H)$.

Theorem 2: If there exist onto e-homomorphism $f: G \to H$ then $\omega_e(G) \leq \omega_e(H)$.

Proof: Let $f: G \to H$be onto e-homomorphism.

Suppose e-clique number of G is n and e-clique number of H is m .

Suppose $n > m$ Then there are n different vertices $(a_1, a_2, \dots \dots, a_n)$ in graph G which are pair wise adjacent to each other then we must get $f(a_i)$ is adjacent to $f(a_j)$ for $i \neq j$ in H. which Contradict our assumption that $n > m$.Hence$n \leq m$.i.e.$\omega_e(G) \leq \omega_e(H)$.

Theoreom3: A Mapping $f : V(G) \to V(H)$ is e-homomorphism if and only if each fiber is e-Independent set.

Proof:

Let $f: V(G) \to V(H)$ be e-homomorphism. Assume that the end vertices x and y of fiber $f^{-1}(a)$For some $a \epsilon V(H)$ are adjacent in G. But $f(x)$ and $f(y)$ are equal in H.As $f: G \to H$ is e-homomorphism $f(x) \neq f(y) \neq a$.Thus,each fiber is e-independent.

Let $f: G \to H$ is a function such that each fiber is e-independent set. Let x and y be end vertices of semi edge in G.Then x and y must be in different fiber set as each fiber is e-independent Hence $x \in f^{-1}(a)$ and $y \in f^{-1}(b)$. Hence $f(x)$ and $f(y)$ are adjacent in H.

REFERENCES

[1] Biggs N. (1993), Algebraic Graph Theory, Cambridge University.

[2] Harary F,(2000)*:Graph Theory,* Narosa Publishing House.

[3] Hedertimini S(1965): Homomorphism of graphs, Technical Report.

[4] Hell P. & Nesetrril J(2004) :Graphs and Homomorphisms, Oxford University Press.

[5] Godsil C. & Royle G, (2001): Algebraic Graph Theory, springer-verlag.

[6] Sampathkumar, E., (2000): Semi graphs and their Applications, DST Project report.

[7] West D. B, (, 2002) :Introduction to Graph theory , Prentice- Hall Inc.

[8] Wilson R. J,(2002), Introduction to Graph theory, Pearson Educationpre. Ltd.

[9] Uchat P.D., Sutaria M,(2017) An Algebraic Characteristic of Line Graph ,NEWEST International Multidisciplinary Referred Journal ,Volume1, Issue1. ISSN 2349-3372.