International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 12 (2018) pp. 10871-10877


# Portfolio of Financial Options as Currency Hedging Strategy in Colombia

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**Abstract**

The purpose of this document is to mitigate the exchange risk for investors who buy US dollars from Colombian pesos. The risk of change to which dollar-buying investors are exposed is determined by the participation in cash flows from the change in foreign currencies, but currency hedging strategies with limits reduced this risk. Coverage is made with purchase options and with a portfolio of options to determine the best coverage strategy. The Geometric Brownian Motion is applied to the modeling of the price of the currency. Monte Carlo simulation results are obtained for three months of projection where it is found that the two hedging strategies mitigate exchange risk, but the best results are the options portfolio.

**Keywords:** currency hedging, portfolio of financial options, Brownian Geometric Motion.

## INTRODUTION

The risk is the possibility of losses generated by changes in the factors that affect the value of an asset [1], [2]. Exchange rate risk is common in international trade operations and refers to uncertainties caused by changes in the exchange rate. This fluctuation in exchange rates has implications for business decisions. Therefore, it is important that multinational companies cover the exchange risk. In this way, exchange risk management in the currency options market can help avoid or control the uncertainty of the exchange rate [3].

The imperfections in the capital market are used to argue the relevance of the corporate risk management function. With corporate coverage, the value of the company is increased by reducing the volatility of the expected cash flow and making it possible to face a lower probability of default and, therefore, obtain lower bankruptcy costs and financial difficulties, without sacrificing tax advantages for debt financing. In this way, companies maintain optimal investment and financing plans to take advantage of attractive investment opportunities, using more internal financing than external financing, since the latter is more expensive due to market imperfections. For this reason, hedges increase the value of the company with the decrease in the cost of debt, a component of the discount rate of expected cash flows [4].

On the other hand, in the scientific literature there are studies that demonstrate the benefits of coverages on companies. Joseph [5] focused on UK companies identifying that hedging operations have a limited set of techniques to cover the risk. Judge [6] determined that companies with predominant sales abroad perform exchange hedges but only when there are expectations of a financial crisis. Tai [7] concluded that half of the industries and most of the banks in the United States have exposure to exchange risk. Finally, the research by Domínguez and Tesar [8] studied the exposure to foreign exchange risk and its effect on the value of the company through the analysis of exports and the multinational state of the companies.

In finance, an option is a contract that gives the owner the right to buy or sell an underlying asset at a specific price or also called strike price on or before a specific date. In contrast, the seller of this option has the obligation to comply with the transaction when the owner chooses to exercise the option. An option that gives the owner the right to buy at a strike price is known as a call option or call option; On the other hand, an option that gives the owner the right to sell at a strike price is known as a put or put option. The purchase of an option is called a long position and the sale of an option is called a short position. Generally, a call option would be exercised when the strike price is lower than the market price of the underlying asset, while a put option would be exercised when the strike price is higher than the market value. The market price of the underlying asset is also known as the spot price or spot price. When the expiration date of the option elapses without it being exercised, the buyer loses the premium or the price of the option paid to the seller of the option [9].

The options are originally designed to mitigate the risk of the change in the price of the underlying assets through the hedges. The traditional bullish hedging strategies are with call options or with a portfolio of options such as butterfly, straddle and spread. Portfolios of options are more attractive to cover than with a single option [9]. Coverage strategies with combination of options involve long positions and short positions in purchase and sale options. The strategy known as Seagull strategy involves the sale of two put options and the purchase of a call option, each option a different strike price. The choice of the combination of options depends on the option of the investor on the possible variations in the value of the underlying asset and of the protection preferences [10].

The objective of this paper is to mitigate the exchange risk from the point of view of the investor who buys US dollars from Colombian pesos (COP). Two coverage strategies are proposed: the first with call options and the second with a portfolio of options. With the Monte Carlo simulation, the price behavior is determined with coverage for 50,000 possible scenarios per month, for three months, modeled with the Geometric Brownian Motion. The following section presents the methods used to apply the two coverages, then the results and finally the conclusions.

## MATERIALS AND METHODS

Exchange coverage from the perspective of the buyer of US dollars was carried out with a portfolio of European options on the currency. The exchange risk associated with the purchase of dollars generates losses when the price of the exchange rate increases, therefore the coverage was made with the gull strategy that is satisfied with a portfolio of options where the net result provides compensation when the price of the asset underlying increases. Additionally, coverage was made with a European call option for the purpose of making comparisons and determining the best coverage strategy.

In this way, the gull strategy consists of a portfolio of three options: buy a call option with an intermediate strike price ($K_2$), sell a put option with a lower strike price ($K_1$) and sell a call option with a higher strike price ($K_3$), as shown in Figure 1. In Figure 1 the net result of the three options is shown with the red line, by the form of the net result, this strategy is known as gull strategy. To obtain this result, the three options must have the same underlying asset and maturity.

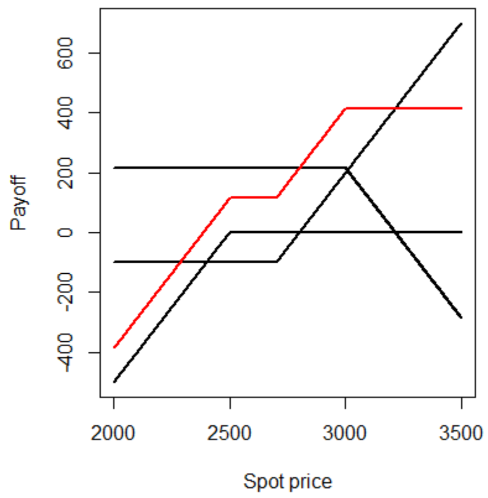


**Figure 1.** Seagull strategy.

The price with coverage with the options portfolio is obtained by means of equation 1, where the first component represents the purchases to the spot market on the due date ($S_t$), for this reason it is negative, after the three compensations of the options and finally the premiums where the call option premium is paid with $K_2$ ($c_{K_2}$) and the premiums are received from the other two options ($p_{K_1}$ and $p_{K_3}$). As the result of the above gives a negative number, then the price with coverage is the absolute value.

$$Hedge = abs\left(-S_t + \max(S_t - K_2; 0) + min(S_t - K_1; 0) + \min(K_3 - S_t; 0) - c_{K_2} + p_{K_1} + p_{K_3}\right) \quad [1]$$

On the other hand, the coverage with the call option is similar to the coverage with the options portfolio, changes that equation 2 is shorter because only the procedure with an option is performed. Equation 1 shows the calculation of the price with coverage with a call option with a strike price equal to $K_2$.

$$Hedge = abs\left(-S_T + \text{máx}(S_T - K_2; 0) - c_{K_2}\right) \quad [2]$$

To determine the benefits of the coverage with the options portfolio in multiple scenarios, the spot price was modeled with the Geometric Brownian Motion. The average of the 50,000 spot prices corresponds to the expected price in the scenario without coverage, in addition, with each of these prices the prices with coverage with the equations were calculated. The averages of the prices with coverage represent the prices with expected coverage where two prices would be obtained for each month, one for the coverage with the options portfolio and another one for the coverage with the call option. Finally, since the options are European, the Black-Scholes method was used to evaluate the options.

### Geometrical Brownian Motion

The geometric, exponential or Brownian process is defined by the stochastic differential equation (see equation 3).

$$dS(t) = \mu S(t) + \sigma S(t) dW(t) \quad [3]$$

Where S(t) is the value of the process at time t, μ is the drift, σ is the volatility and dW(t) is a Standard Brownian Motion. S(t) is also called Exponential Brownian Motion, since S(t) takes the exponential form of dW(t) [11]. Knowing S(0) at time t = 0, the solution to the equation is given by equation 4 [12].

$$S(t) = S(0)e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma d(t)} \quad [4]$$

### Black-Scholes Model

Merton and Scholes won the Nobel Prize in Economics in 1955 for the model commonly called Black-Scholes, the model was published in 1973. This work had a great impact on the financial market and among academics when establishing the pricing principles of options It led to the development of hedging strategies promising the assured growth of financial assets. The Black-Scholes model is a mathematical model that allows similar the price of financial assets and determine the fair prices of certain financial derivatives such as European options [13], [14]. The model is based on the following eight assumptions [15].

1. The price of the asset behaves like a geometric Brownian Motion. Where μ is the average of the yields or also called drift, σ is the standard deviation of the price returns.
2. The risk-free interest rate is a constant during the due date.
3. Investors can borrow and lend at the risk-free rate.
4. The asset does not pay dividends during the due date.
5. The stock market does not present frictions, where there are no taxes and no transaction costs.
6. There are no risk-free arbitrage opportunities in the market.
7. The assets are in constant negotiation.
8. The option is of a European type, which is not enforceable until the expiration date.

On the expiration date T, the value of the options is given by the compensation of equation 5 for purchase options and by equation 6 for put options.

$$Call\ payoff = Max(S_t - K; 0) \quad [5]$$

$$Put\ payoff = Max(K - S_t; 0) \quad [6]$$

According to the Black-Scholes model, the European call price is given by equation 7 [14].

$$call = S_0 N(d_+) - Ke^{-rt} N(d_-) \quad [7]$$

Where

$$d_{\pm} = \frac{log\left(\frac{S_0}{K}\right) \pm \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}} \quad [8]$$

In equation 8, N() is the Gaussian probability distribution function with zero mean and variance equal to one.

The price of the European put option is obtained by the put-call parity that appears in equation 9 [14].

$$put = call + Ke^{-rt} - S_0 \quad [9]$$

The price of the put option can also be found with equation 10.

$$put = Ke^{-rt} N(-d_-) - S_0 N(-d_+) \quad [10]$$

For currency options, the Black-Scholes model has the following variation by entering the foreign risk-free rate ($r_f$) (see equations 11 and 12).

$$call = S_0 e^{-r_f t} N(d_+) - Ke^{-rt} N(d_-) \quad [11]$$

$$put = Ke^{-rt} N(-d_-) - S_0 e^{-r_f t} N(-d_+) \quad [12]$$

The premium prices for the hedges were calculated with equations 11 and 12.

After modeling the spot price and calculating the option premiums, a Monte Carlo simulation was performed with 50,000 iterations to determine the possible price results with coverage with the options portfolio and with the call option. The modeling and simulation were carried out for three months, calculating in each month the prices without coverage corresponding to the expected spot price and the two prices with coverage. To determine the benefits of coverage with options, the percentiles of 5% and 95% and the standard deviation for both the price without coverage and the prices with coverage were calculated for each month of projection. It is expected that with the coverage the distance between the percentiles decreases and in turn the standard deviation. If the above occurs, it would show that the risk is reduced with coverage, in this case the exchange risk, because the volatility of the projected scenarios would be lower.

On the other hand, monthly historical data of the exchange rate for five years from April 2013 to March 2018 were used, obtaining 60 data. Because the models used are continuous, logarithmic performances were calculated. With these yields the average represents the drift (μ) and the standard deviation represents σ for the Geometric Brownian Motion. For the risk free rates of Colombia (r), the interbank rates (IBR) valid for one and three months were used, the same was done for the free rates of the United States ($r_f$), the current US Treasury rates were used to one and three months. The free risk rates valid for two months were found using the Bootstrap method, this is a method that assumes linearity between two rates, between the current rate for a month and the current rate for three months, the rate for the second month is multiplying both rates by 0.5 and adding the two results. Then, the rates were converted to continuous time and divided by 12 to have them in the same units as μ and σ.

## RESULTS AND DISCUSSIONS

The modeling of the price of the exchange rate was made with the Geometric Brownian Motion, being a continuous model, the input data must be continuous. For this reason, the average of the logarithmic or drift yields of the 60 months resulted in 0.710% and the standard deviation equal to 4.199%. The free local (r) and foreign risk rates ($r_f$) are shown in table 1, where the rates with validity of one and three months are known real rates and the rates valid for two months were found by the Bootstrap method. Table 1 shows the risk-free rates continuous compound monthly (C.C.M).

**Table 1.** Risk-free rates.

| | **1 month** | **2 months** | **3 months** |
|---|---|---|---|
| r [C.C.M] | 0.363% | 0.358% | 0.353% |
| $r_f$ [C.C.M] | 0.014% | 0.014% | 0.014% |

With the above data, drift, standard deviation and risk-free rates, the price of the exchange rate was modeled for the next three months. This model was applied a Monte Carlo simulation with 50,000 iterations for each month where the expected price was found and the percentiles of 5% and 95%. Figure 2 shows the results of the exchange rate that represents the behavior of the price without coverage for the buyer of US dollars. In Figure 2, month zero corresponds to the last known real price of the time series which is COP 2,780.47. According to the results, it is expected that the price of the currency will remain stable without variation in the following three months; however, the percentiles of 5% and 95% show that as time progresses, the possible results are more volatile, this is evidenced because the distance between the percentiles increases. The foregoing indicates that although the price is expected to remain unchanged, there are scenarios with a 5% confidence that the dollars that will be purchased will be more expensive. This indicates the exchange risk presented by the buyers of the US dollars. The scenario at risk corresponds to the 95% percentile where in the three-month forecast an upward trend is expected, in other words, with a confidence of 5%, it is expected that the dollars bought in the spot market will be more expensive.

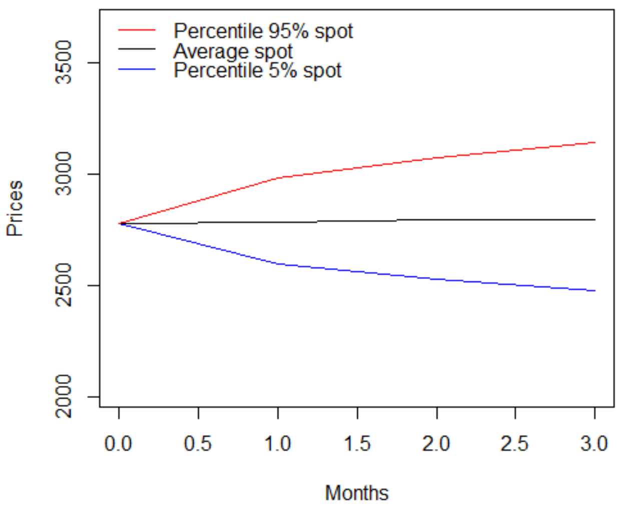


**Figure 2.** Price behavior without coverage.

On the other hand, Table 2 shows the standard deviations for each month of the price without coverage. It is observed that as the months pass the risk increases, this is also proven by the increase in the distance between the percentiles.

**Table 2.** Standard deviations price without coverage.

| | **1 month** | **2 months** | **3 months** |
|---|---|---|---|
| Price without coverage | COP 116.1763 | COP 166.3116 | COP 202.7968 |

Consequently, as there is exchange risk for the buyers of the currency, the hedges take on value to mitigate this risk. To mitigate the exchange risk, it is proposed to evaluate two hedges, the first is through a European call option for each month and the second one, covering each month with a portfolio of European options.

**Coverage with call option**

To cover three months three strike prices are needed, these were used equal to the average plus one standard deviation of the prices without coverage of each month. These strike prices are shown in table 3. If the spot price closes above the strike price in each projection month, the call options would give compensation equal to the difference between the spot price and the strike price, otherwise, no There would be compensation because the buyer of the option would prefer to buy the currency at a lower price, which means that the prices with coverage will stabilize.

**Table 3.** Prices strikes for coverage with call options.

| | **1 month** | **2 months** | **3 months** |
|---|---|---|---|
| Prices strikes | COP 2,902.153 | COP 2,959.877 | COP 3,000.292 |

The premiums of the call options increase each time the maturity increases, this is shown in table 4. These values correspond to the value that the buyer of the currency must pay to be able to hedge the exchange risk with financial options. The payment of premiums ensures compensation when the spot price is bullish above strike prices. The premiums were calculated by the Black-Scholes method.

**Table 4.** Premium call options.

| | **1 month** | **2 months** | **3 months** |
|---|---|---|---|
| Premium call options | COP 10.5683 | COP 14.6839 | COP 18.7499 |

Using simulated unhedged prices and applying call options for each month of projection, in other words, applying equation 2, you get the price with coverage with call options. Figure 3 shows the results for this coverage, the expected price as well as the price without coverage do not present variations; however, the expected price is closer to the 95% percentile.

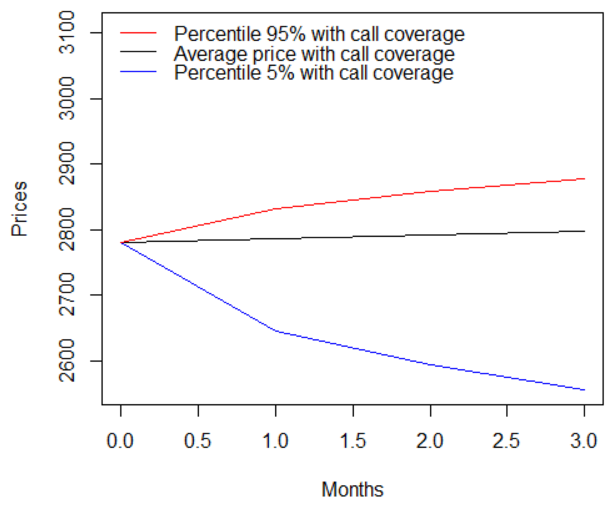


**Figure 3.** Prices with coverage with call options

To determine whether the exchange risk is mitigated with the coverage with the options, figure 4 shows a comparison between figures 2 and 3. The expected prices of the scenarios without coverage and the scenario with call coverage are similar, but the distance between the percentiles with coverage is lower in each month. The result shows a lower volatility of the price resulting from the purchase of the dollars if the coverage is made with call options. In addition, with a confidence of 5%, the purchase of the dollars with coverage would be at lower prices than without coverage.

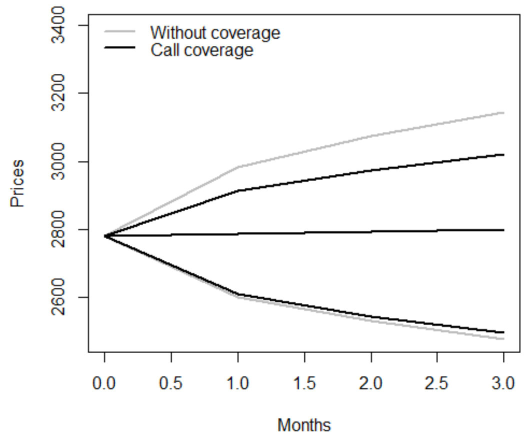


**Figure 4.** Prices without coverage and with call coverage.

The standard deviation for each month with the call coverage also indicates that the exchange risk is mitigated because the result is lower, although it increases over time, but it is always lower than the non-hedged scenario, on average a 15% decrease in the standard deviation. This is observed in table 5.

**Table 5.** Standard deviation prices with call coverage.

| | **1 month** | **2 months** | **3 months** |
|---|---|---|---|
| Call coverage | COP 99.5753 | COP 141.2579 | COP 171.4370 |

**Coverage with options portfolio**

The portfolio of options that was used to make the coverage needs three different strikes prices where one has to be in the middle of the other two. The intermediate strike price was used equal to the strike price of the call coverage, that is, $K_2$ is equal to the average plus one standard deviation of the prices without coverage, $K_1$ corresponds to the average and $K_3$ to the average plus two standard deviations. Table 6 shows these prices strikes. As the prices without coverage are more dispersed over time, the prices strikes are higher.

**Table 6.** Prices strikes for coverage with the options portfolio.

| | **1 month** | **2 months** | **3 months** |
|---|---|---|---|
| $K_1$ | COP 2,785.983 | COP 2,793.566 | COP 2,979.749 |
| $K_2$ | COP 2,902.153 | COP 2,959.877 | COP 3,000.292 |
| $K_3$ | COP 3,018.335 | COP 3,126.189 | COP 3,203.089 |

With the previous strike prices, the premiums of the options portfolio were calculated, consisting of selling a put option with a lower strike price ($K_1$), buying a call option with an intermediate strike price ($K_2$) and selling a call option with the highest strike price ($K_3$). In this way, the premiums calculated by the Black-Scholes method are found in table 7, as well as the result of selling two options and buying one. The positive sign in the premiums represents the income from the sale of the options and the negative sign the income from the purchase of the options, the total indicates that with the portfolio of options money is received, this by selling two options and buying one. This is initially an advantage over call coverage as you do not have to pay but receive money from the start of coverage.

**Table 7.** Premium portfolio of options.

| | **1 month** | **2 months** | **3 months** |
|---|---|---|---|
| Put premium $K_1$ | COP 46.1072 | COP 66.1423 | COP 80.0146 |
| Call premium $K_2$ | COP – 10.5683 | COP – 14.6839 | COP – 18.7499 |
| Call premium $K_3$ | COP 1.3382 | COP 1.9257 | COP 2.6482 |
| **Total** | **COP 36.8771** | **COP 53.3840** | **COP 63.9129** |

The price with coverage with the portfolio of is using equation 1 to each of the simulated scenarios, the result of the average and the percentiles of 5% and 95% are observed in figure 5. The prices with expected coverage are more close to the 5% percentile, indicating that there is a greater probability that the prices of the purchase of the dollars after making coverage would be lower. This is different from call coverage where prices tend to the 95th percentile with higher prices.

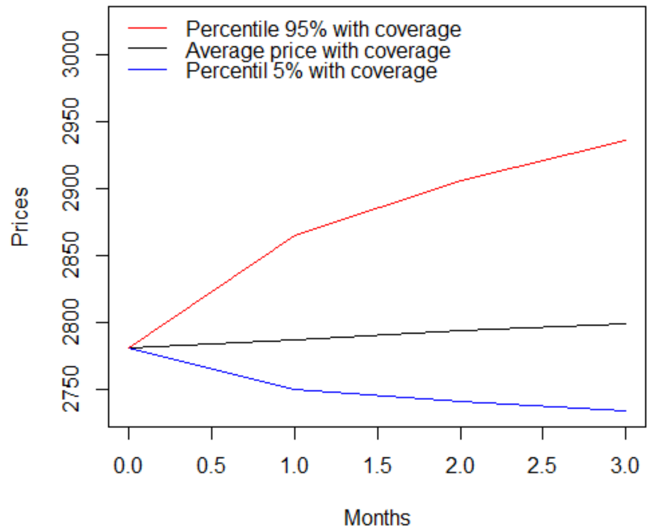


**Figure 5.** Prices with coverage with options portfolio.

The scenarios of the purchase of dollars with this coverage have lower volatility because the standard deviation is lower than the prices without coverage and call coverage. Table 8 shows these results where on average the standard deviation is lower than the uncovered scenario at 57% and with the scenario with call coverage at 50%. Therefore, it is better to make the coverage with the options portfolio than with just call options.

**Table 8.** Standard deviations prices with coverage with options portfolio.

| | **1 month** | **2 months** | **3 months** |
|---|---|---|---|
| Coverage with options portfolio | COP 49.2498 | COP 71.2903 | COP 87.1617 |

Likewise, Figure 6 shows a comparison of the three scenarios: no coverage, call coverage and coverage options portfolio. The best scenarios are obtained using the options portfolio because it has the least distance between the percentiles and it is emphasized that the average price of the three scenarios is the same, where the same result of the purchase of the dollars is expected, but with a 5% confidence if they have less harmful scenarios with the options portfolio.

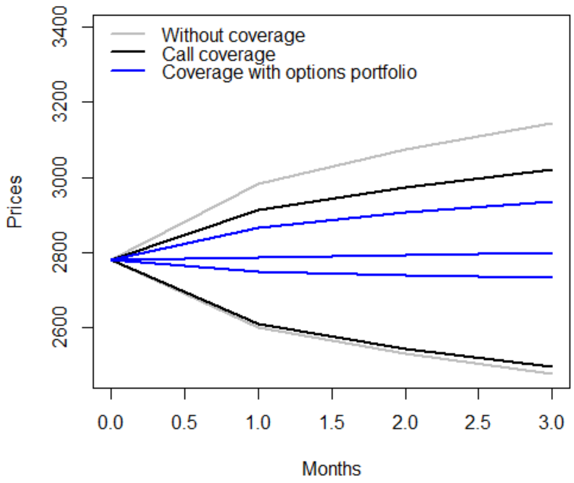


**Figure 6.** Comparison of the three scenarios.

## CONCLUSIONS

This document uses the financial options on currencies to mitigate the exchange rate risk for a buyer of the currency. Exposure to fluctuations in the exchange rate is known as currency risk, in the case of buyers, they have direct and negative effects when the price of the currency increases. Through financial options, the buyer can reduce this effect. Two alternatives of coverage are proposed: the first with purchase option and the second with a portfolio of options. The coverages were applied for an investor who buys US dollars paying with Colombian pesos (COP).

To determine the effect of hedges on the purchase price after hedging transactions, the price is determined with coverage for three months from the modeling of the currency price with the Geometric Brownian Motion and Monte Carlo simulation. The options used are European options, so the valuation is calculated by the Black-Scholes method.

According to the modeling and simulation of the spot price of the currency, the 95% percentile shows that the investor will buy US dollars at a higher price over time with a confidence of 5%. The foregoing demonstrates the presence of foreign exchange risk so that a way to mitigate it takes value.

The findings show that with the two alternatives of hedging the exchange risk is mitigated because the standard deviation of the price with coverage decreases, although, with the coverage with the options portfolio, better results are obtained. The average prices expected are equal without coverage and with the two coverages, but the 95% percentile with call coverage is lower and even more so with the coverage with the options portfolio.

With coverage with only purchase options you must always pay the premium value to be able to make the coverage; however, the options portfolio is made up of the sale of two options and the purchase of an option granting a greater benefit when generating an initial income for the premiums.

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