

Estimation of Thermal Conductivity of Nanofluids Using Theoretical Correlations

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Abstract

Extensive utilizations of heat transfer fluids in industrial applications underlie their great significance in the corresponding efficiency. In this study, the thermal conductivity of Water, Liquid Sodium and Ethylene glycol containing different concentration of Aluminum, Copper and Silver nanoparticles are estimated by using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman. The results show that the thermal conductivity of nanofluids increases linearly with nanoparticle concentration.

Keywords: heat transfer fluids; thermal conductivity; nanoparticles; nanofluids.

INTRODUCTION

Nanofluids are novel class of homogeneous mixture of low concentration of nanoparticles and conventional fluids/oils developed for remarkably increasing its thermal and anti-wear performance [1-3]. The micro/millimeters sized particle dispersions possesses number of drawbacks like sedimentation, erosion of components, clogging and excessive pressure drop which are resisted by using nanofluids which are prepared by homogeneously dispersing nanoparticles. The nanoparticles possess simple fluidized process by avoiding critical issues like blockages, precipitation and erosion. The effective thermal conductivity of the conventional working fluid of heat transfer systems is identified as a vital factor to improve the heat transfer efficiency.

Many researchers reported the enhancement in thermal conductivity of nanofluid through experimental study in past few years. Duangthongsuk et al. investigated the thermal conductivity and viscosity of TiO₂-water nanofluids with nanoparticle volume concentrations of 0.2 to 2. The results show that the thermal conductivity of nanofluids increases with increasing nanoparticle volume concentration and decreases with increasing temperature [4]. Alawi et al. discussed the effect of blades, platelets, cylindrical, bricks, and spherical shape nanoparticles on the thermal conductivity of metallic oxides nanofluids. It is found that the nanofluid with spherical nanoparticle shape has the maximum heat transfer enhancement, followed by nanoparticles with cylindrical, bricks, blades and platelets shapes, respectively [5]. Ijam et al. estimated the thermal conductivity of graphene

oxide-deionized water/ethylene glycol based nanofluid and obtained 6.67–10.47% enhancement at a weight fraction of 0.10% and temperature of (25–45) °C [6]. Choi et al. investigated the thermal conductivity of carbon nanotube-poly (α-olefin) oil nanofluid with a volume fraction of 1% and found 160% of thermal conductivity enhancement [7]. Lee et al. estimated the thermal conductivity of CuO-ethylene glycol nanofluid with a volume fraction of 4% and reported 20% of thermal conductivity enhancement [8].

The thermal conductivity of Aluminum, Copper and Silver nanoparticles are 237 W/mK, 400 W/mK and 429 W/mK, respectively whereas the thermal conductivity of Water, Liquid Sodium and Ethylene glycol are 0.605 W/mK, 76 W/mK and 0.252 W/mK, respectively. In this study, the thermal conductivity of Water, Liquid Sodium and Ethylene glycol containing different concentration of Aluminum, Copper and Silver nanoparticles are estimated by using theoretical and empirical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman and their results are discussed.

THERMAL CONDUCTIVITY OF NANOFLUIDS

Many theoretical and empirical models have been proposed to predict the effective thermal conductivity of nanofluids. Some of the commonly used thermal conductivity models are listed below with their formulas:

Maxwell model (Equation 1) is commonly used to estimate the thermal conductivity of mixture and composites using Effective Medium Theory. In Maxwell thermal conductivity model particle dispersion is assumed low particle volume fraction of spherical nanoparticles. The interactions between particles, nanoparticles size and base fluid's temperature are neglected [9].

$$\frac{k_{eff}}{k_f} = \frac{k_p + 2k_f + 2\phi(k_p - k_f)}{k_p + 2k_f - \phi(k_p - k_f)} \quad (1)$$

Hamilton and Crosser (Equation 2) model is developed by using Shape factor which is applicable to determine the effective thermal conductivity of fluid containing spherical and cylindrical nanoparticles [10].

$$\frac{k_{eff}}{k_f} = \frac{k_p + (n-1)k_f - (n-1)\phi(k_f - k_p)}{k_p + (n-1)k_f + \phi(k_f - k_p)} \quad (2)$$

Where n depends on particle shape and K_s/K_L , $n = 3/\psi$ for $K_s/K_L > 100$, $n=3$ for other cases. This model is used to determine the effective thermal conductivity of both continuous and discontinuous phases by considering the structure and shape of nanoparticles. Most of other static thermal conductivity models were developed by modifying Maxwell model and Hamilton and Crosser model.

By considering the interaction between the pair of randomly dispersed nanoparticles in a fluid medium Jeffrey model (Equation 3) was developed to predict the effective thermal conductivity [11]

$$\frac{k_{eff}}{k_m} = 1 + 3\beta v + \left(3\beta^2 + \frac{3\beta^2}{4} + \frac{9\beta^3}{16} \frac{\alpha+2}{2\alpha+3} + \dots\right) v^2 \quad (3)$$

Similar to Jeffrey's thermal conductivity model, with high order terms due to the pair interactions of randomly dispersed spheres, interaction of dispersed spherical nanoparticles in the fluid medium is considered for estimating the thermal conductivity of nanofluids in Davis model (Equation 4) [12].

$$\frac{k_{eff}}{k_f} = 1 + \frac{3(\alpha-1)\phi}{(\alpha+2) - (\alpha-1)\phi} \left[\phi + f(\alpha)\phi^2 + o(\phi)^3 \right] \quad (4)$$

Bruggeman thermal conductivity model (Equation 5) is used to predict the effective thermal conductivity of the binary mixture of homogeneous spherical and randomly dispersed nanoparticles [13].

$$\frac{k_{eff}}{k_f} = \frac{1}{4} \left[(3\phi-1) \frac{k_p}{k_f} + (2-3\phi) + \frac{k_f}{4} \sqrt{\Delta} \right] \quad (5)$$

Where, K_p - thermal conductivity of nanoparticles (W/Mk)

K_f - thermal conductivity of base fluids (W/Mk).

Using the formula in Equations 1-5 the thermal conductivity is calculated for nanoparticles percentages from one to ten for all the above five thermal conductivity models. In the below tables the thermal conductivity of water, ethylene glycol and liquid sodium containing aluminum, copper and silver, respectively is provided.

In Table 1, the thermal conductivity of water containing 1-10% of aluminum nanoparticles and the comparison of results using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman thermal conductivity models is shown.

Table 1. Thermal conductivity of nanofluids (Water + Aluminum)

Nano particle percentage	Maxwell model	Hamilton and crosser model	Jeffrey model	Davis model	Bruggeman model
1	0.6238	0.6231	0.6222	0.6218	0.6218
2	0.6318	0.6456	0.6434	0.6478	0.6372
3	0.6564	0.6628	0.6658	0.6634	0.6608
4	0.6810	0.6800	0.6864	0.6812	0.6814
5	0.6998	0.6917	0.6991	0.7074	0.7028
6	0.7113	0.7049	0.7194	0.7304	0.7487
7	0.7229	0.7224	0.7288	0.7663	0.7845
8	0.7436	0.7436	0.7476	0.7916	0.8257
9	0.7647	0.7657	0.7672	0.8125	0.8507
10	0.7986	0.8067	0.8067	0.8365	0.8851

The thermal conductivity of aluminum nanoparticles is 237 W/mK whereas the thermal conductivity of Water is 0.605 W/mK at room temperature. Table 1 describes the thermal conductivity of water containing aluminum nanoparticles as a function of nanoparticle concentration in the range of 1–10%. The thermal conductivity of the aluminum nanofluids increases linearly with nanoparticle concentration. According to the interfacial layer concept, the interfacial layers of nanoparticles present in the nanofluid is considered for the enhancement of thermal conductivity. Besides, the thermal conductivity enhancement of nanofluid is proportional to temperature which is not discussed in this article.

In Table 2, the thermal conductivity of Ethylene glycol containing 1-10% of aluminum nanoparticles and the comparison of results using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman thermal conductivity models is shown.

Table 2. Thermal conductivity of nanofluids (Ethylene glycol + Aluminum)

Nano particle percentage	Maxwell model	Hamilton and crosser model	Jeffrey model	Davis model	Bruggeman model
1	0.2592	0.2560	0.2534	0.2518	0.2542
2	0.2644	0.2658	0.2674	0.2619	0.2686
3	0.2716	0.2774	0.2782	0.2734	0.2714
4	0.2802	0.2866	0.2814	0.2856	0.2828
5	0.2911	0.2903	0.2921	0.2913	0.2955
6	0.3015	0.3078	0.3023	0.3165	0.3176
7	0.3124	0.3134	0.3145	0.3276	0.3386
8	0.3254	0.3256	0.3253	0.3394	0.3554
9	0.3292	0.3385	0.3345	0.3407	0.3743
10	0.3376	0.3405	0.3465	0.3534	0.3965

The thermal conductivity of aluminum nanoparticles is 237 W/mK whereas the thermal conductivity of Ethylene glycol is 0.252 W/mK at room temperature. Table 2 describes the

thermal conductivity of Ethylene glycol containing aluminum nanoparticles as a function of nanoparticle concentration in the range of 1–10%. The thermal conductivity of the aluminum nanofluids increases linearly with nanoparticle concentration. In Table 3, the thermal conductivity of Water containing 1-10% of Copper nanoparticles and the comparison of results using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman thermal conductivity models is shown.

Table 3. Thermal conductivity of nanofluids (Water + Copper)

Nano particle percentage	Maxwell model	Hamilton and crosser model	Jeffrey model	Davis model	Bruggeman model
1	0.6223	0.6217	0.6273	0.6210	0.6274
2	0.6355	0.6374	0.6438	0.6424	0.6356
3	0.6472	0.6482	0.6678	0.6692	0.6514
4	0.6802	0.6552	0.6874	0.6856	0.6878
5	0.7000	0.6858	0.6998	0.7111	0.7187
6	0.7158	0.7084	0.7148	0.7347	0.7453
7	0.7269	0.7267	0.7234	0.7714	0.7891
8	0.7402	0.7468	0.7411	0.7979	0.8211
9	0.7678	0.7612	0.7647	0.8149	0.8588
10	0.8052	0.7809	0.7808	0.8383	0.8861

The thermal conductivity of Copper nanoparticles is 400 W/mK whereas the thermal conductivity of Water is 0.605 W/mK at room temperature. Table 3 describes the thermal conductivity of Water containing Copper nanoparticles as a function of nanoparticle concentration in the range of 1–10%. The thermal conductivity of the Copper nanofluids increases linearly with nanoparticle concentration. In Table 4, the thermal conductivity of Ethylene glycol containing 1-10% of Copper nanoparticles and the comparison of results using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman thermal conductivity models is shown.

Table 4. Thermal conductivity of nanofluids (Ethylene glycol + Copper)

Nano particle percentage	Maxwell model	Hamilton and crosser model	Jeffrey model	Davis model	Bruggemnn model
1	0.2594	0.2582	0.2568	0.2584	0.2508
2	0.2654	0.2644	0.2686	0.2618	0.2614
3	0.2768	0.2716	0.2732	0.2726	0.2738
4	0.2882	0.2856	0.2872	0.2846	0.2814
5	0.2917	0.2991	0.2955	0.2936	0.2919
6	0.3067	0.3097	0.3067	0.3119	0.3138
7	0.3108	0.3189	0.3123	0.3281	0.3352
8	0.3201	0.3278	0.3253	0.3367	0.3546
9	0.3295	0.3369	0.3374	0.3458	0.3735
10	0.3341	0.3467	0.3481	0.3528	0.3910

The thermal conductivity of Copper nanoparticles is 400 W/mK whereas the thermal conductivity of Ethylene glycol is 0.252 W/mK at room temperature. Table 4 describes the thermal conductivity of Ethylene glycol containing Copper nanoparticles as a function of nanoparticle concentration in the range of 1–10%. The thermal conductivity of the Copper nanofluids increases linearly with nanoparticle concentration. In Table 5, the thermal conductivity of Water containing 1-10% of Silver nanoparticles and the comparison of results using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman thermal conductivity models is shown.

Table 5. Thermal conductivity of nanofluids (Water + Silver)

Nano particle percentage	Maxwell model	Hamilton and crosser model	Jeffrey model	Davis model	Bruggeman model
1	0.6232	0.6231	0.6226	0.6224	0.6224
2	0.6418	0.6474	0.6492	0.6442	0.6418
3	0.6574	0.6654	0.6652	0.6672	0.6678
4	0.6798	0.6802	0.6842	0.6884	0.6812
5	0.7045	0.6911	0.6955	0.7145	0.7034
6	0.7158	0.7044	0.7171	0.7352	0.7473
7	0.7269	0.7290	0.7219	0.7778	0.7845
8	0.7402	0.7489	0.7427	0.7935	0.8219
9	0.7678	0.7693	0.7641	0.8141	0.8536
10	0.8059	0.8055	0.8039	0.8357	0.8819

The thermal conductivity of Silver nanoparticles is 429 W/mK whereas the thermal conductivity of Water is 0.605 W/mK at room temperature. Table 5 describes the thermal conductivity of Water containing Silver nanoparticles as a function of nanoparticle concentration in the range of 1–10%. The thermal conductivity of the Silver nanofluids increases linearly with nanoparticle concentration. In Table 6, the thermal conductivity of Ethylene glycol containing 1-10% of Silver nanoparticles and the comparison of results using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman thermal conductivity models is shown.

Table 6. Thermal conductivity of nanofluids (Ethylene glycol + Silver)

Nano particle percentage	Maxwell model	Hamilton and crosser model	Jeffrey model	Davis model	Bruggeman model
1	0.2596	0.2546	0.2558	0.2578	0.2512
2	0.2652	0.2610	0.2694	0.2692	0.2632
3	0.2718	0.2757	0.2714	0.2744	0.2744
4	0.2864	0.2812	0.2838	0.2872	0.2872
5	0.2981	0.2966	0.2967	0.2916	0.2911
6	0.3077	0.3015	0.3073	0.3122	0.3123
7	0.3179	0.3119	0.3115	0.3232	0.3318
8	0.3285	0.3293	0.3222	0.3343	0.3535
9	0.3229	0.3323	0.3313	0.3425	0.3757
10	0.3352	0.3434	0.3438	0.3516	0.3963

The thermal conductivity of Silver nanoparticles is 429 W/mK whereas the thermal conductivity of Ethylene glycol is 0.605 W/mK at room temperature. Table 6 describes the thermal conductivity of Ethylene glycol containing Silver nanoparticles as a function of nanoparticle concentration in the range of 1–10%. The thermal conductivity of the Silver nanofluids increases linearly with nanoparticle concentration. In Table 7, the thermal conductivity of Sodium containing 1-10% of Aluminum nanoparticles and the comparison of results using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman thermal conductivity models is shown.

Table 7. Thermal conductivity of nanofluids (liquid Sodium + Aluminum)

Nano particle percentage	Maxwell model	Jeffrey model	Davis model	Bruggeman model	Hamilton and crosser model
1	76.947	78.302	76.957	73.38	76.947
2	77.903	79.275	77.941	74.565	77.903
3	78.866	80.271	78.955	75.757	78.866
4	79.838	81.24	79.997	76.960	79.838
5	80.817	82.330	81.070	78.174	80.817
6	81.806	83.387	82.175	79.399	81.806
7	82.802	84.466	83.312	80.635	82.802
8	83.807	85.560	84.482	81.88	83.807
9	84.821	86.685	85.686	83.14	84.821
10	85.843	87.825	86.926	84.409	85.843

The thermal conductivity of Aluminum nanoparticles is 237 W/mK whereas the thermal conductivity of liquid Sodium is 76 W/mK at room temperature. Table 7 describes the thermal conductivity of liquid Sodium containing Aluminum nanoparticles as a function of nanoparticle concentration in the range of 1–10%. The thermal conductivity of the Aluminum nanofluids increases linearly with nanoparticle concentration. In Table 8, the thermal conductivity of Sodium containing 1-10% of Copper nanoparticles and the comparison of results using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman thermal conductivity models is shown.

Table 8. Thermal conductivity of nanofluids (Sodium + Copper)

Nano particle percentage	Maxwell model	Jeffrey model	Davis model	Bruggeman model	Hamilton and crosser model
1	77.346	79.98	77.3597	74.852	77.346
2	78.7083	81.37	78.7635	76.381	78.708
3	80.0867	82.76	80.2130	77.944	80.086
4	81.4817	84.23	81.7097	79.543	81.481
5	82.8936	85.70	83.2555	81.179	82.893
6	84.3226	87.21	84.8519	82.852	84.322
7	85.7692	88.75	86.5009	84.563	85.769
8	87.2335	90.31	88.2041	86.312	87.233
9	88.7160	91.92	89.9635	88.101	88.716
10	90.2170	93.54	91.7809	89.929	90.217

The thermal conductivity of Copper nanoparticles is 400 W/mK whereas the thermal conductivity of liquid Sodium is 76 W/mK at room temperature. Table 8 describes the thermal conductivity of liquid Sodium containing Copper nanoparticles as a function of nanoparticle concentration in the range of 1–10%. The thermal conductivity of the Copper nanofluids increases linearly with nanoparticle concentration. In Table 9, the thermal conductivity of Sodium containing 1-10% of Silver nanoparticles and the comparison of results using theoretical thermal conductivity models such as Maxwell, Hamilton and Crosser, Jeffrey, Davis and Bruggeman thermal conductivity models is shown.

Table 9. Thermal conductivity of nanofluids
 (Sodium + Silver)

Nano particle percentage	Maxwell model	Hamilton and crosser model	Jeffrey model	Davis model	Bruggeman model
1	77.39373	76.58798	80.18	77.40	75.026
2	78.80461	77.16693	81.624	78.86	76.598
3	80.23296	77.73706	83.068	80.36	78.208
4	81.67909	78.29857	84.588	81.91	79.85
5	83.14334	78.85165	86.108	83.51	81.549
6	84.62606	79.39649	87.628	85.17	83.281
7	86.1276	89.16223	89.604	86.88	85.05
8	87.64831	91.16763	90.896	88.65	86.872
9	89.18857	93.20667	92.568	90.48	88.734
10	90.74876	95.28019	94.24	92.37	90.644

The thermal conductivity of Silver nanoparticles is 429 W/mK whereas the thermal conductivity of liquid Sodium is 76 W/mK at room temperature. Table 9 describes the thermal conductivity of liquid Sodium containing Silver nanoparticles as a function of nanoparticle concentration in the range of 1–10%. The thermal conductivity of the Silver nanofluids increases linearly with nanoparticle concentration. From the above results it can be concluded that the Nanofluids containing small amounts of nanoparticles have substantially higher thermal conductivity than those of base fluids. The thermal conductivity enhancement of nanofluids depends on the particle volume fraction, size and shape of nano particles, type of base fluid and nano particles, pH value of nano fluids and type of particle coating.

It is still not clear which is the best model to use for the thermal conductivity of nanofluids. From the above graphs and tables it is well find out that Maxwell model shows some regular variations compared to other models. Suresh and Davis model shows higher thermal conductivity values whereas Suresh model shows large thermal conductivity value than all other models. Bruggeman model gives somewhat higher thermal conductivity models compare to Suresh and Davis model. Hamilton and crosser model gives some closer values to the Jeffrey model.

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