

A New Edge Detection Method for Digital Images based on Interval Arithmetic and Fuzzy Logic

Hassan B. M. El-Owny^{1,2}, Yasser A. Nada¹

¹ Computer science department, Taif University, Taif, KSA.

² Asswan University, Asswan, Egypt.

Abstract

In this paper, we introduce a new method based on model the grayscale values by grayscale intervals, where the actual grayscale values bounded by grayscale intervals. Then we modify the Prewitt method for edge detection to the new situation of the image, this representation leads to the edges in the form of an interval valued fuzzy set. The performance and the applicability of the proposed method are introduced, and compared with classical methods on grayscale images.

Keywords: edge detection, interval arithmetic, fuzzy theory

INTRODUCTION

The value of a pixel in a gray-level image indicates the amount of white or black present at that specified position in the image. However, one always assumes that these values are certain, although in practice the measured values might be uncertain. The uncertainty regarding the value is an immediate fact if one takes into account that any device will round captured values up or down to the finite set of allowed values. Hence, we can never have a full certainty about the tone (intensity) of a pixel. In edge detection field, the problems arising out of this uncertainty appear themselves clearly, since occasionally not even two human can reach an agreement on where the boundary between two objects is.

Most image processing tasks requires us to find an algorithm which approximate a true image as closely as possible. This process approximates some real situation, and carries some uncertainty [1, 2]. The uncertainty grows if several takes of an image reveal different values for some pixels. This might be the case under identical recording circumstances, and will surely arise when these circumstances change (e.g., due to weather conditions). In order to take this uncertainty into account in such a manner that it is incorporated in the image model and can be processed together with the image, new image models are required [3]. Examples are image reconstruction (registration, tomography reconstruction, deblurring), encoding (discretization, compression, hierarchical representation) and low-level scene analysis (edge detection, segmentation, classification). It can be useful to work with grayscale intervals instead of grayscale values, where the interval represents the set to which the actual grayscale value belongs [3].

The goal of this work is to model the grayscale values by grayscale intervals, where the interval represents the set to which the actual grayscale value belongs. Then adapt the Prewitt method for edge detection to the new condition of the image, this representation leads to the edges in the form of an interval-valued and fuzzy set. The proposed method test on

several images and also compares with conventional edge detectors such as Prewitt edge detector.

The remainder of this work is organized as follows. Section 2 is devoted to literature review in edge detection. Section 3 introduces the proposed method, and the extension of the Prewitt method for edge detection to interval valued images. Section 4 includes experimental tests. Section 5 contains results and discussion. Section 6 presents some brief conclusions.

LITERATURE REVIEW

Edge detection has been used extensively in areas related to image and signal processing [4]. Its use includes pattern recognition, image segmentation, and scene analysis [5, 6]. The edges are also use to locate the objects in an image and measure their geometrical features. Hence, edge detection is an important identification and classification tool in computer vision. This topic has attracted many researchers and several achievements have been made to investigate new and more robust techniques.

A large number of studies have been published in the field of image edge detection [7-28], which attests to its importance within the field of image processing. Examples of approaches to edge detection include algorithms such as the Sobel and Prewitt edge detectors which are based on the first order derivative of the pixel intensities [4, 5]. The Laplacian-of-Gaussian (LoG) edge detector is another popular technique, using instead the second order differential operators to detect the location of edges [7, 8, 9]. However, all of these algorithms tend to be sensitive to noise, which is an intrinsically high frequency phenomenon. To solve this problem the Canny edge detector was proposed, which combines a smoothing function with zero crossing based edge detection [10]. Many fuzzy, crisp and entropies edge detection methods have been proposed by researchers [11, 13-18, 26, 28].

Some of the related work in fuzzy set theory and interval valued fuzzy set are mentioned. Abdallah et al. [11] proposed an edge detection approach based on fuzzy logic without determining the threshold value. J. Lopera et al. [12] used Jensen-Shannon divergence of gray level histogram obtained by sliding a double window over an image for edge detection. Khan et al. [13] suggested fuzzy logic based edge detection algorithm for gray scale images. T. W. Tao and W. E. Thomson [14] used gradient approximations as input variables. They used two fuzzy sets, small and large, as linguistic variables and 16 fuzzy rules. K. Ho et al. [15] used fuzzy edge detector for edge detection. They used several fuzzy templates and then edge detects an image using a similarity measure. I. Haq [16] developed a methodology that

was able to detect edges effectively in smooth and noisy clinical images. This technique employs a 3x3 mask guided by fuzzy rule set for edge detection in noisy images. Moreover, for smooth clinical images an extra mask of contrast adjustment is integrated with the edge detection mask based on fuzzy logic to intensify the smooth images. S. Uguz et al. [17] discussed the application of fuzzy cellular automata rules that optimized by practical swarm optimization method to the problem of edge detection. P. Bibiloni et al. [18] presented fuzzy mathematical morphology framework for edge detection in images.

PROPOSED METHODOLOGY

In image processing, an image is refer to a continuous function, denoted by $f(x, y)$, where the value of f at (x, y) in the coordinates space gives the brightness (intensity) of the image [3, 4]. To do the computational processing we need to digitalize the image. The digitalization of the value quantification it is called of gray levels and the digitalization of the space coordinates (x, y) it is called sampling of the image. This stage approximates the continuous image in the form of a matrix; each element of this matrix is denominated of pixel. To convert a continuous image in a digital image it is naturally a stage of uncertainty, the intensity of gray tones of the pixel in a digital image will never correspond the existent in the nature. Intervals arithmetic [30-33] and fuzzy sets [34, 35] appear as appropriate tools for low level feature extraction.

Interval arithmetic

Interval arithmetic is a powerful tool to determine the effects of uncertain data [30-33]. It can deal with numbers that vary within a range. The basic concepts of interval arithmetic are discussed as follows [32, 33].

An interval number $X = [x_1, x_2]$ is the set of real numbers x such that $x_1 \leq x \leq x_2$. Here x_1 and x_2 are known as the lower limit and upper limit of the interval number, respectively. For a real interval $X = [x_1, x_2]$ define the mid-point

$$\text{mid}(X) = (x_1 + x_2)/2,$$

and the radius

$$\text{rad}(X) = (x_2 - x_1)/2$$

Let $X = [x_1, x_2]$ and $Y = [y_1, y_2]$ be the two interval numbers and α is a positive scalar. The basic arithmetic operations of addition, subtraction, multiplication, and division of these two interval numbers are defined as follows [32]:

$$X + Y = [x_1 + y_1, x_2 + y_2];$$

$$X - Y = [x_1 - y_2, x_2 - y_1];$$

$$X \cdot Y = [\min(x_1y_1, x_1y_2, x_2y_1, x_2y_2), \max(x_1y_1, x_1y_2, x_2y_1, x_2y_2)];$$

$$X \div Y = X \cdot Y^{-1} \text{ if } 0 \notin Y;$$

$$\alpha \cdot X = [\alpha \cdot x_1, \alpha \cdot x_2]$$

Definition of real intervals and operation with such intervals can be found in a number of references [31, 32]

Fuzzy set theory

The theory of Fuzzy set is proposed by Zadeh [34, 35]. In recent times, many applications of this theory can be found, such that in signal and image processing, pattern recognition,

operation research, artificial intelligence, expert systems, and robotics [28, 35]. In fuzzy set theory, each elements has a degree of membership μ ranging from zero to one, where the degree of non-membership γ is just automatically equal to one minus the degree of membership ($\gamma=1-\mu$).

Uncertainty and interval images representation

We consider an image A to be a matrix of size $M \times N$, with $S = \{1, \dots, M\} \times \{1, \dots, N\}$ be the set of their locations. The value of a pixel at a location $(i, j) \in S$, $A(i, j)$ is a value in $\{0, \dots, 255\}$. To handle the uncertainty in a pixel, we will use an interval representation of the image pixels. For an image A, the interval value for each pixel (i, j) in an interval valued image IA is obtained as:

$$IA(i, j) = [\max(0, A(i, j) - 1), \min(255, A(i, j) + 1)] \quad (1)$$

Using the formulation of equation (1), we can assign to each location in the digital image an interval, which represent all the brightness values. The cameraman image and the lower and upper bounds of its interval representation are shown in figure 1



Original Image Lower bound image Upper bound image
Figure 1: The cameraman image and its interval representation

Prewitt method and its application to interval images

The Prewitt method for edge detection is a discrete differentiation operator [4], which estimates the gradient for the intensity change along one of the axis (vertical and horizontal) as shown in figure 2. The estimation of the change is used for generating the gradients. Gradients are vectors representing the direction and strength of the intensity changes at each point of the image.

-1	0	+1
-1	0	+1
-1	0	+1

Horizontal Operator

+1	+1	+1
0	0	0
-1	-1	-1

Vertical Operator

Figure 2: The 3x3 kernels of the Prewitt operator

For example, the normalized estimation of the vertical intensity change at a point of the image ($V(i, j) \in [-1, 1]$), is obtained as

$$V(i, j) = \frac{1}{3 \cdot 255} \sum \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{pmatrix} * \begin{pmatrix} A(i-1, j+1) & A(i, j+1) & A(i+1, j+1) \\ A(i-1, j) & A(i, j) & A(i+1, j) \\ A(i-1, j-1) & A(i, j-1) & A(i+1, j-1) \end{pmatrix}$$

Where * represents the convolution operator. By similar way, we can compute the horizontal estimation($H(i, j)$).
 The magnitude for the gradient is calculated as (Euclidean norm):

$$\|G\| = \sqrt{H^2 + V^2}$$

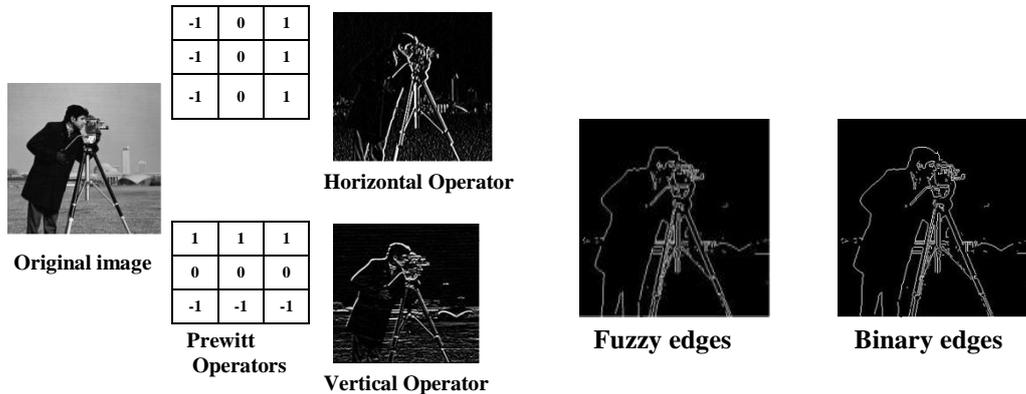


Figure 3: The Prewitt edge detection method.

When we have interval representation images, then we can apply different option of Prewitt operators. One of them, we can apply the operator individually on the lower and upper bound of the intervals, as the scalar images themselves. So, we will apply the basic interval operations as before. In this way, both the vertical and horizontal intensity changes are expressed as subintervals of $[-1, 1]$. As a result, estimation of the gradient at a point in an image will no longer be a vector in the $[-1, 1]^2$ space, but an area, as shown in figure 4. This area can be seen as a projection of the uncertainty about intensity of the pixel in the image.

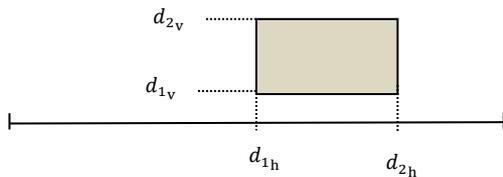


Figure 4: Area representation

We are not certain about the initial data, so the uncertainty is increased when measuring the gradient. The gradient is placed somewhere in the gray area (see figure 4), while its exact location remains unknown. To obtain the binary edges, we

The intermediate representation of the edges in the shape of a fuzzy set is generated by the gradient magnitudes.

transform the area into a segment. To do that, we calculate the center of gravity of the area

$$c = \left[\frac{d_{1h} + d_{2h}}{2}, \frac{d_{1v} + d_{2v}}{2} \right],$$

where d is Euclidean norm. Then, the gradient will be the points in the intersection of the area with the straight line passing through the origin c (note that, in case c concur with the origin, we have a zero gradient).

In order to follow the Prewitt method as closely as possible, we use directly an interval valued fuzzy set, instead of computing a classical fuzzy set. The degree of interval valued membership will be specified by $[\|g_1\|, \|g_2\|]$, where g_1 and g_2 are the closet and furthest point to the origin, respectively. We use the following operator to turn the interval valued fuzzy set representation of the edges into a fuzzy set representation

$$L_\theta[x_1, x_2] = x_1 + \theta(x_2 - x_1), \theta \in [0, 1],$$

This method for edge detection is called the interval valued Prewitt method (IVP); the illustration of this method is shown in figure 5.

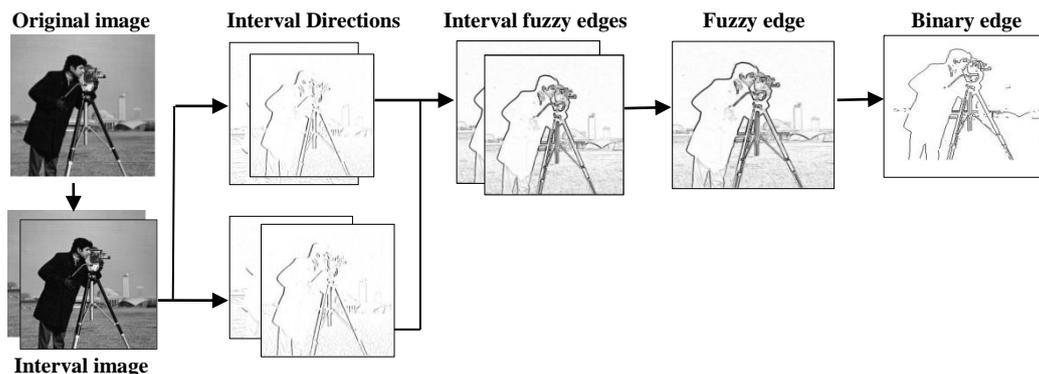


Figure 5: Interval valued Prewitt method for edge detection

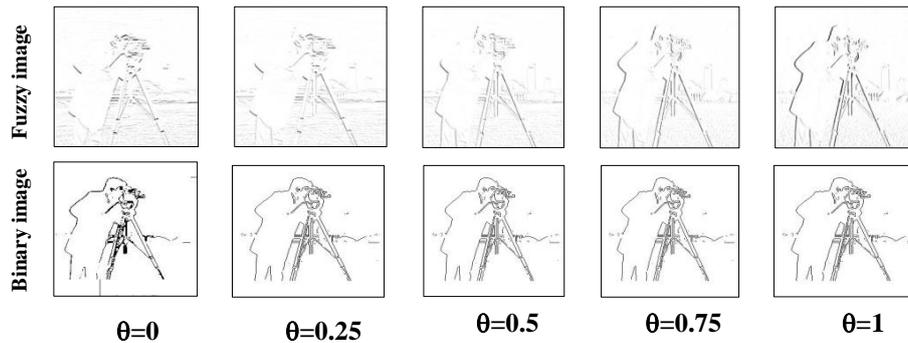


Figure 6: Fuzzy and binary edge images obtained by using different values of θ

EXPERIMENTAL DESIGN

The aim of this section is to examine the effect of using interval valued images for edge detection. For this purpose, we have compared the results obtained by the new method with original Prewitt method. This section also describes images datasets and performance measurements.

Images datasets

In our experiment we used the test and validation subset of the Berkeley Segmentation Dataset [36]. These datasets (BSDS500) consist of 300 images. The images divided into 200 and 100 images, together with hand-made solutions.

Quantitative performance measurement

There are different methods for edges evaluation [37], all of them are finding the similarity between two images. The first one corresponds to the edge detection, and the second, which is the image with the ideal edge (reference image).

In this work, we compare the results of the new method with that generated by a human, which we take as reference image. For an objective comparison of the position accuracy of the edge detection method, Pratt's Figure of Merit (FOM) is one of the most used metric, which used as a quantitative measure, and it is defined as:

$$FOM = \frac{1}{\max(N_i, N_A)} \sum_{i=1}^{N_A} \frac{1}{1 + \beta d_i^2}$$

Where N_i and N_A denote to the number of actual edges point in reference image and detected edges image, respectively d_i refers to the Euclidean distance between pixel i in detected edges image and the nearest point in the reference image, and β is a constant factor.

RESULTS AND DISCUSSION

We present in this section the results and discussion of the IVP method. To carry out the binary edges, we use non-maximum suppression method, which proposed by Kitchen and Rosenfeld [38] and the unimodal thresholding method, as proposed by P. L. Rosin [39]. We observe that the results by the Prewitt method can be widely improved by the interval valued Prewitt method. Some results of the IVP method on the cameraman image are shown in figure 6. From this figure, we observe that the initial value of θ produce bad result, but the performance rapidly increases. Then, when the value of θ tends to one the performance of the Prewitt and IVP methods

becomes identical. We compare IVP method and the Prewitt method on two sets of the BSDS500, using different values of θ . Figure 7 displays the comparison between the performance of the IVP and Prewitt methods on the image set

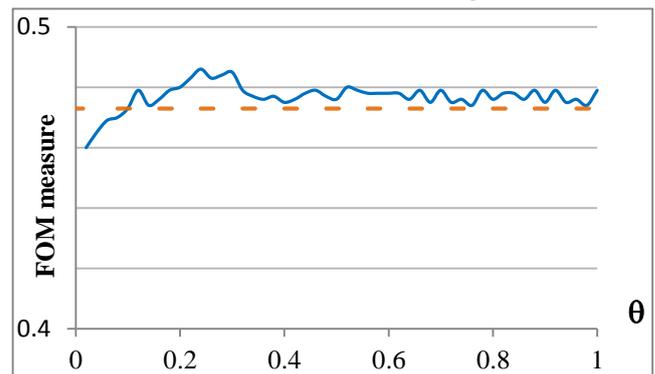


Figure 7: The performance between the results of IVP (solid line) and Prewitt (dashed line) methods of the BSDS500

CONCLUSION

We have analyzed the error in digital images. We proposed a grayscale intervals, where the actual grayscale values bounded by grayscale intervals. Also, we adapted the Prewitt method for edge detection to the new situation of the image, this representation leads to the edges in the form of an interval valued fuzzy set. Finally, we presented a quantitative performance and the applicability of the proposed method and compared with the classical Prewitt method on grayscale images.

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