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Modeling of Determination of Effective Stress of Soil

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Abstract

When the soil has not reached critical equilibrium, if we do not assume that the soil is elastic, elastoplastic material, have no relationship between stress and strain to determine the effective stress state in the soil. Therefore, the purpose of this paper is to present the methodology for establishing additional condition $Z=\int_V \frac{1}{G}\tau_{max}^2 dV \to \min$ for launching a new model determining effective stress state in the soil. The problem of determining the stress state in soil is a nonlinear planning problem solved by the finite difference method in the Matlab programming language. In order to test the new theory, the authors have compared the results with the classic solution of the bearing capacity of the soil under the strip footing when not considering the weight itself with the highly plasticity sticky soil ($\varphi = 0^0$, $c \neq 0$) which is exactly the result of Prandtl solution (p_{th}^{tt} =5,14c).

Keywords: Stress, effective stress, model, soil.

INTRODUCTION

Currently, models for determining stress states in soil are elastic, elastoplastic or limit equilibrium theory methods for determining critical stress states. The state of stress in the case when the soil has not reached the critical point, there is no suitable solution for the real working properties of the soil in that state. This paper presents the method of building additional conditions $Z = \int_V \frac{1}{G}\tau_{max}^2 dV \to \min$ for launching a new model determining effective stress state in the soil. The problem of determining the stress state in soil is a nonlinear planning problem solved by the finite difference method in the Matlab programming language. In order to test the new theory, the authors have compared the results with the classic solution of the bearing capacity of the soil under the strip footing when not considering the weight itself with the highly plasticity sticky soil ($\varphi = 0^0$, $c \neq 0$) which is exactly the result of Prandtl solution (p_{th}^{tt} =5,14c).

ESTABLISHING THEORETICAL BASIS FOR DETERMINING THE EFFECTIVE STRESS IN THE SOIL

The theory of effective stress in the soil

Consider a soil element in a plane problem that is affected by the stresses σ_z, σ_x, τ_{xz} and the unit weight γ (Figure 1), satisfying the equilibrium equations:

$$\begin{aligned} &\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0 \\ &\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} - \gamma = 0 \end{aligned} \qquad (1)$$

If the soil is elastic, elastoplastic material or in critical state, to determine the stress in the soil, system (1) has one more equation. For example, soil is an elastic material that has a continuous equation, system (1) becomes:

$$\begin{aligned} &\nabla\left(\sigma_x + \sigma_z\right) = 0 \\ &\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0 \\ &\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} - \gamma = 0 \end{aligned} \qquad (2)$$

Thus, system (2) has three equations for identifying three stresses $\sigma_z(x,z)$, $\sigma_x(x,z)$ *and* $\tau_{zx}(x,z)=\tau_{xz}(x,z)$.

If we do not assume that the soil is elastic, elastoplastic or non-critical, the problem is not defined. There are only two equations (1) that have three hidden functions $\sigma_z(x,z)$, $\sigma_x(x,z)$ and $\tau_{zx}(x,z) = \tau_{xz}(x,z)$. The problem of determining the state of stress in the soil of the system (1) has countless solutions.

What additional conditions can be added to determine the effective stress state in the soil. The author would be presented the following.

The prosesses of formation, existence, compaction, dilatancy, contractancy, consolidation of soil are the processes that lead to stable or stable formation. Stable conditions of the soil are expressed through the following stress conditions: The stress state of the point M is expressed through the Mohr circle, assuming the circle (1) in Figure 2. The maximum tangential stress τ_{max1} of the point M mathematically is the radius of the Mohr circle (1). Under the effect of load, point M is in steady state, the circle Mohr has the smallest radius, τ_{max1} is the smallest. Thus, when the soil is most stable, the stressed Mohr circle has the greatest tangential stress τ_{max} (circular radius) is minimum. The additional condition for determining the stress state in the soil is $\min(\tau_{max})$.

With soil material, the suitable durability is Morh-Coulomb. Therefore, the Morh circle shows the stress in the soil must always be below the shear strength line S. The maximum stress state in the soil is circle (2) in Figure 2 and the maximum tangential stress is τ_{max2}.

In least squares (which can be understood as formulas), the problem is of the form:

$$Z=\int_V \frac{1}{G}\tau_{max}^2 dV=\int_V \frac{1}{G}\left[\left(\frac{\sigma_x-\sigma_z}{2}\right)^2+\left(\frac{\tau_{xz}+\tau_{zx}}{2}\right)^2\right]dxdz \to \min \qquad (3)$$

where: τ_{max} - the maximum shear stress at the point under consideration; V - integrating domain limit, the volume of the soil mass under consideration; and G - sliding modulus of soil.

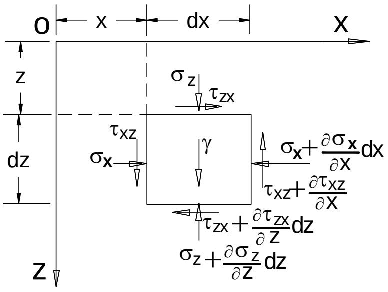


Figure 1. Stress on soil elemnet

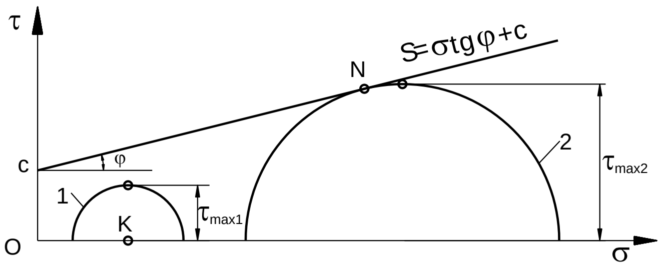


Figure 2. Stress state in soil

Functional (3) is an additional condition for determining the state of stress in the soil. The stress state in (3) must satisfy the equilibrium conditions:

$$\begin{aligned} &\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{zx}}{\partial z}=0 \\ &\frac{\partial\sigma_z}{\partial z}+\frac{\partial\tau_{xz}}{\partial x}-\gamma=0 \end{aligned} \qquad (4)$$

Thus, the problem of determining the stress state in the soil is the extreme problem of the objective function (3) with constraints (4). This is a nonlinear planning problem that finds the stress state satisfying both the equilibrium equation and ensuring the tangential τ_{max} minimum.

In the individual case, it can be considered as a differential problem. Putting on non-binding form by writing nonsense Lagrange function expansion:

$$\int_V \left\{ \begin{aligned} &\frac{1}{G}\left[\left(\frac{\sigma_x-\sigma_z}{2}\right)^2+\left(\frac{\tau_{xz}+\tau_{zx}}{2}\right)^2\right]+\lambda_1(x,z)\left(\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{zx}}{\partial z}\right) \\ &+\lambda_2(x,z)\left(\frac{\partial\sigma_z}{\partial z}+\frac{\partial\tau_{xz}}{\partial x}-\gamma\right) \end{aligned} \right\} dV \to \min \qquad (5)$$

where λ_1 and λ_2 - Lagrange factor, is a function of x and z, two unknown functions; σ_x, σ_z, τ_{xz}, τ_{zx} are the functions of coordinates x and z. Stress components σ_x, σ_z, τ_{xz}, τ_{zx} in soil mechanics problems arising from element equilibrium conditions and in continuous environments should be continuous functions.

If (5) is a differential problem with σ_x, σ_z, τ_{xz}, τ_{zx} are the variables and uses the differential calculus for the objective function (5), we obtain the following equations:

$$\begin{aligned} &\frac{1}{2G}\left(\sigma_x-\sigma_z\right)=\frac{\partial\lambda_1}{\partial x} \\ &\frac{1}{2G}\left(\sigma_z-\sigma_x\right)=\frac{\partial\lambda_2}{\partial z} \\ &\frac{1}{2G}\left(\tau_{xz}+\tau_{zx}\right)=\frac{\partial\lambda_1}{\partial z} \\ &\frac{1}{2G}\left(\tau_{xz}+\tau_{zx}\right)=\frac{\partial\lambda_2}{\partial x} \\ &\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{zx}}{\partial z}=0 \\ &\frac{\partial\sigma_z}{\partial z}+\frac{\partial\tau_{xz}}{\partial x}-\gamma=0 \end{aligned} \qquad (6)$$

Taking the second derivative, $\tau_{xz} = \tau_{zx}$, system (6) has 3 following equations:

$$\begin{aligned} &\nabla\left(\sigma_x-\sigma_z\right)=0 \\ &\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{zx}}{\partial z}=0 \\ &\frac{\partial\sigma_z}{\partial z}+\frac{\partial\tau_{xz}}{\partial x}-\gamma=0 \end{aligned} \qquad (7)$$

From the first equation, we find that a solution of (7) is $\sigma_x = \sigma_z$. Combining with $\tau_{xz} = 0$ This stress state tests Mohr-Coulomb strength conditions are always satisfied, the soil is always stable.

ESTABLISHING A PROBLEM TO DETERMINE THE STATE OF STRESS IN THE SOIL

The problem of determining the effective stress in the soil is the problem of finding the extremes of the objective function (3) rewritten below:

$$Z=\int_V \frac{1}{G}\tau_{max}^2 dV \to \min \qquad (8)$$

The stress state in the soil must satisfy the following constraints:

+ Two balance equations (4)

+ Soil without tension:

$$\sigma_x \geq 0 \text{ and } \sigma_z \geq 0 \qquad (9)$$

+ Mohr-Coulomb strength condition:

$$f(k)=\tau-\sigma.tg\varphi-c\leq 0 \qquad (10)$$

or

$$\left(\frac{\sigma_1-\sigma_3}{2}\right)-\left(\frac{\sigma_1+\sigma_3}{2}\right)\sin\varphi-c\cos\varphi\leq 0$$

+ The boundary conditions:

Boundary conditions on the surface at the location without external load effects of stress components are placed at points on the horizontal surface is:

$$\sigma_z=0;\ \tau_{xz}=0;\ \sigma_x \text{ - unknown} \qquad (11)$$

The problem of determining the state of stresses in the soil is the extreme problem of the objective function (8) with constraints (9), (10), (11). This is a nonlinear planing problem

DEVELOPING A METHOD FOR SOLVING THE STRESS PROBLEM IN SOIL BY FINITE DIFFERENCE

Differential Diagram

Dividing the ground into differentiated grid as shown in Figure 3. At each node, there are hidden unknowns that are stresses σ_x, σ_z, τ_{xz}. Let Δx, Δz be the size of the difference grid.

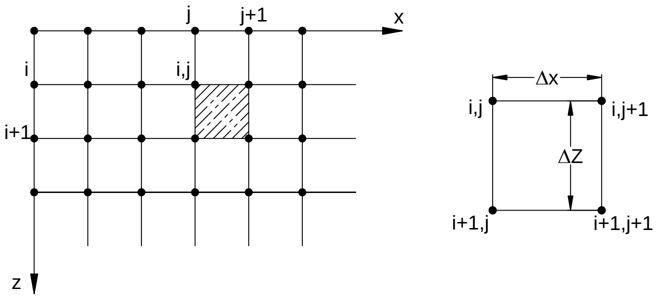


Figure 3. Grid and differential diagram

Differential diagram is selected so that the stresses of the difference grid nodes in the scope of calculation as well as the boundary are both within the equilibrium equation and in the objective function.

Equilibrium equation:

Considering the grid cell is determined by the 4 nodes: (i,j), (i,j+1), (i+1,j), (i+1,j+1) and the area is F=Δx.Δz. The equilibrium equation is written for the midpoint of the grid and for the area F=Δx.Δz of the element so equation (4) has the following form:

$$\left(\frac{\sigma_x^{(i+1,j+1)}+\sigma_x^{(i,j+1)}}{2}-\frac{\sigma_x^{(i+1,j)}+\sigma_x^{(i,j)}}{2}\right)\frac{1}{\Delta z}\Delta x\Delta z+$$
$$\left(\frac{\tau_{xz}^{(i+1,j+1)}+\tau_{xz}^{(i+1,j)}}{2}-\frac{\tau_{xz}^{(i,j+1)}+\tau_{xz}^{(i,j)}}{2}\right)\frac{1}{\Delta x}\Delta x\Delta z=0; \quad (12)$$

$$\left(\frac{\sigma_z^{(i+1,j+1)}+\sigma_z^{(i+1,j)}}{2}-\frac{\sigma_z^{(i,j+1)}+\sigma_z^{(i,j)}}{2}\right)\frac{1}{\Delta z}\Delta x\Delta z+$$
$$\left(\frac{\tau_{xz}^{(i+1,j+1)}+\tau_{xz}^{(i,j+1)}}{2}-\frac{\tau_{xz}^{(i+1,j)}+\tau_{xz}^{(i,j)}}{2}\right)\frac{1}{\Delta x}\Delta x\Delta z-\gamma\Delta x\Delta z=0.$$

The conditions of soil without tension: Condition (9) for each grid node will be:

$$\sigma_x^{(i,j)}\geq 0 \text{ and } \sigma_z^{(i,j)}\geq 0. \quad (13)$$

Objective function:

In order to have the objective function (8) in the form of the difference, we note the tangential stress τ_{max} and the normal stress at the point under consideration are determined for the flat problem as follows:

$$\tau_{max}=\frac{\sigma_1-\sigma_3}{2};\ \sigma=\frac{\sigma_1+\sigma_3}{2} \quad (14)$$

and

$$\sigma_{1,3}=\left(\frac{\sigma_x+\sigma_z}{2}\right)\pm\sqrt{\left(\frac{\sigma_x-\sigma_z}{2}\right)^2+\tau_{xz}^{\ 2}} \quad (15)$$

Replace (14) into (15), then substitute (8) and write the objective function in terms of the difference for the average stress on each of the different grid cells in the form:

$$\int_S \frac{1}{G}\tau_{max}^2 dx.dz \approx \sum_i\sum_j \frac{1}{G}\left[\left(\frac{\frac{\sigma_x^{(i,j)}+\sigma_x^{(i,j+1)}}{2}-\frac{\sigma_z^{(i,j)}+\sigma_z^{(i,j+1)}}{2}}{2}\right)^2+\left(\frac{\tau_{xz}^{(i,j)}+\tau_{xz}^{(i,j+1)}}{2}\right)^2\right]\Delta x\Delta z$$
$$+\sum_i\sum_j \frac{1}{G}\left[\left(\frac{\frac{\sigma_x^{(i+1,j)}+\sigma_x^{(i+1,j+1)}}{2}-\frac{\sigma_z^{(i+1,j)}+\sigma_z^{(i+1,j+1)}}{2}}{2}\right)^2+\left(\frac{\tau_{xz}^{(i+1,j)}+\tau_{xz}^{(i+1,j+1)}}{2}\right)^2\right]\Delta x\Delta z$$
$$+\sum_i\sum_j \frac{1}{G}\left[\left(\frac{\frac{\sigma_x^{(i,j)}+\sigma_x^{(i+1,j)}}{2}-\frac{\sigma_z^{(i,j)}+\sigma_z^{(i+1,j)}}{2}}{2}\right)^2+\left(\frac{\tau_{xz}^{(i,j)}+\tau_{xz}^{(i+1,j)}}{2}\right)^2\right]\Delta x\Delta z$$
$$+\sum_i\sum_j \frac{1}{G}\left[\left(\frac{\frac{\sigma_x^{(i,j+1)}+\sigma_x^{(i+1,j+1)}}{2}-\frac{\sigma_z^{(i,j+1)}+\sigma_z^{(i+1,j+1)}}{2}}{2}\right)^2+\left(\frac{\tau_{xz}^{(i,j+1)}+\tau_{xz}^{(i+1,j+1)}}{2}\right)^2\right]\Delta x\Delta z \rightarrow \min \quad (16)$$

Slider module G calculated as G = const and G changes linearly as shown in Figure 4b and Figure 4c.

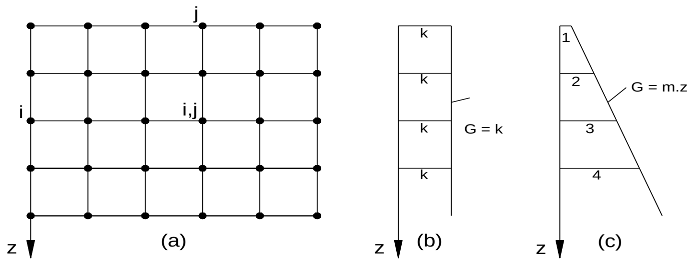


Figure 4. Slider module G in depth

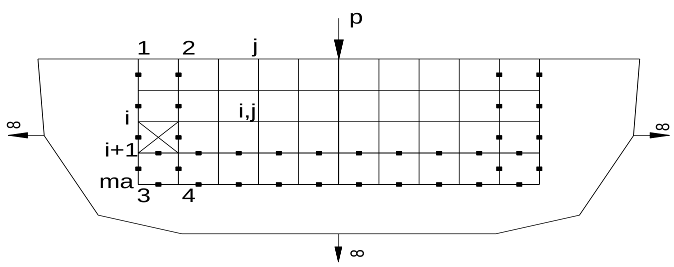


Figure 5. Condition on boundary of finite soil

Mohr-Coulomb strength condition:

Mohr-Coulomb strength condition (10) in terms of the difference for each node::

$$f(k)=\sqrt{\frac{(\sigma_x^{(i,j)}-\sigma_z^{(i,j)})^2}{4}+\tau_{xz}^{(i,j)2}}-\frac{(\sigma_x^{(i,j)}+\sigma_z^{(i,j)})}{2}\sin\varphi-c.\cos\varphi\leq 0 \quad (17)$$

Boundary conditions at the surface of the soil mass:

$$\sigma_z^{(i,j)}=0;\ \tau_{xz}^{(i,j)}=0\ ;\ \sigma_x^{(i,j)} \text{ - unknown} \quad (18)$$

The boundary conditions of the soil mass for the infinite plane

Due consideration is only finite soil mass of half infinite plane, the condition on the boundary of the mass is to be considered.
Consider the left boundary of the soil mass (Fig. 5), when the distance from the force point is greater, the stress state on the 1-3 boundary is approximately the stress state on the 2-4 cross section. The equilibrium equation is written in the form of the difference using the average stress per each square grid edge (square markers), so the boundary conditions (1-2) of the soil mass are written in the least squares form as follows:

+ For compression stress σ_x:

$$\sum_{i=1,ma-1} \frac{1}{G}\left(\frac{\sigma_x^{(i,1)}+\sigma_x^{(i+1,1)}}{2}-\frac{\sigma_x^{(i,2)}+\sigma_x^{(i+1,2)}}{2}\right)^2 \Delta x \Delta z \to \min \quad (19a)$$

+ For compression stress σ_z:

$$\sum_{i=1,ma-1} \frac{1}{G}\left(\frac{\sigma_z^{(i,1)}+\sigma_z^{(i+1,1)}}{2}-\frac{\sigma_z^{(i,2)}+\sigma_z^{(i+1,2)}}{2}\right)^2 \Delta x \Delta z \to \min \quad (19b)$$

+ For tangential stress τ_{xz}:

$$\sum_{i=1,ma-1} \frac{1}{G}\left(\frac{\tau_{xz}^{(i,1)}+\tau_{xz}^{(i+1,1)}}{2}-\frac{\tau_{xz}^{(i,2)}+\tau_{xz}^{(i+1,2)}}{2}\right)^2 \Delta x \Delta z \to \min \quad (19c)$$

As such, the problem of nonlinear planning is in the form of the difference, which consists of the objective function (16), the constraints being two equation (12), the soil without tension condition (13), the Mohr -Coulomb (17), boundary conditions (18), (19a), (19b), (19c).

DETERMINATION OF EFFECTIVE STRESSES IN THE SOIL DUE TO THE EFFECT OF THE LOAD DISTRIBUTED EVENLY OVER THE HORIZONTAL GROUND (WITHOUT CONSIDERING THE EFFECT OF THE WEIGHT ITSELF)

Purpose: Apply the theory of stress determination in the studied soil, solving by the finite difference method as described above for the specific case to prove the correctness of the solution to the problem by finite difference.

Calculated data: Soil with unit cohesion c=30 kPa, internal friction angle $\varphi=10^0$, volume weight γ = 17 kN/m^3, slider modulus G changes linearly according to the depth of the rule as shown in Figure 4c; uniformly distributed load intensity p = 50 kPa.

The results of calculating the vertical stres σ_z and the horizontal stress σ_x of the soil columns in depth are shown in Figures 6a and 6b.

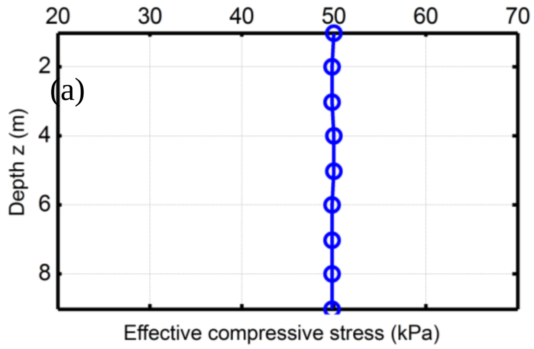
Effective compression stress σ_z

Tangential stress value τ_{xz} at the computational nodes is almost zero.

Application and validation of the results of effective stress determination with finite difference to determine the load bearing capacity of the soil under strip footing - the horizontal ground

METHOD OF DETERMINING THE BEARING CAPACITY OF THE GROUND UNDER THE LOWER LIMIT THEOREM

The ground is affected by an infinite and longitudinal flat strip of load so the problem is constructed to find the limit load as a flat problem with the calculation diagram as shown in Figure 7 (a).



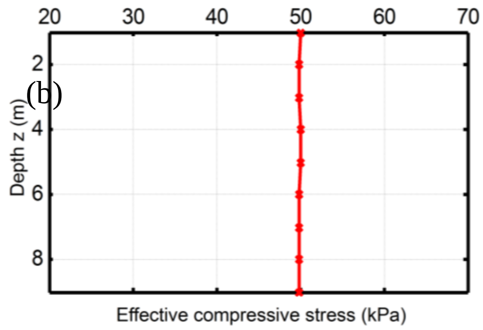


Figure 6a. Effective compression stress σ_x; Figure 6b. Effective compression stress σz

Differential Diagram: Because the ground is horizontal, the ground is considered to be a semi-space affected by an infinite flat strip of load, so the author chooses the diagram as symmetry and the differential grid diagram as shown in Figure 7b and Figure 7c. The ordinal number of differential net nodes according to the axis Oz varies in about 1÷m and Ox axis is 1÷n (n_0 – middle nodes in about 1÷n). Differential diagrams used in math is the central difference.

In this case it is possible to imagine that each point in the soil is likely to have a sliding strain (Mohr-Coulomb stability under (10), then f(k) = 0). Therefore, the condition of the critical stress state is written in the form of the least squares as follows:

$$Z_1 = \frac{1}{G}\left[\sqrt{\frac{(\sigma_x-\sigma_z)^2}{4}+\tau_{xz}^2}-\frac{(\sigma_x+\sigma_z)}{2}\sin\varphi - c\cos\varphi\right]^2 \to \min \quad 20)$$

In this method, the intensity of the load p at the edge of the foundation is hidden (at the edge of the foundation there is a stress concentration so the contact stress is greatest) and the stress state is critical to determine the bearing capacity of the ground as the maximum value of p (p_{max}). However, because of the distribution load p not contained in the objective function, in this problem, the objective function of the p-force at the edge of the foundation with the goal of pmax (when moving to the min problem with the sign (-)).Thus, the objective function of the force p at the edge of the foundation is as follows:

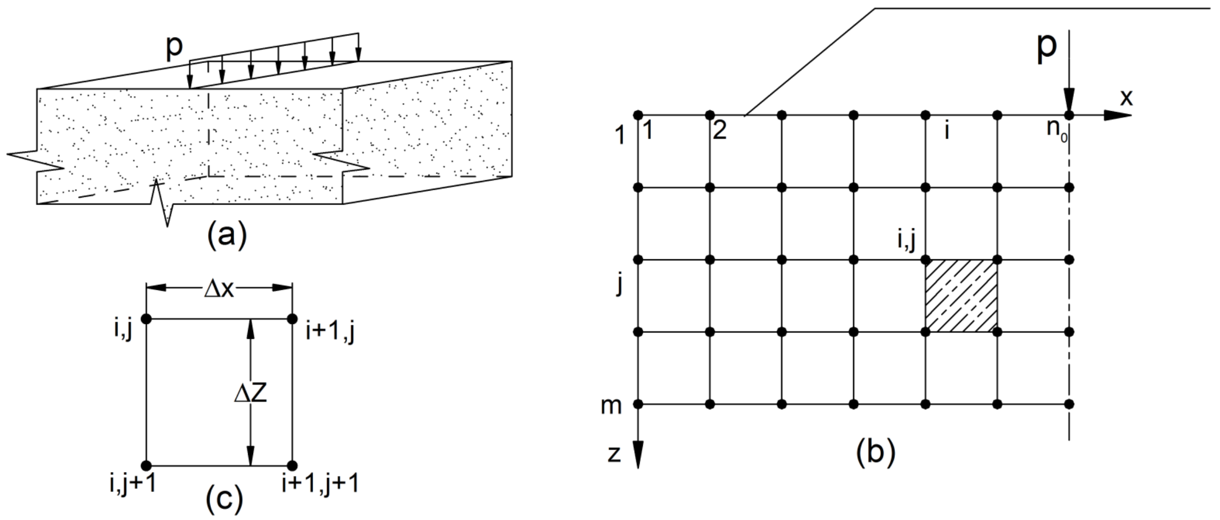


Figure 7. Calculation diagram
(a) - Model of soil mass calculation; (b) and (c) - Differential Grid and Grid Size

$$Z_2 = \int_V \frac{1}{G}\tau_{\max}^2 dV - \frac{p_{\max}^2}{G}\Delta V \to \min \qquad (21)$$

where ΔV is the area associated with the force point.

Soil conditions at steady state:

$$Z_3 = \int_v \frac{1}{G}\tau_{\max}^2 dv = \int_v \frac{1}{G}\left[\frac{(\sigma_x - \sigma_z)^2}{4} + \tau_{xz}{}^2\right]dv \to \min \qquad (22)$$

The stress state in the ground beneath the strip footing must satisfy the following constraints:

+ Two balance equations (4);

+ Mohr-Coulomb strength conditions (10);

+ The boundary conditions of the problem are the stress state of the nodes in the lower, upper and lateral sides of the mesh, as follows: At the top of the block: the nodes are not affected by the load only the unknown is σ_x and σ_z=0 và $\tau_{xz}=\tau_{zx}$=0; the nodes are affected by the load only the unknown isσ_x and σ_z= p và $\tau_{xz}=\tau_{zx}$ = 0; the stress at the remaining boundary points is unknown.

Thus, the problem of determining the bearing capacity of the ground is the minimum problem of (20), (21), (22) with constraints (4), (10) and boundary conditions.

DETERMINE THE BEARING CAPACITY OF GROUND WHEN NOT CONSIDERING THE WEIGHT ITSELF TO COMPARE WITH PRANDTL SOLUTION

Purpose: Compare with Prandtl solution

Use the problem posed above, the author conducted with the size of difference mesh m = 8, n = 15 and received the results corresponding to two different instances of the ground as below.

Case c ≠ 0; φ = 0, γ = 0

• At the pressure value of the load p < 4c, the stresses in all nodes of the differential mesh meet the inequality condition f(k) < 0 in equation (10). The ground is in a stable state.

• At the pressure value of the load p = 4c, the plastic flow appeared (equilibriumcondition reached in equation (10), f (k) = 0) at points corresponding nodes (i = 8, j = 1) and (i = 8, j = 2) as shown in Figure 8a. The isomorphic lines f(k) of the Mohr-Coulomb strength condition are shown in Figure 8b.

• At the pressure value of load p = 4.3c, the plastic flow at the nodes appears as p = 4.0c..

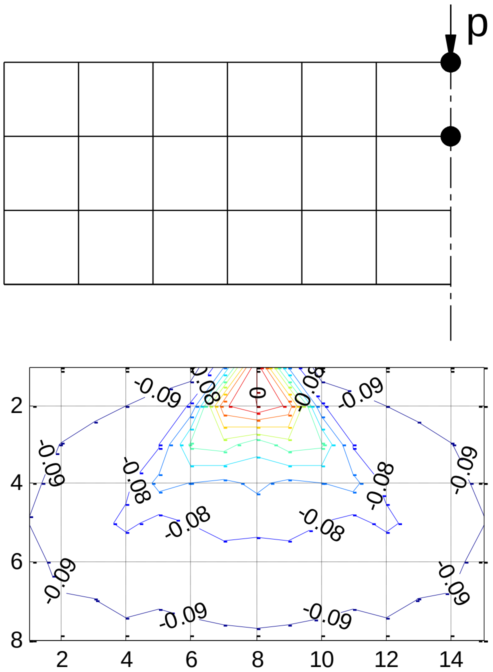


Figure 8 (a) Plastic flow points**, (b)** The isomorphic lines f (k)

• At the pressure value of the load p = 4.6c, the plastic flow appeared (equilibrium condition reached in equation (10), f (k) = 0) at three corresponding nodes (i = 8, j = first); (i = 8, j = 2) and (i = 7, j = 2) as shown in Figure 9a. The isomorphic lines f(k) of the Mohr-Coulomb strength condition is shown in Figure 9b.

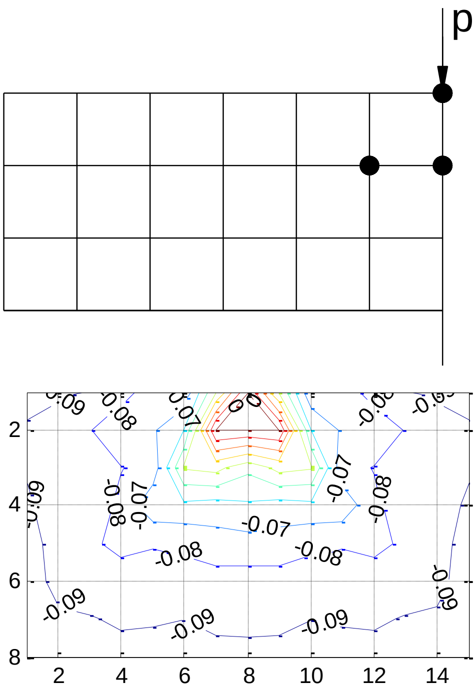


Figure 9 (a) Plastic flow points,
(b) The isomorphic lines f (k)

• At the pressure value of the load p = 5c, the plastic flow appeared (equilibrium condition reached in equation (10), f (k) = 0) at four corresponding nodes (i = 8, j = 1); (i = 8, j = 2); (i = 7, j = 2) and (i = 7, j = 1) as shown in Figure 10a. The isomorphic lines f(k) of the Mohr-Coulomb strength condition is shown in Figure 10b.

• At the pressure value of the load p = 5.23c, the plastic flow appeared (equilibrium condition reached in equation (10), f (k) = 0) at four corresponding nodes (i = 8, j = first); (i = 8, j = 2); (i = 7, j = 2) and (i = 7, j = 1) as shown in Figure 11a. The isomorphic lines f (k) of the Mohr-Coulomb strength condition are shown in Figure 11b.

+ Case of load p = 5.24c or greater - problem with no solution.

Remark:

+ From the isomorphic lines f (k) in Figures 8b, 9b, 10b, and 11b, we see the nonlinearity of the stress state when increasing the external load.

+ The value of the load pressure p = 5.23c is the limit load value calculated - when soil structure destruction appear.

+ The determined limit load p_{gh}^{tt}=5.23c versus restricted by Prandtl $p_{gh}^{Prandtl}$ = 5.14c wrong number is 1.75%.

Case c ≠ 0; φ ≠ 0, γ = 0:

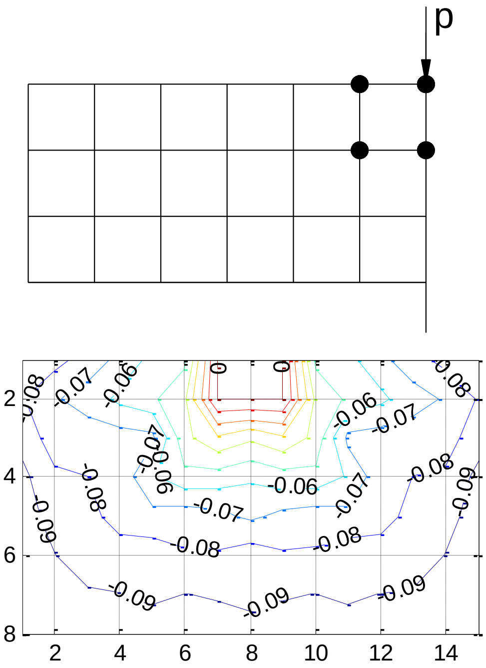


Figure 10 (a) Plastic flow points,
(b) The isomorphic lines f (k)

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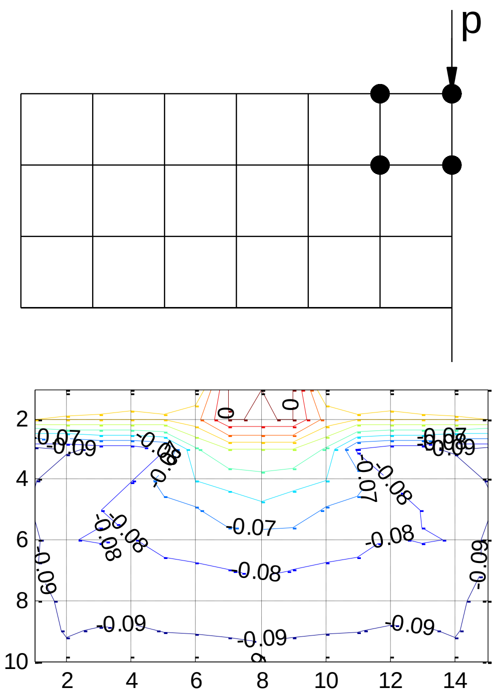

Figure 11(a) Plastic flow points, **(b)** The isomorphic lines f (k)

The method of solving also follows the same principle as in the case of 2.2.1. The different cases of calculations are the cases for different internal friction angles (assuming angle φ varies from1÷31), the unit cohesion c is constant. Corresponding to each case calculation, we determine the limit charge calculation p_{th}^{tt} .

The formula to determine the load limit calculation p_{gh}^{tt} is built according to the formula of Prandtl as follows::

$$p_{gh}^{tt} = N_c^{tt} \ .c \qquad (23)$$

where N_c^{tt} - calculated bearing capacity coefficient .

According to Prandtl, coefficient of bearing capaci Prandtl $N_c^{Prandtl}$ determined by the following formula:

$$N_c^{Prandtl} = (\frac{1+\sin\varphi}{1+\sin\varphi} e^{\pi tg\varphi} - 1)\cot g\varphi \qquad (24)$$

where φ is the internal friction angle of soil.

According to the equation (23), the computed results are computed by the coefficients of load bearing capacity. Comparing the value of bearing capacity coefficient determined with the bearing capacity coefficients under Prandtl according to formula (24), we have the results as in Table 1.

Table 1. Comparison of bearing capacity coefficients N_c^{tt} and $N_c^{Prandtl}$

φ	N_c^{tt}	$N_c^{Prandtl}$	**Difference (%)**
0	5.23	5.142	1.71
1	5.42	5.379	0.76
2	5.6	5.632	0.57
3	5.84	5.900	1.02
4	6.06	6.185	2.02
5	6.3	6.489	2.91
6	6.54	6.813	4.01
7	6.81	7.158	4.86
8	7.11	7.527	5.54
9	7.41	7.922	6.46
10	7.71	8.345	7.61
11	8.08	8.798	8.16
12	8.45	9.285	8.99
13	8.85	9.807	9.76
14	9.28	10.370	10.51
15	9.74	10.977	11.27

φ	N_c^{tt}	$N_c^{Prandtl}$	**Difference (%)**
16	10.22	11.631	12.13
17	10.72	12.338	13.11
18	11.35	13.104	13.39
19	11.98	13.934	14.02
20	12.68	14.835	14.53
21	13.42	15.815	15.14
22	14.26	16.833	15.29
23	15.06	18.049	16.56
24	15.94	19.324	17.51
25	16.98	20.721	18.05
26	18.52	22.254	16.78
27	19.95	23.942	16.67
28	21.45	25.803	16.87
29	22.98	27.860	17.52
30	24.5	30.140	18.71
31	26.92	32.671	17.60

Comment: The comparing results between the calculated bearing capacity coefficient N_c^{tt} and Prandtl bearing capacity coefficient as Table 1 shows, the difference between the calculated results of two methods fluctuate in about 1÷19% Increase the value of the internal friction angle.

Nhận xét: Kết quả so sánh hệ số sức chịu tải tính toán và hệ số sức chịu tải theo Prandtl $N_c^{Prandlt}$ như trên Bảng 1 cho thấy, sự chênh lệnh kết quả tính giữa hai phương pháp dao động trong khoảng từ 1÷19% following the increasing of the internal friction angle φ.

DISCUSSION AND CONCLUSIONS

As we all know, the stress state is the state which leads to form stability or forms stability state of soil. When soil is the most stable, the largest shear stress τ_{max} (the largest shear stress "τ_{max}" is radius of stress Mohr circle) is minimal. Therefore, the author added additional conditions to determine the stress state in the soil min (τ_{max}) is reasonable and correct. Main contributions of this study are (1) the soil is a three-phase material, using Terzaghi's effective stress principle, adding additional conditions min (τ_{max}) to obtain the full equation to determine the state of stress in the soil in general, (2) the problem solved with conditions: soil without tension, Mohr-Coulomb failure criterion and bound conditions. The problem is nonlinear programming. By calculating the dummy variable, we obtain the system of equations (7) which is the case for determining the stress state in the soil. The explainations as to the variational and stress Mohr circle indicate that the problem has root, on mechanism side, it is the only root, (3) applying the new theory presented above, we can: have enough conditions to determine the stress state in the soil, taking into account the special properties of the soil such as soil without tension, Mohr-Coulomb failure criterion, increased hardness in depth, (4) using the above theory, the authors have studied the determination of the stress state in the case of the effect of the load distributed evenly on the ground with intensity p, normal stress value $\sigma_z = \sigma_x = p$ and no change in depth, tangential stress $\tau_{xz} \approx 0$. That proves that the problem is valid, (5) in order to test the new theory of determining the effective stress state, the authors have compared the results with respect to the classical solution of the load bearing capacity of the soil under the strip footing when not considering its weight with highly sticky soil ($\varphi = 0^0$, $c \neq 0$) correct by the results of the Prandtl solution ($p_{th}^{tt} = 5,14c$). This result is one of the proofs to show that the construction of additional conditions to determine the stress state in soil of the author is correct.

Using researched theory to determine stress states for different cases and solve the application problems of soil mechanics such as stress state in the embankment, stress state in the slope, state stresses the ground under the foundation, calculates the stability of the slope, determines the bearing capacity of the ground and considers the effect of the cap concrete on the bearing capacity of the ground. Using the researched theory to study the stress state of soil in the spatial problem

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