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Modeling and Solving of Multi-Product Inventory Lot-Sizing with Supplier Selection under Quantity Discounts

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Abstract

This paper proposes a multi-product and multi-period inventory lot-sizing problem with supplier selection under vehicle capacity and both all-units and incremental quantity discounts. In addition, we consider that buyer's limited storage capacity. The objectives are to minimize total costs, where the costs include purchase cost of the products, ordering cost for the suppliers, transportation cost for the suppliers and holding cost of the remaining inventory in each period. It is assumed that demand of multi-product is known over a planning horizon. The problem is formulated as a mixed integer linear programming (MILP) model. Finally, a numerical example is provided to illustrate the solution procedure. The results, the buyer needs to determine what products to order in what quantities with which suppliers in which periods, in order to satisfy overall demand and minimize the total costs.

Keywords: Inventory Lot-Sizing, Vehicle Capacity, Quantity Discounts, Mixed Integer Linear Programming

INTRODUCTION

Nowadays, because of the global market competition, the firm can accomplish competitiveness by reducing total logistics costs of purchasing. The importance of the purchasing, is one of the most strategic activities in supply chain management (SCM), provides opportunities to reduce costs and increase profits of companies [1]. The determination of lot-sizing and the supplier selection play important roles in the purchasing strategy and affect the inventory costs [2]. In the past several decades, inventory lot-sizing problem and supplier selection has been studied in the literature. Inventory lot-sizing with supplier selection is a fast-growing offspring of two major problem parents in the field of SCM [3]. Supplier selection is considered a very important issue in inventory lot-sizing problems because of the probable differences between the purchasing costs that may arise from choosing different suppliers in each period. Inventory lot-sizing and supplier selection are two important decisions any purchasing of the buyer has to make. With regard to inventory lot-sizing problem and supplier selection, the buyer needs to consider minimizing the inventory costs [4]. In addition, the buyer often faces multiple suppliers and quantity discount. When quantity discount is considered and the order made by the buyer is large, the supplier reduces the unit price under discount schedule [5], [6]. Quantity discounts can be part of a pricing strategy and can be a powerful incentive to motivate buyers to increase the amount of their ordered quantities.

Therefore, the problems of inventory lot-sizing and supplier selection have been verified as having a grave impact on the supply chain, and attracted many researchers for several years [7]. Soon afterwards, [5], [6], [8], [9] proposed a lot-sizing model with single product multiple supplier and quantity discounts to minimize total cost over the planning horizon. The problem was first formulated as a mixed integer linear programming (MILP) model. This model is not able to consider the multi-product of each supplier and vehicle capacity of each supplier. One of the important modifications we consider in this paper is that of introducing the multi-product and multi-period inventory lot-sizing with supplier selection under alternative quantity discounts and vehicle capacity.

This paper extends the work of [9], [5] [10], model. We formulate the multi-product and multi-period inventory lot-sizing with supplier selection considering vehicle capacity and quantity discounts (all-units and incremental discounts). For all-units quantity discounts, if the order size belongs to a specified quantity level, the reduced purchase price is applied to all-unit starting from the first unit. Different with incremental quantity discounts, the reduced purchase price is applied to the units inside the price break quantity [11]. The problem is formulated as a MILP model and is solved with optimization package LINGO 12. The objectives are to minimize total costs, where the costs include purchase cost of the products, ordering cost for the suppliers, transportation cost for the suppliers and holding cost of the remaining inventory in each period. The results, the buyer needs to determine what products to order in what quantities with which suppliers in which periods, in order to satisfy overall demand and minimize the total costs.

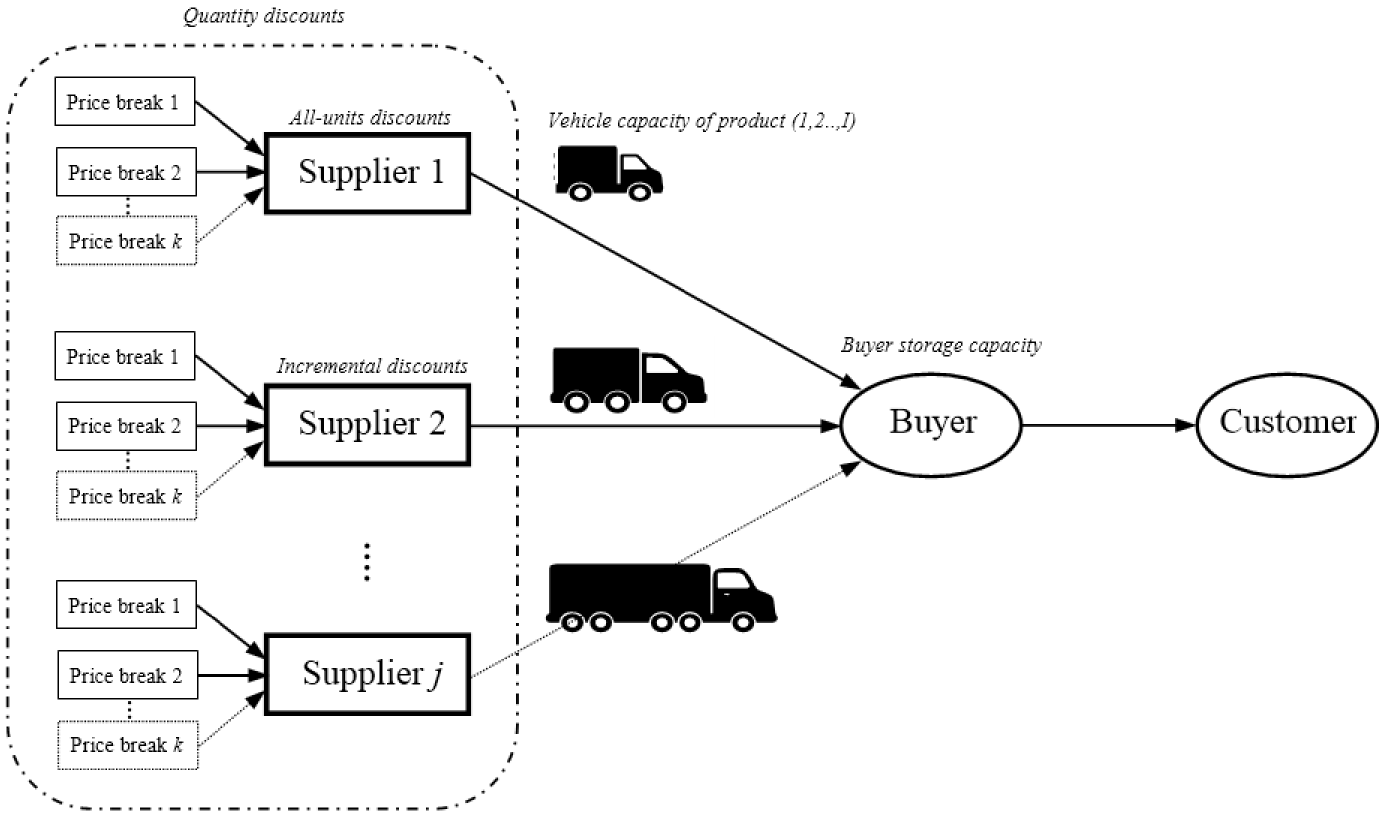

Figure 1: The structure of the problem under consideration

MODEL FORMULATION

Model assumptions

In this section, the problem can be briefly considered in Figure 1. The buyer aim is to minimize total costs, where the costs include purchase cost of the products, ordering cost for the suppliers, transportation cost for the suppliers and holding cost of the remaining inventory in each period. Each supplier offers quantity discounts (all-units and incremental discounts) to motivate the buyer for purchasing large quantity. In addition, we consider that the capacity each vehicle of the supplier transport the products to the buyer. The proposed MILP model formulation for the problem. Consequently, the model assumptions, parameters and decision variables, a mathematical formulation are presented in following sub-sections.

Model assumptions

- Demand of products are known and deterministic.
- All requirements must be fulfilled in the period in which they occur: shortage or backordering is not allowed.
- The suppliers offer quantity discounts (all-units and incremental discounts) of each product. Each supplier offer one quantity discounts only.
- Ordering cost from supplier does not depend on the variety and quantity of products involved.
- Holding cost of product per period is product-dependent.
- Transportation cost is known for the capacity each vehicle of the supplier.
- The buyer's limited storage capacity.
- Initial inventory of the first period and the inventory at the end of the last period are assumed to be zero (I_{10} = 0; I_{20} = 0; I_{30} = 0).

Model notations

Notations used in the formulation of the model are as follows:

Indices:

i Product (*i* = 1,…, *I*).

j Supplier (*j* = 1,…, *J*), Where *j* = 1 to supplier *j'* for all-units quantity discounts, and supplier *j* = *j'* + 1 to *J* for incremental quantity discounts.

k Price break (1 = *k*,…, *K*).

t Time period (*t* = *t*,…, *T*).

Parameters:

D_{it} Demand of product *i* in period *t*.

P_{ijk} Purchase cost under discount schedule based on the quantity level of product *i* from supplier *j* with price break *k*.

Q_{ijk} The upper bound quantity of product *i* from supplier *j* with price break *k*.

I_{it} Expected ending inventory level of product *i* in period *t*.

Y_{it} Expected beginning available inventory level of product *i* in period *t*.

H_i Holding cost of product *i* per period.

O_j Ordering cost from supplier *j*.

τ_j Transportation cost per vehicle from supplier *j*.

V_j Vehicle capacity from supplier *j*.

N_{jt} Number of vehicles from supplier *j* in period *t*.

s_i Storage space required for product *i*.

S Total storage capacity of the buyer.

M A large number.

Decision variables:

X_{ijkt} Number of product *i* ordered from supplier *j* with price break *k* in period *t*.

F_{jt} A binary variable, set equal to 1 if purchase quantity from supplier *j* in period *t*, 0 otherwise.

U_{ijkt} A binary variable, set equal to 1 if purchase quantity of product *i* ^{i} from supplier *j* with price break *k* in period *t*, 0 otherwise.

Objective Function

$$\text{Min} = \sum_{j}^{J}\sum_{t}^{T} O_j F_{jt} + \sum_{j}^{J}\sum_{t}^{T} \tau_j N_{jt} + \sum_{i}^{I}\sum_{j}^{J}\sum_{k}^{K}\sum_{t}^{T} P_{ijk} X_{ijkt} + \sum_{i}^{I}\sum_{j=j'+1}^{J}\sum_{k}^{K}\sum_{t}^{T} U_{ijkt}\left[\sum_{k'=1}^{k-1} P_{ijk'}\left(Q_{ijk'} - Q_{ijk'-1}\right) - \left(P_{ijk} Q_{ijk-1}\right)\right] + \sum_{i}^{I}\sum_{t}^{T} \frac{H_i}{2}\left(2Y_{it} - D_{it}\right) \quad (1)$$

Subject to

$$I_{it} = Y_{it} - D_{it}, \qquad \forall i, \forall t \qquad (2)$$

$$Y_{it} = I_{it-1} + \sum_{j}^{J}\sum_{k}^{K} X_{ijkt}, \qquad \forall i, \forall t \qquad (3)$$

$$\sum_{k}^{K} X_{ijkt} \le M \times F_{jt}, \qquad \forall i, \forall j, \forall t \qquad (4)$$

$$Q_{ijk-1} U_{ijkt} \le X_{ijkt} \le Q_{ijk} U_{ijkt}, \qquad \forall i, \forall j, \forall k, \forall t \qquad (5)$$

$$N_{jt} = \left\lceil \frac{\sum_{i}^{I}\sum_{k}^{K} S_i X_{ijkt}}{V_j} \right\rceil \qquad \forall j, \forall t \qquad (6)$$

$$\sum_{i}^{I} Y_{it} S_i \le S \qquad \forall t \qquad (7)$$

$$F_{jt} \in \{0, 1\}, \qquad \forall j, \forall t \qquad (8)$$

$$U_{ijkt} \in \{0, 1\}, \qquad \forall i, \forall j, \forall k, \forall t \qquad (9)$$

$$X_{ijkt}, \ge 0, \text{ and Integer} \qquad \forall i, \forall j, \forall k, \forall t \qquad (10)$$

In the formulation, the objective function as shown in Equation (1) that consists of four parts: the total cost (TC) of ordering cost for the suppliers, transportation cost for the suppliers, purchase cost of the products and holding cost for remaining inventory in each period. In Equation (2) the expected endings inventory of product *i* in period *t*. In Equation (3) the beginning available inventory level of product *i* in period *t*. In Equation (4) is to set the ordering cost from supplier *j* in period *t*. In Equation (5) set the purchase between lower bound quantity and upper bound quantity price break *k* of product *i* from supplier *j* in period *t*. In Equation (6) is number of vehicles from supplier *j* in period *t*. In Equation (7) the buyer storage capacity in period *t*. Finally, Equations (8) – (10) are used to force non-negative integer values and binary variable 0 or 1 in the model.

NUMERICAL EXAMPLE

Example data

The effectiveness of propose MILP model is demonstrated through a numerical example. The data used in these examples consists of the demands of three products over a planning horizon of five periods as shown in Table 2. The ordering cost, transportation cost and vehicle capacity of each supplier is show in Table 3.The quantity discounts are all-units and incremental discounts for each supplier in Table 4.

Table 1: Demands of three products over a planning horizon of five periods

(D_{it})	Period (t)				
	1	2	3	4	5
Product 1	230	1,750	650	1,410	2,950
Product 2	465	1,510	2,410	515	1,850
Product 2	500	510	475	1,000	700

Table 2: Ordering cost, transportation cost, vehicle capacity of each supplier and types of quantity discounts

	Ordering cost	*Transportation cost*	*Vehicle capacity*	*Quantity discounts*
Supplier 1	$200	$50	25	All-units
Supplier 2	$250	$60	30	Incremental
Supplier 3	$270	$70	35	All-units

Table 3: Holding cost and storage space

	Holding cost	Storage space
Product 1	$0.1	0.2
Product 2	$0.2	0.3
Product 3	$0.3	0.5

Total storage capacity of the buyer is 2,000.

Table 4: The quantity discounts of each supplier

	Product 1			*Product 2*			*Product 3*		
	Price break (k)	*Quantity level* (Q_{1jk})	*Price units* (P_{1jk})	*Price break* (k)	*Quantity level* (Q_{2jk})	*Price units* (P_{2jk})	*Price break* (k)	*Quantity level* (Q_{3jk})	*Price units* (P_{3jk})
Supplier 1	1	0-1,100	$3.00	1	0-2,000	$2.98	1	0-1,999	$2.88
	2	1,101-2,200	$2.93	2	2,001-4,000	$2.83	2	2,000-3,599	$2.63
	3	2,201-3,400	$2.82	3	4,001-6,000	$2.72	3	3,600-4,099	$2.52
	4	3,401 or more	$2.75	4	6,001 or more	$2.65	4	4,100 or more	$2.45
Supplier 2	1	0-999	$3.12	1	0-999	$2.78	1	0-999	$2.68
	2	1,000-2,599	$2.92	2	1,000-1,999	$2.62	2	1,500-2,499	$2.52
	3	2,600-4,099	$2.89	3	2,000-2,999	$2.59	3	2,500-3,499	$2.49
	4	4,100 or more	$2.76	4	3,000 or more	$2.46	4	3,500 or more	$2.33
Supplier 3	1	0-2,000	$3.14	1	0-999	$2.68	1	0-2,999	$2.83
	2	2,001-4,000	$2.96	2	1,000-1,999	$2.52	2	3,000-3,999	$2.60
	3	4,001-6,000	$2.79	3	2,000-2,999	$2.49	3	4,000-4,999	$2.58
	4	6,001 or more	$2.66	4	3,000 or more	$2.33	4	5,000 or more	$2.41

Numerical results

In this section, we solved a numerical example of the proposed model. To solve this, the MILP model and the optimal solution are found. The model is coded in optimization package like LINGO 12 and was run on computer with 1.80 GHz Intel processor Core i3, and 4 GB RAM. The results of the numerical example with two products, two suppliers and five periods can be determined.

The solution obtained by the MILP model is shown in Table 5. The MILP model can provide a satisfactory solution with a very short elapsed runtime. The computational time is 6 seconds. Figure 2 shown result of the numerical example solvers by LINGO 12. An example of the purchase cost of supplier 1 the buyer needs to determine to order quantities from supplier 1, X_{1132} = 2,400 units of product 1 is 2,400 x 2.82 = $6,768.00. The supplier 2 the buyer needs to determine to order quantities from supplier 2, X_{1211} = 230 units of product 1 is 230 x 3.12 = $717.6.00. The supplier 3 the buyer needs to determine to order quantities from supplier 3, X_{2333} = 2,925 units of product 2 is 2,925 x 2.49 = $7,283.25. Thus, the total costs from supplier 1, supplier 2 and supplier 3 is $59,532.65.

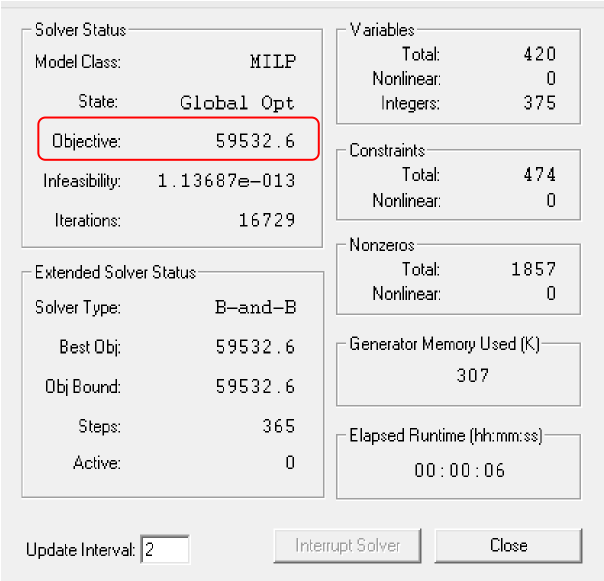

Figure 2: The results best objective solvers by LINGO 12

Table 5: The results of the numerical example

	Supplier 1	*Supplier 2*	*Supplier 3*
Number of orders (X_{ijkt})	$X_{1132} = 2{,}400$, $X_{1144} = 4{,}360$, $X_{3114} = 1{,}000$	$X_{1211} = 230$, $X_{2211} = 465$, $X_{2222} = 1{,}510$, $X_{2225} = 1{,}850$, $X_{3211} = 500$, $X_{3212} = 510$, $X_{3215} = 700$	$X_{2333} = 2{,}925$, $X_{3313} = 475$
Beginning inventory (Y_{it})	$Y_{11} = 230$, $Y_{12} = 2{,}400$, $Y_{13} = 650$, $Y_{14} = 4{,}360$, $Y_{15} = 2{,}950$	$Y_{21} = 465$, $Y_{22} = 1{,}510$, $Y_{23} = 2{,}925$, $Y_{24} = 515$, $Y_{25} = 1{,}850$	$Y_{31} = 500$, $Y_{32} = 510$, $Y_{33} = 475$, $Y_{34} = 1{,}000$, $Y_{35} = 700$
Ending inventory (I_{it})	$I_{12} = 650$, $I_{14} = 2{,}950$	$I_{23} = 515$	
Purchase cost	$46,166.45		
Ordering cost	$1,370.00		
Transportation cost	$10,031.62		
Holding cost	$1,964.53		
Total cost (TC)	$59,532.60		

CONCLUSIONS

In this paper, we proposed a multi-product and multi-period inventory lot-sizing problem with supplier selection under vehicle capacity and quantity discounts. Inventory lot-sizing with supplier selection under quantity discounts are important decisions any buyer has to make. A buyer needs to determine what products to order in what quantities with which suppliers in which periods, in order to satisfy overall demand and minimize the total cost. Selecting the right suppliers over multi-product and multi-period decision horizon becomes a major challenge for a buyer when suppliers offer quantity discounts, and with regard to vehicle capacity of each supplier. In addition, we consider that buyer's limited storage capacity.

The objectives are to minimize total costs, where the costs include purchase cost of the products, ordering cost for the suppliers, transportation cost for the suppliers and holding cost of the remaining inventory in each period. It is assumed that demand of multiple products is known over a planning horizon. The problem is formulated as a MILP model and is solved with optimization package LINGO 12. The MILP model can provide a satisfactory solution with a very short elapsed runtime is 6 seconds. Finally, a numerical example is provided to illustrate the solution procedure.

For future research, a model that takes into account stochastic demand or lead time, and safety stock is considered. In addition, the budget constraint is considered for the total costs.

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