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Comparative Study of Computational Material Models

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Abstract

The present work aims to investigate the framework for predicting the best model for a hyperelastic material like rubber, which is isotropic and incompressible. Different material models like Mooney-Rivlin, Yeoh, Neo-Hookean and Ogden have been considered. Increase in the usage of rubber in different applications from motor tyres to aerospace etc, drags the interest to study its mechanical properties. In general, Uniaxial test, Biaxial test, Simple planar test, Pure shear test and their combinations are useful to study the complete behaviour of the hyperelastic material. But in the present work, only uniaxial tests have been considered to compare with different computational models in ANSYS. However, reducing number of experiments might lead to inaccurate material parameters in strain energy function and it cannot replicate exactly the behaviour in other modes but it will give a reasonable approximation.

Material parameters for all models have been determined by curve fitting the stress-stretch relationship curves for respective models. By using these parameters, a rectangular model was simulated in ANSYS. The stress-strain curves obtained for respective models from the simulated data were compared with the Treloar's uniaxial test data [5]. After comparison of various models, the Mooney-Rivlin 3parameter model was found to fit perfectly with the Treloar's uniaxial test data within the range of 0-2.5 strain. Error obtained for the Mooney-3P was 0.037516 followed by 4.5956822 for Yeoh-1 order, 2.1949167 for Yeoh-2 order, 1.0093652 for Yeoh-3 order and 7.0425402 for Neo-Hookean model.

Keywords: Hyperelastic material, Uniaxial test, Curve fitting.

INTRODUCTION

Hyperelastic materials like rubber are used extensively in many applications and products, either in pure form or in combination with other materials. The study of mechanical properties of hyperelastic material is necessary to understand their behaviour in all loading conditions. The most attractive property of rubbers is their ability to experience large deformation under small loads. So, the linear theory of elasticity, especially Hooke's law, used since the 17th century, was an inadequate approach to study the mechanical properties of hyperelastic materials as proved by Mooney [1] and Rivlin [2] by their experimental measurements of the stress-strain relationship for rubber materials. A new classical theory of elasticity was developed to study the mechanical properties of those materials whose stress-strain curves are non-linear.

Boyce and Arruda [3] compared five models and Steinmann et al. [4] evaluated performance and validity of the models using Treloar's experimental data [5]. Shahzad *et al*. [6] aimed to characterize hyper-elastic material and to determine strain energy function (SEF) for an indigenously developed rubber. More recently, Marckmann and Verron [7] considered experimental data of two different authors and gave ranking to twenty-one material models. Kim *et al.* [8] brought comparison among Neo-Hookean Model, Mooney-Rivlin Model, and Ogden Model for Chloroprene Rubber. Rey *et al.* [9] investigated on effects of temperature on mechanical properties of Silicon Rubber. Yoshida *et al.* [10] developed a mathematical model for high-damping rubber materials. Böl & Reese [11] proposed a new finite element based simulation method for polymer networks. Vandenbroucke *et al.* [12] presented an original phenomenological model, named Hyperelasto-Visco-Hysteresis (HVH model). Guo and Sluys [13] worked on simulation of rubber-like material behaviour by means of the finite element method. Yoshida *et al*. [14] studied on a three-dimensional finite element modelling of high damping rubber bearings. Ali *et al.* [15] reviewed several classical continuum mechanics models for incompressible and isotropic materials based on strain energy function.

The non-linear theory of elasticity, which constitutes the theoretical basis for the study of hyperelastic materials such as rubber uses a strain-energy function (W). The strain-energy function is based on three strain invariants I_1, I_2, and I_3 to describe the mechanical behaviour of this class of materials.

Strain-Energy Function (W):

The energy stored in a material per unit of reference volume as a function of strain a point is called Strain-energy function.

$$W = f(I_1, I_2, I_3) \tag{1}$$

Where I_1, I_2, and I_3 are three invariants of Right Cauchy Green deformation tensor ($C = F^T F$) defined in terms of principle stretch ratios λ_1, λ_2, and λ_3 are given below

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 \qquad (2)$$

$$I_3 = \lambda_1^2\lambda_2^2\lambda_3^2$$

If hyperelastic materials are considered incompressible then $I_3 = \det(C) = 1$; hence only two independent strain measures namely I_1 and I_2 remain. This implies that 'W' is a function of I_1 and I_2 only. In the undeformed state $I_1=3$ and $I_2=3$; hence the SEF can be written as:

$$W = W(I_1 - 3, I_2 - 3) \qquad (3)$$

Strain energy functions for the respective models:

For detailed study of strain energy functions and stress-stretch relationship equations for the respective material models reader can refer [16] and [17].

Mooney–Rivlin material model:

$$W = \frac{\mu_1}{2} * (I_1 - 3) - \frac{\mu_2}{2} * (I_2 - 3) \qquad (4)$$

$$\frac{\partial W}{\partial I_1} = \frac{\mu_1}{2} = c_1; \frac{\partial W}{\partial I_2} = \frac{-\mu_2}{2} = c_2;$$

So that we may rewrite the above expression in the form:

$$W = \sum_{i=1}^{2} c_i (I_i - 3) \qquad (5)$$

Yeoh material model:

$$W = \sum_{i=1}^{3} c_i (I_i - 3)^i \qquad (6)$$

The material constants c_1, c_2 and c_3 are the parameters that we want to fit in our work.

Neo–Hookean material model:

$$W = c_1 (I_i - 3) \qquad (7)$$

The constant c_1 allows us to know the shear modulus by the relation

$$\mu = 2c_1. \qquad (8)$$

Stress-stretch relations for the respective models:

Mooney–Rivlin model:

$$\sigma_{Mooney-3P} = 2c_{10}\left(\lambda - \frac{1}{\lambda}\right) + 2c_{01}\left(1 - \frac{1}{\lambda^3}\right) + 6c_{11}\left(\lambda^2 - \lambda - 1 + \frac{1}{\lambda^2} + \frac{1}{\lambda^3} + \frac{1}{\lambda^4}\right) \qquad (9)$$

Constraints

$c_{10} + c_{01} \geq 0$ and $c_{11} \geq 0$

Yeoh model:

$$\sigma_{Yeoh} = 2(\lambda^2 - \tfrac{1}{\lambda})(c_1 + 2c_2(I_1 - 3) + 3c_3(I_1 - 3)^2 \qquad (10)$$

Neo–Hookean model:

$$\sigma_{Neo-Hookean} = 2c_1\left(\lambda^2 - \frac{1}{\lambda}\right) \qquad (11)$$

Where,

λ = stretch = $\frac{l + \Delta l}{l}$

l=length of the specimen; Δl = change in length

Objectives:

The objectives of the present study include the following:

- Curve fit the uniaxial test data to determine parameters for various models.
- Using the parameters define the various hyper elastic models.
- Simulate the uniaxial test in ANSYS by creating an FE model.
- Determine the performance of each material model.

Scope of the Study:

The scope of the study is to determine the material parameters for respective models through curve fitting. Treloar's uniaxial test data [5] is considered to determine the material parameters. Curve fitting can be done in ANSYS or MS Excel or MATLAB. In the present work curve fitting was done in MS Excel (2016). These parameters obtained were necessary to understand the mechanical behaviour of hyperelastic materials like rubber.

Next, we compared the stress-strain curves obtained for respective material models with the Treloar's test data [5]. Stress-strain curves were obtained, after the simulation of the model in ANSYS. This simulation was carried out on 2D-rectangular model with one end clamped (all degrees of freedom zero) and the other end free along X-axis i.e. (displacement in Y-axis and Z-axis is zero).

Curve Fitting Procedure:

Parameters for the respective models were determined in curve fitting procedure. These were the parameters which were used to simulate the model in ANSYS APDL 16.0.

Determination of Parameters can be done in two ways:

i. Directly by giving test data as input in ANSYS APDL16.0.
ii. Manually by considering stress-stretch relationship equations of the respective models.

In the first method, if the accuracy is poor the error cannot be reduced. But in the second method the error can be calculated and can be reduced to give the best fit.

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In the present work, parameters were determined with the Treloar's data [5] for uniaxial test using stress-stretch relation of respective models in MS Excel.

Curve Fitting Procedure in MS Excel:

Calculated $\sigma_{(Mooney\text{-}3p)}$ using equation (9)

Error = $(\sigma_{(Mooney\text{-}3p)} - \sigma_{(Treloar\ data)})^2$

Table 1: $\sigma_{(Mooney\text{-}3p)}$ and Error values before curve fitting.

Strain	$\sigma_{Treloar\ data}$	$\sigma_{Mooney-3P}$	Error	Stretch(λ)
0	0	0	0	1
0.0068694	3.8806E-06	0.068051892	0.004630532	1.006869
0.1234078	0.142425655	1.106768843	0.929957783	1.123408
0.2400473	0.225532513	2.115415061	3.571656045	1.240047
0.3910341	0.308658768	3.592340927	10.78256852	1.391034
0.5969566	0.403679048	6.073735436	32.14953944	1.596957
0.8784499	0.494787661	10.41492268	98.40907887	1.87845
1.1462178	0.577979925	15.54516683	224.0166839	2.146218
1.4071162	0.661168266	21.44950999	432.1551518	2.407116
2.0044436	0.847359284	38.21518409	1396.354331	3.004444
2.56741	1.041439556	58.05073346	3250.059592	3.56741
3.013636	1.211727914	76.51399915	5670.432053	4.013636
3.7205715	1.575925811	110.6920048	11906.3187	4.720572
4.3244514	1.947974127	144.6559409	20365.56379	5.324451
4.715445	2.280358957	168.9856374	27790.64987	5.715445
5.1200633	2.679974845	196.097366	37410.28722	6.120063
5.373647	3.024144915	214.0913877	44549.38096	6.373647
		Total Error =	153141.0658	

Table 2: Material parameters for Mooney-3P before curve fitting

parametres	
C10	1
C01	1
C11	1

Table: 3 $\sigma_{(Mooney\text{-}3p)}$ and error values after curve fitting

Strain	σ(Treloar data)	σ(Mooney-3p)	Error	Stretch(λ)
0	0	0	0	1
0.0068694	3.8806E-06	0.011768767	0.000138413	1.006869
0.1234078	0.142425655	0.167438953	0.000625665	1.123408
0.2400473	0.225532513	0.266980497	0.001717935	1.240047
0.3910341	0.308658768	0.350548183	0.001754723	1.391034
0.5969566	0.403679048	0.423644727	0.000398628	1.596957
0.8784499	0.494787661	0.493847698	8.83531E-07	1.87845
1.1462178	0.577979925	0.55350588	0.000598979	2.146218
1.4071162	0.661168266	0.615220611	0.002111187	2.407116
2.0044436	0.847359284	0.788998493	0.003405982	3.004444
2.56741	1.041439556	1.004981272	0.001329206	3.56741
3.013636	1.211727914	1.215063417	1.11256E-05	4.013636
3.7205715	1.575925811	1.619916018	0.001935138	4.720572
4.3244514	1.947974127	2.036208828	0.007785362	5.324451
4.715445	2.280358957	2.340451067	0.003611062	5.715445
5.1200633	2.679974845	2.684029887	1.64434E-05	6.120063
5.373647	3.024144915	2.914257337	0.01207528	6.373647
		Total Error =	0.037516013	

Table 4: Material parameters for Mooney-3P after curve fitting

parametres	
C10	-0.056
C01	0.3273
C11	0.0148

Repeat the same procedure for all the respective models to calculate material parameters and error.

Create and Simulate Finite Element Model in ANSYS:

Rectangular geometry

- The dimension of this model was specified in pre-processing phase by using 3D-coordinate system in ANSYS APDL16.0.
- Length along X-axis is 40mm, breadth along y-axis is 13mm and thickness along Z-axis is 4mm were specified.
- Direct modelling method was used to construct the geometry by using the area tool in pre-processing section of ANSYS APDL16.0.
- Smart meshing tool was used for meshing.

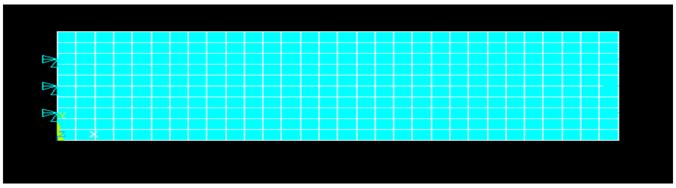

Figure 1. Rectangular Geometry with Quadrilateral mesh and loading.

RESULTS AND CONCLUSIONS

Curve Fitting Results for Treloar's Uniaxial Test Data:

Parameters obtained after the curve fitting for respective models were used to simulate the non-linear hyperelastic model in ANSYS APDL16.0.

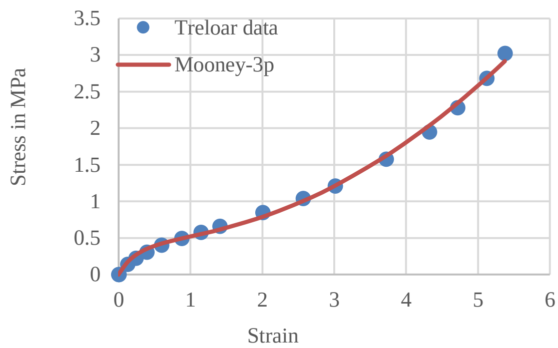

Figure 2. Curve fitting data for Mooney-3parameter

Strain range is reduced for both Mooney-3P to get best fit curves. From this we analyse that this computational model is suitable for the strain value below 5.0.

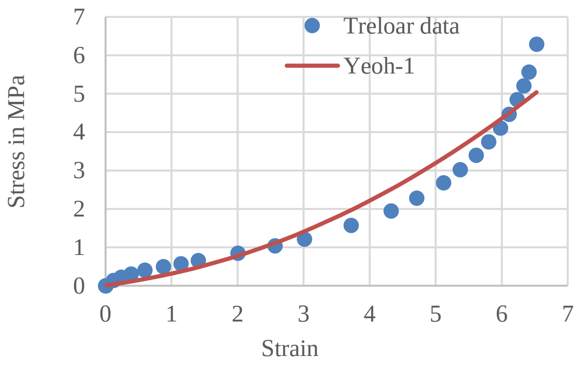

Figure 3: Curve fitting data for Yeoh-1order

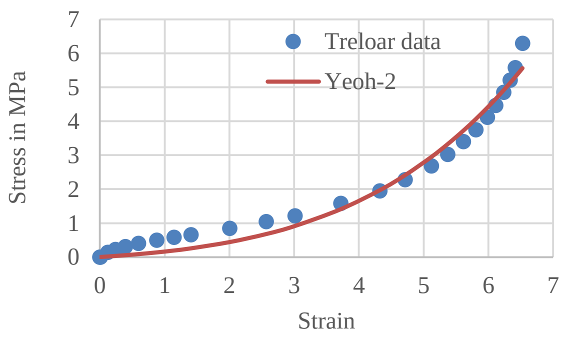

Figure 4: Curve fitting data for Yeoh-2order

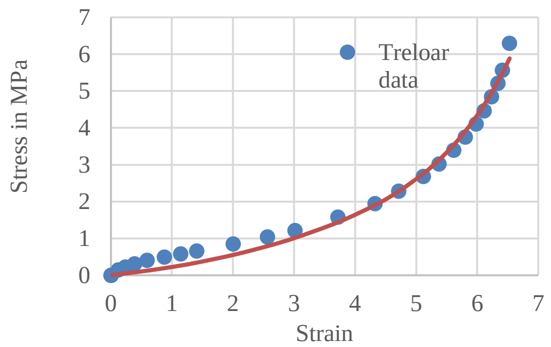

Figure 5: Curve fitting data for Yeoh-3order

From the figure5 it is clear that Yeoh-3order is suitable for large strain values i.e. above 4.0.

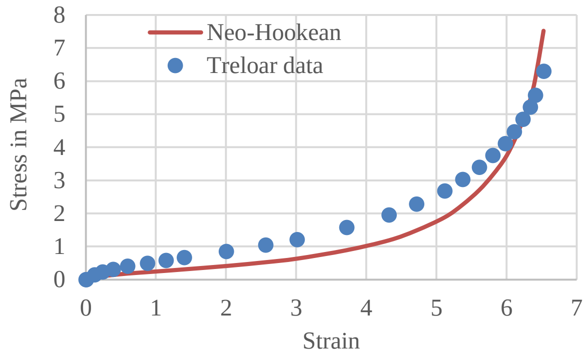

Figure 6: Curve fitting data for Neo-Hookean model

From all the above curve fitting data for different computational models Mooney-3parameter model fitted with least error.

Respective material parameters and the error obtained during curve fitting were mentioned in the table 5.

Material parameters for the respective Models:

Table 5: Material Parameters and error values for the respective models

Material models	Parameters		Error
Mooney-Rivlin-3P	C_10	-0.05624902	0.037516
	C_01	0.32734975	
	C_11	0.01483209	
Yeoh-1	C_1	0.04453756	4.5956822
Yeoh-2	C_1	0.02199498	2.1949167
	C_2	0.00025214	
Yeoh-3	C_1	0.03151799	1.0093652
	C_2	0.00018431	
	C_3	0.0001181	
Neo-Hookean	C_1	0.07082235	7.0425402

From the above table it was observed that Mooney-3 parameter model fitted perfectly followed by Yeoh-3order, Yeoh-2order, Yeoh-1order and Neo-Hookean.

Comparison of Simulated Models Stress-Strain Curves with Treloar's Test Data:

Stress-strain curves drawn for the results obtained after simulating uniaxial tension test by applying 100mm displacement in X-direction and zero displacement in Y-direction at one end and zero displacement at another end in all directions of the rectangular geometry for 10sec are given below. For the given loading condition, the maximum strain developed is 2.5. So, the comparison of stress-strain curves of the respective computational models with the Treloar's uniaxial test data to predict the best model is considered with in the strain range of 0 – 2.5.

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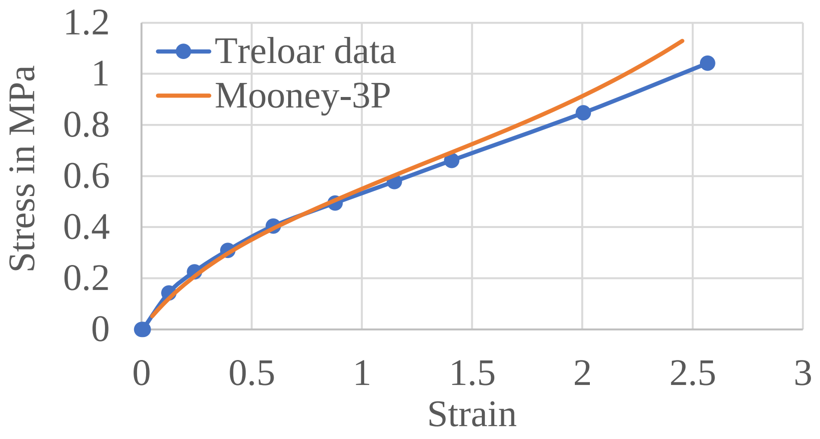

Figure 7: Comparison of Mooney-3P stress-strain curve with Treloar's Data

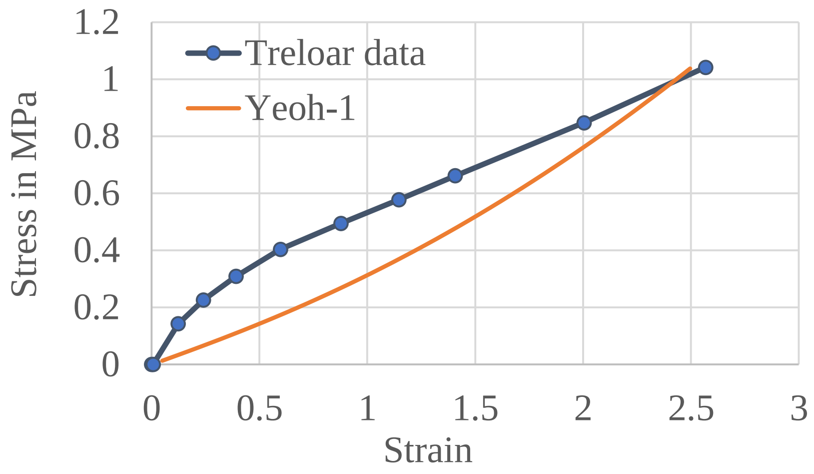

Figure 8: Comparison of Yeoh-1order stress-strain curve with Treloar's Data

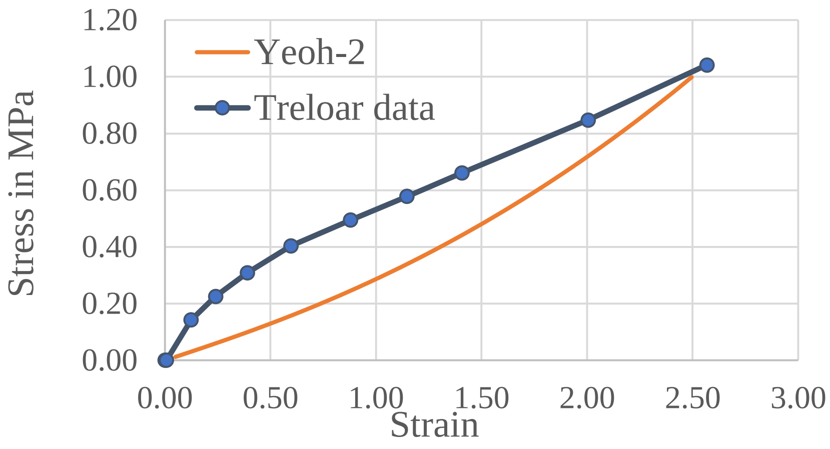

Figure 9: Comparison of Yeoh-2order stress-strain curve with Treloar's Data

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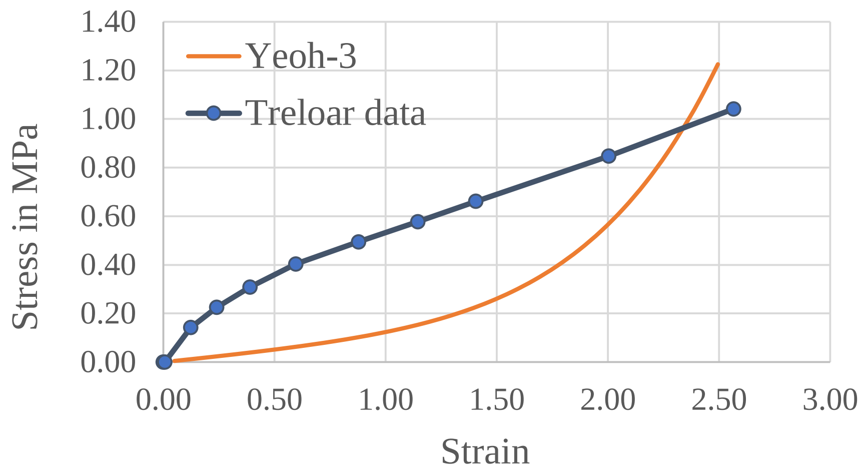

Figure 10: Comparison of Yeoh-3order stress-strain curve with Treloar's Data

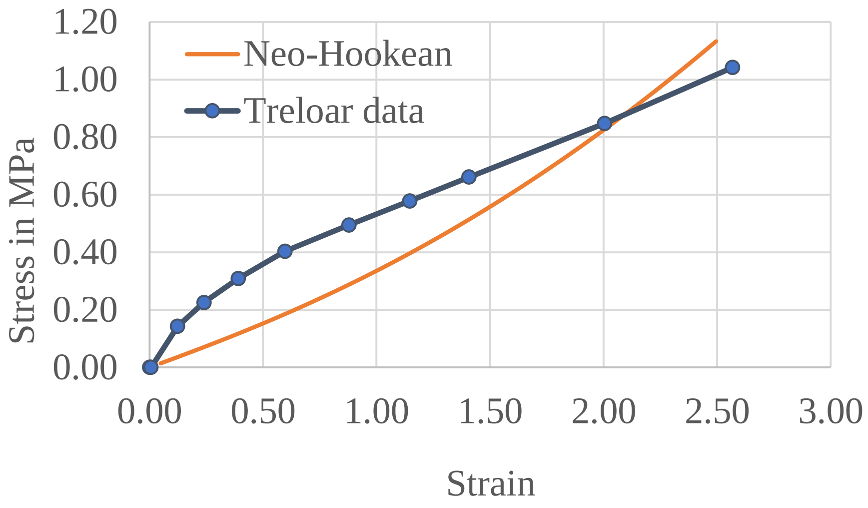

Figure 11: Comparison of Neo-Hookean stress-strain curve with Treloar's Data

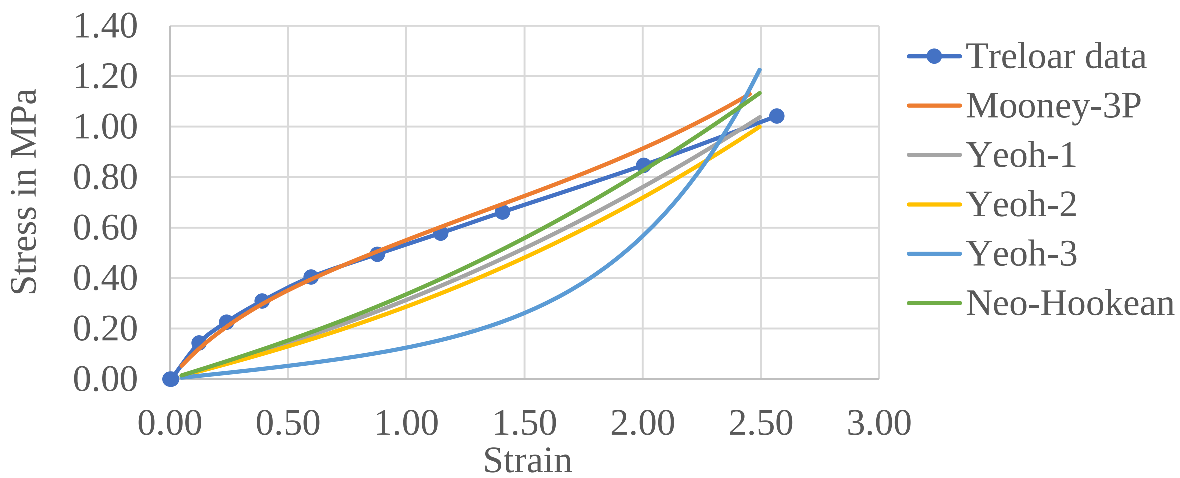

Figure 12: Comparison of all respective model's stress-strain curves with Treloar's Data

CONCLUSIONS

By observing the fig:12 it was clear that Mooney-3P model almost fitted perfectly with the Treloar's data. The best results were obtained with Mooney-3P. Yeoh-(order1,2 and 3) and Neo-Hookean models showed comparatively poor performance. All the tested models except Mooney-3P gave high error.

For Mooney-3P model the error obtained was 0.038. Even though the error was very less the stress-strain curve obtained after simulation did not fit perfectly by giving those parameters that were directly obtained from the curve fitting as input to simulate the model. So, this model required adjustment of material parameters. After adjusting the values of material parameters, we came to know that C-01 and C-11 parameters showed much impact on the curve. By reducing C-01 and C-11 values by 1.37 times and 3 times the model fitted the Treloar's data curve perfectly.

For Yeoh-1order model the error obtained was 4.6. Adjustment was not done for this model. Because the change in value showed almost zero impact on the curve. This model showed poor performance.

For Yeoh-2order model the error obtained was 2.2 which was comparatively less than Yeoh-1order. But it was not appreciable value. After adjusting the values of material parameters, we came to know that C-1 and C-2 parameters showed much impact on the curve. Even after dividing C-1 and C-2 values with 0.55 and 3 the model did not fit the Treloar's data curve perfectly.

For Yeoh-3order model the error obtained was 1.01 which was comparatively less than Yeoh-(1and 2order). But it was also not appreciable value. After adjusting the values of material parameters, we came to know that C-1 and C-2 parameters showed much impact on the curve. Even after dividing C-1 and C-2 values with 2 and 4 the model did not fit the Treloar's data curve perfectly.

For Neo-Hookean model the error obtained was 7.04 which was comparatively more than remaining models. After adjusting the values of material parameters, we came to know that C-1 parameter showed much impact on the curve. Even after dividing C-1 value with 1.5 the model did not fit the Treloar's data curve perfectly.

FINAL REMARKS

Material parameters obtained for respective models as shown in table:5 underwent some adjustments to fit the stress-strain curve drawn for the simulated data with Treloar's data. Based on the requirement, values of the material parameters can be either decreased or increased. This was possible because they were considered as arbitrary values during curve fitting. This type of adjustment was necessary in this work because the respective model did not fit perfectly with those values that were directly obtained from the curve fitting. But adjustment for material parameters was not compulsory for all works. By adjusting the values, one can identify that particularly which parameter effected the shape of the curve.

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