International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 10 (2018) pp. 8116-8125

An Interactive Intuitionistic Fuzzy Non-Linear Fractional Programming Problem

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Abstract

This paper develops a new interactive method for solving non-linear fractional programming problem based on the intuitionistic fuzzy set theory. In this interactive method, if the decision maker (DM) specifies the degree α of the α-level sets then a max-min problem can be solved by using Zimmermann's min operator and the satisficing solution of the DM can be derived from among an α-Pareto optimal solution set by updating the degree α based on the current values of the membership function and the non – membership function. Firstly, fuzzy intuitionistic in the coefficients can be characterized by fuzzy numbers based on α- cut analysis according to the degree α which specified by the DM. Secondly, intuitionistic fuzzy non-linear fractional programming problem (IFNLFPP) is transformed into an equivalent crisp multi-objective non-linear fractional programming problem (MONLFPP). Also, by using the concept of fuzzy mathematical programming approach, MONLFPP can be reduced into a single objective non-linear programming problem (NLPP) which can be solved easily by using any suitable NLPP algorithm. The proposed procedure is illustrated by a numerical example.

Keyword: Fuzzy sets, Interactive decision making, Triangular intuitionistic fuzzy number Intuitionistic fuzzy non-linear fractional programming problem.

INTRODUCTION

In real life decision making situations, the decision makers often face problems in making decision from linear or non-linear fractional programming problems (FPP_s). The objectives are generally conflicted, non-commensurable and fuzzy in nature and many considerations of the vague nature of uncertainty should be taken in the formulation of the problem.

Naturally the objective functions and constraints are uncertain in their nature and involve many fuzzy parameters. These fuzzy parameters are characterized as fuzzy numbers introduced by Sakawa [1-4]. Many researchers have investigated different kinds of fuzzy non-linear fractional programming problem (FNLFPP). The FNLFPP can be classified into two categories such as NLFPP with fuzzy goals and NLFPP with fuzzy coefficients.

The concept of fuzzy sets (FS) seems to be most appropriate to deal with such imprecise date, introduced by Zadeh [5] and Bellman and Zadeh [6] which proposed the definition of fuzzy decision. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. Therefore, the theory of intuitionistic fuzzy set (IFS) is expected to play an important role in modern mathematics in general as it represents a generalization of fuzzy set.

The notation of IFS was first defined by Atanassov [7, 8, 9] as a generalization of Zadeh's [5] fuzzy set. The knowledge and semantic representation of IFS become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness and the hesitation margin by Atanassov [10]. Szmidt and Kacprzyk [11, 12] showed that IFS are pretty useful in situations when description of a problem by a linguistic variable given in terms of a membership function only seems too rough.

In a recent review, Hezibah [13] gave a Taylor series approach to intuitionistic fuzzy multi-objective non-linear programming problem (IFMONLPP). Singh and Yadav [14] developed an approach for solving non-linear programming problem in intuitionistic fuzzy environment. Singh and Yadav [15] proposed an approach for solving intuitionistic fuzzy linear fractional programming problem (IFLFPP). Raouf, Ali Hassan and Hezam [16] investigated method for solving fractional programming problems under uncertainty (FPPU) using Sperm Motility algorithm.

In this paper, in order to deal with non-linear fractional programming problem with fuzzy parameters characterized by fuzzy numbers, the concept of α- Pareto optimality is introduced by depending on the basis of α - level sets of fuzzy numbers. Then an interactive decision making method to derive the satisficing solution of the decision maker efficiently from among an α - Pareto optimal solution set is presented as a generalization of the results obtained in Sakawa et al. [1-4]. Also, IFNLFPP is considered with the coefficients of objective function and the set of constraints are triangular intuitionistic fuzzy numbers. The given IFNLFPP is transformed into deterministic multi-objectives non-linear fractional programming problem (MONLFPP). Next, by using fuzzy mathematical programming approach [17, 18, 19], MONLFPP is transformed and reduced to a single objective NLPP. Finally, a numerical example is given to illustrate the feasibility of this method.

Brief Introduction of Intuitionistic Fuzzy Sets

Definition 1[5]: Let X be a non empty set. A fuzzy set A drawn from X is defined as $A=\{<x,\mu_A(x)>: x\in X\}$, where $\mu_A(x): X\to[0,1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.

Definition 2[21, 22]**:** A real fuzzy number $\tilde{P}$ is a continuous fuzzy subset from the real line R whose triangular membership function $\mu_{\tilde{P}}(P)$ is defined by as:

(1) A continuous mapping from R to the closed interval [0,1], as shown in figure1.

(2) $\mu_{\tilde{P}}(P)=0$ for all $P\in(-\infty,P_1]$.

(3) Strictly increasing on $[P_1,P_2]$.

(4) $\mu_{\tilde{P}}(P)=1$ for $P=P_2$.

(5) Strictly decreasing on $P\in[P_2,P_3]$.

(6) $\mu_{\tilde{P}}(P)=0$ for all $P\in[P_3,+\infty)$.

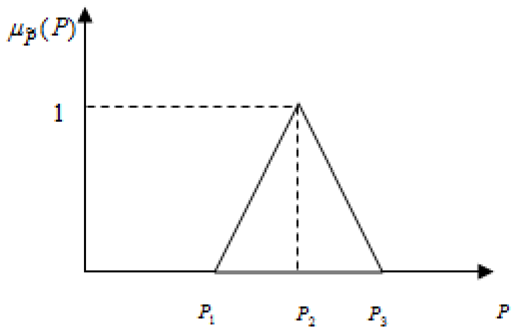

Figure 1: Membership Function of Fuzzy Number $\tilde{P}$

Definition 3[16]**:** The α - level set of the fuzzy number $\tilde{P}$ is defined as the ordinary set $L_\alpha(\tilde{P})$ for which the degree of membership function exceeds the level α, $\alpha\in[0,1]$, where,

$$L_\alpha(\tilde{P})=\{P\in R \mid \mu_{\tilde{P}}(P)\geq\alpha\} \tag{1}$$

Definition 4[14,15]**:** Let X be a universe of discourse. Then an intuitionistic fuzzy set (IFS) $\tilde{A}^I$ in X is defined by a set of ordered triples :

$\tilde{A}^I=\{<x,\mu_{\tilde{A}^I}(x),\nu_{\tilde{A}^I}(x)>: x\in X\}$,

where $\mu_{\tilde{A}^I}, \nu_{\tilde{A}^I}: X\to[0,1]$ are functions such that $0\leq\mu_{\tilde{A}^I}(x)+\nu_{\tilde{A}^I}\leq 1,\ \forall x\in X$. The value $\mu_{\tilde{A}^I}(x)$ represents the degree of membership and $\nu_{\tilde{A}^I}(x)$ represents the degree of non – membership of the element $x\in X$ being in $\tilde{A}^I$. $h(x)=1-\mu_{\tilde{A}^I}(x)-\nu_{\tilde{A}^I}(x)$ is degree of hesitation of the element $x\in X$ being in $\tilde{A}^I$.

Definition 5[14,15]**:** An IFS $\tilde{A}^I=\{<x,\mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)>: x\in X\}$ is called an intuitionistic fuzzy number (IFN) if the following hold:

(1) There exists $m\in R$ such that $\mu_{\tilde{A}^I}(m)=1$ *and* $\nu_{\tilde{A}^I}(m)=0$ where m is the mean value of $\tilde{A}^I$.

(2) $\mu_{\tilde{A}^I}$ *and* $\nu_{\tilde{A}^I}$ are piecewise continuous functions from R to the closed interval [0,1] and $0\leq\mu_{\tilde{A}^I}(x)+\nu_{\tilde{A}^I}(x)\leq 1\ \forall x\in R$ where

$$\mu_{\tilde{A}^I}(x)=\begin{cases} g_1(x), & m-\tilde{a}\leq x<m,\\ 1, & x=m\\ h_1(x), & m<x\leq m+\tilde{b},\\ 0, & otherwise, \end{cases} \tag{2}$$

and

$$\nu_{\tilde{A}^I}(x)=\begin{cases} g_2(x), & m-\tilde{a}'\leq x<m;\ 0\leq g_1(x)+g_2(x)\leq 1\\ 0, & x=m,\\ h_2(x), & m<x\leq m+\tilde{b}';\ 0\leq h_1(x)+h_2(x)\leq 1\\ 1, & otherwise. \end{cases} \tag{3}$$

Here m is the mean value of $\tilde{A}^I$; *a*, *b* are the left and right spreads of membership function $\mu_{\tilde{A}^I}$, respectively; $\tilde{a}'$ and $\tilde{b}'$ are the left and right spreads of non-membership function $\nu_{\tilde{A}^I}$ respectively; g_1 and h_1 are piecewise continuous, strictly increasing, and strictly decreasing function in $[m-\tilde{a},m)$ and $(m,m+\tilde{b}]$, respectively; g_2 and h_2 are piecewise continuous, strictly decreasing, and strictly increasing function in $[m-\tilde{a}', m]$ and $[m, m+\tilde{b}']$, respectively. The IFN $\tilde{A}^I$ is represented by:

$\tilde{A}^I = (m; a, b; a', b')$.

Definition 6[14, 15]**:** A triangular intuitionistic fuzzy number (TIFN) $\tilde{A}^I$ is an IFN with the membership function $\mu_{\tilde{A}^I}$ and non-membership function $v_{\tilde{A}^I}$ given by :

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-a}{b-a} & , \quad a < x \le b, \\ 1 & , \quad x = b \quad , \\ \frac{c-x}{c-b} & , \quad b \le x < c, \\ 0 & , \quad otherwise, \end{cases} \tag{4}$$

and

$$v_{\tilde{A}^I}(x) = \begin{cases} \frac{b-x}{b-a'} & , \quad a' < x \le b, \\ 0 & , \quad x = b \quad , \\ \frac{x-b}{c'-b} & , \quad b \le x < c', \\ 1 & , \quad otherwise. \end{cases} \tag{5}$$

where $a' \le a \le b \le c < c'$. This TIFN is denoted by:

$\tilde{A}^I = (a, b, c; a', b, c')$.

Definition 7[14]**:** When $a = a'$ and $c = c'$ then the TIFN reduces to a triangular fuzzy number (TFN) denoted by $\tilde{A} = (a, b, c)$.

Definition 8[14]**:** Arithmetic operations on TIFNs:

Let $\tilde{A}^I = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}_1', \tilde{a}_2, \tilde{a}_3')$ and $\tilde{B}^I = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3; \tilde{b}_1', \tilde{b}_2, \tilde{b}_3')$,

Addition:

$\tilde{A}^I \oplus \tilde{B}^I = (\tilde{a}_1 + \tilde{b}_1, \tilde{a}_2 + \tilde{b}_2, \tilde{a}_3 + \tilde{b}_3; \tilde{a}_1' + \tilde{b}_1', \tilde{a}_2 + \tilde{b}_2, \tilde{a}_3' + \tilde{b}_3')$.

Subtraction:

$\tilde{A}^I \Theta \tilde{B}^I = (\tilde{a}_1 - \tilde{b}_3, \tilde{a}_2 - \tilde{b}_2, \tilde{a}_3 - \tilde{b}_1; \tilde{a}_1' - \tilde{b}_3', \tilde{a}_2 - \tilde{b}_2, \tilde{a}_3' - \tilde{b}_1')$.

Multiplication: $\tilde{A}^I \otimes \tilde{B}^I = (\tilde{l}_1, \tilde{l}_2, \tilde{l}_3; \tilde{l}_1', \tilde{l}_2', \tilde{l}_3')$,

where $\tilde{l}_1 = \min\{\tilde{a}_1\tilde{b}_1, \tilde{a}_1\tilde{b}_3, \tilde{a}_3\tilde{b}_1, \tilde{a}_3\tilde{b}_3\}$,

$\tilde{l}_3 = \max\{\tilde{a}_1\tilde{b}_1, \tilde{a}_1\tilde{b}_3, \tilde{a}_3\tilde{b}_1, \tilde{a}_3\tilde{b}_3\}$,

$l_1' = \min\{\tilde{a}_1'\tilde{b}_1', \tilde{a}_1'\tilde{b}_3', \tilde{a}_3'\tilde{b}_1', \tilde{a}_3'\tilde{b}_3'\}$,

$l_3' = \max\{\tilde{a}_1'\tilde{b}_1', \tilde{a}_1'\tilde{b}_3', \tilde{a}_3'\tilde{b}_1', \tilde{a}_3'\tilde{b}_3'\}, l_2 = \tilde{a}_2\tilde{b}_2$.

Division:

$\tilde{A}^I \oslash \tilde{B}^I = (\tilde{a}_1/\tilde{b}_3, \tilde{a}_2/\tilde{b}_2, \tilde{a}_3/\tilde{b}_1; \tilde{a}_1'/\tilde{b}_3', \tilde{a}_2/\tilde{b}_2, \tilde{a}_3'/\tilde{b}_1')$ Scalar multiplication

1. $k\tilde{A}^I = (k\tilde{a}_1, k\tilde{a}_2, k\tilde{a}_3, k\tilde{a}_1', k\tilde{a}_2, k\tilde{a}_3'), k > 0$.

2. $k\tilde{A}^I = (k\tilde{a}_3, k\tilde{a}_2, k\tilde{a}_1; k\tilde{a}_3', k\tilde{a}_2, k\tilde{a}_1'), k < 0$.

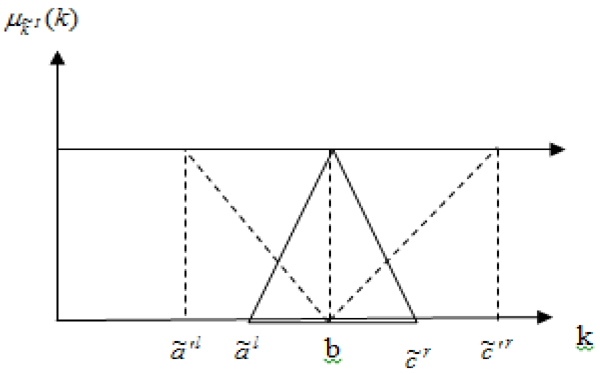

Figure 2: Triangular Intuitionistic Fuzzy Number

Problem Formulation and Solution Concepts

The general mathematical model of IFNLFPP is considered as follows:

$$Max \;\; Z(\tilde{x}^I) = \frac{f(x, \tilde{A}^I)}{g(x, \tilde{B}^I)}, \tag{6}$$

$$s.t. \;\; h_{k_1}(x, \tilde{C}^I) \le \tilde{D}_{k_1}^I, \quad k_1 = 1.....s_1,$$

$$h_{k_2}(x, \tilde{E}^I) \ge \tilde{J}_{k_2}^I \quad, k_2 = s_1 + 1,....s_2,$$

$$h_{k_3}(x, \tilde{I}^I) = \tilde{L}_{k_3}^I, \quad k_3 = s_2 + 1,.....s,$$

$$x_r \ge 0, \quad r = 1,2,\ldots,n,$$

$$g(x, \tilde{B}^I) \ne 0.$$

where x is n-dimensional decision variable vector $x = (x_1, x_2, \ldots, x_n)$,

$f(x, \tilde{A}^I)$ and $g(x, \tilde{B}^I) \ne 0, h_{k_1}(x, \tilde{C}^I), h_{k_2}(x, \tilde{E}^I)$ and $h_{k_3}(x, \tilde{I}^I)$ respectively are supposed to be real valued continuous non-linear functions with IFNs. The parameters $\tilde{A}^I, \tilde{B}^I, \tilde{C}^I, \tilde{D}^I, \tilde{E}^I, \tilde{J}^I, \tilde{I}^I$ and $\tilde{L}$ respectively are assumed to be TIFNs.

In this section, the procedure for solving an interactive IFNLFPP can be developed where all coefficients are TIFNs. This problem differs from the crisp problem by parametric values. In crisp or non-fuzzy models, the parameters are known exactly. Therefore, for a certain degree of α, as in definition 3, which specified by the DM, problem (6) can be reformulated as the following intuitionistic non-fuzzy α-non-linear fractional programming problem (α- INLFPP) with intuitionistic non-fuzzy numbers (α-IFNs)as:

$$Max \qquad Z(x) = \frac{F(x, A)}{G(x, B)}, \tag{7}$$

$$
\begin{aligned}
s.t. \quad & H_{k_1}(x,C) \le D_{k_1}, \; k_1 = 1,\dots,s_1, \\
& H_{k_2}(x,E) \ge J_{k_2}, \; k_2 = s_1+1,\dots,s_2, \\
& H_{k_3}(x,I) = L_{k_3}, \; k_3 = s_2+1,\dots,s, \\
& x_r \ge 0 \quad , \qquad r = 1,2,\dots,n, \\
& G(x,b) \ne 0, \qquad \mu_{\tilde{k}^I}(k) \ge \alpha_k,
\end{aligned}
$$

where k is any coefficient and the parameters A,B,C,D,E,J,I and L respectively are assumed to be non-fuzzy numbers defined as $(\tilde{a}^l, b, \tilde{c}^r; \tilde{a}'^l, b, \tilde{c}'^r)$ in Figure 2..

Solution Procedure for an Interactive IFNLFPP

By using the division in definition (8), problem (7) reduces to an equivalent intuitionistic multi-objective non-linear fractional programming problem (IMONLFPP) as follows:

$$
\begin{aligned}
Max \quad & Z_1(x) = \frac{F(x,a_1)}{G(x,b_3)}, \\
Max \quad & Z_2(x) = \frac{F(x,a_2)}{G(x,b_2)}, \\
Max \quad & Z_3(x) = \frac{F(x,a_3)}{G(x,b_1)}, \\
Max \quad & Z_4(x) = \frac{F(x,a_1')}{G(x,b_3')}, \\
Max \quad & Z_5(x) = \frac{F(x,a_3')}{G(x,b_1')}, \\
s.t. \quad & H_{k_1}(x,C) \le D_{k_1}, k_1 = 1,\dots,s_1, \\
& H_{k_2}(x,E) \ge J_{k_2}, \; k_2 = s_1+1,\dots,s_2, \\
& H_{k_3}(x,I) = L_{k_3}, \; k_3 = s_2+1,\dots,s, \\
& x_r \ge 0, \quad r = 1,2,\dots,n.
\end{aligned} \tag{8}
$$

Let us assume that $Z_1(x)$, $Z_2(x)$, $Z_3(x)$, $Z_4(x)$, *and* $Z_5(x) \ge 0$ feasible region of problem (8). Hence, by using Charnes and Cooper's transformation [19, 20], the above model IMONLFPP can be converted into the following intuitionistic multi-objective non-linear programming problem (IMONLPP) by taking y = t x, t > 0, as follows:

$$
\begin{aligned}
Max \quad & Z_1(y/t) = F(y/t,a_1), \\
Max \quad & Z_2(y/t) = F(y/t,a_2), \\
Max \quad & Z_3(y/t) = F(y/t,a_3), \\
Max \quad & Z_4(y/t) = F(y/t,a_1'), \\
Max \quad & Z_5(y/t) = F(y/t,a_3'), \\
s.t. \quad & G(y/t,b_3) \le 1 \quad , \\
& G(y/t,b_2) \le 1 \quad , \\
& G(y/t,b_1) \le 1 \quad , \\
& G(y/t,b_3') \le 1 \quad , \\
& G(y/t,b_1') \le 1 \quad , \\
& H_{k_1}(y/t,c_1) \le (d_1)_{k_1}, k_1 = 1,\dots,s_1, \\
& H_{k_1}(y/t,c_2) \le (d_2)_{k_1}, k_1 = 1,\dots,s_1, \\
& H_{k_1}(y/t,c_3) \le (d_3^r)_{k_1}, k_1 = 1,\dots,s_1, \\
& H_{k_1}(y/t,c_1') \le (d_1')_{k_1}, k_1 = 1,\dots,s_1, \\
& H_{k_1}(y/t,c_3') \le (d_3^{r'})_{k_1}, k_1 = 1,\dots,s_1, \\
& H_{k_2}(y/t,e_1) \ge (j_1^L)_{k_2}, \; k_2 = s_1+1,\dots,s_2 \quad , \\
& H_{k_2}(y/t,e_2) \ge (j_2)_{k_2}, \; k_2 = s_1+1,\dots,s_2 \quad , \\
& H_{k_2}(y/t,e_3) \ge (j_3)_{k_2}, \; k_2 = s_1+1,\dots,s_2 \quad , \\
& H_{k_2}(y/t,e_1') \ge (j_1^{L'})_{k_2}, \; k_2 = s_1+1,\dots,s_2 \quad , \\
& H_{k_2}(y/t,e_3') \ge (j_3')_{k_2}, \; k_2 = s_1+1,\dots,s_2 \quad , \\
& H_{k_3}(y/t,i_1) = (L_1^L)_{k_3}, k_3 = s_2+1,\dots,s, \\
& H_{k_3}(y/t,i_2) = (L_2)_{k_3}, k_3 = s_2+1,\dots,s, \\
& H_{k_3}(y/t,i_3) = (L_3^r)_{k_3}, k_3 = s_2+1,\dots,s, \\
& H_{k_3}(y/t,i_1') = (L_1^{L'})_{k_3}, k_3 = s_2+1,\dots,s, \\
& H_{k_3}(y/t,i_3') = (L_3^{r'})_{k_3}, k_3 = s_2+1,\dots,s, \\
& y \ge 0, t > 0.
\end{aligned} \tag{9}
$$

Now to solve problem (9), the following algorithm can be developed as:

Algorithm (1):

Step 1: Use the method which proposed by Singh and Yadav [14],then we extend this method to decompose problem (9) into five sub-problems MONLPP$_s$ according to TIFN$_s$ as follows:

(P$_1$):

$$
\begin{aligned}
Max \quad & Z_1(y/t) = F(y/t,a_1), \\
Max \quad & Z_2(y/t) = F(y/t,a_2) \quad , \\
Max \quad & Z_3(y/t) = F(y/t,a_3) \quad , \\
Max \quad & Z_4(y/t) = F(y/t,a_1') \quad , \\
Max \quad & Z_5(y/t) = F(y/t,a_3') \quad , \\
s.t. \quad & G(y/t,b_3) \le 1, \\
& H_{k_1}(y/t,c_1) \le (d_1)_{k_1} \quad , k_1 = 1,\dots,s_1, \\
& H_{k_2}(y/t,e_1) \ge (j_1^L)_{k_2} \quad , k_2 = s_1+1,\dots,s_2, \\
& H_{k_3}(y/t,i_1) = (L_1^L)_{k_3} \quad , k_3 = s_2+1,\dots,s, \\
& y \ge 0, t > 0.
\end{aligned}
$$

(P2):

$$\begin{aligned}
&Max && Z_1(y/t) = F(y/t, a_1),\\
&Max && Z_2(y/t) = F(y/t, a_2)\quad ,\\
&Max && Z_3(y/t) = F(y/t, a_3)\quad ,\\
&Max && Z_4(y/t) = F(y/t, a_1')\quad ,\\
&Max && Z_5(y/t) = F(y/t, a_3')\quad ,\\
&s.t. && G\ (y/t, b_2) \le 1,\\
&H_{k_1}(y/t, c_2) \le (d_2)_{k_1} && , k_1 = 1,..., s_1,\\
&H_{k_2}(y/t, e_2) \ge (j_2)_{k_2} && , k_2 = s_1 + 1,..., s_2,\\
&H_{k_3}(y/t, i_2) = (L_2)_{k_3} && , k_3 = s_2 + 1,..., s,\\
& && y \ge 0, t > 0.
\end{aligned}$$

(P3):

$$\begin{aligned}
&Max && Z_1(y/t) = F(y/t, a_1),\\
&Max && Z_2(y/t) = F(y/t, a_2)\quad ,\\
&Max && Z_3(y/t) = F(y/t, a_3)\quad ,\\
&Max && Z_4(y/t) = F(y/t, a_1')\quad ,\\
&Max && Z_5(y/t) = F(y/t, a_3')\quad ,\\
&s.t. && G(y/t, b_1) \le 1,\\
&H_{k_1}(y/t, c_3) \le (d_3^r)_{k_1} && , k_1 = 1,\dots,s_1,\\
&H_{k_2}(y/t, e_3) \ge (j_3)_{k_2} && , k_2 = s_1 + 1,\dots,s_2,\\
&H_{k_3}(y/t, i_3) = (L_3^r)_{k_3} && , k_3 = s_2 + 1,\dots,s,\\
& && y \ge 0, t > 0.
\end{aligned}$$

(P4):

$$\begin{aligned}
&Max && Z_1(y/t) = F(y/t, a_1),\\
&Max && Z_2(y/t) = F(y/t, a_2)\quad ,\\
&Max && Z_3(y/t) = F(y/t, a_3)\quad ,\\
&Max && Z_4(y/t) = F(y/t, a_1')\quad ,\\
&Max && Z_5(y/t) = F(y/t, a_3')\quad ,\\
&s.t. && G\ (y/t, b_3') \le 1,\\
&H_{k_1}(y/t, c_1') \le (d_1')_{k_1} && , k_1 = 1,..., s_1,\\
&H_{k_2}(y/t, e_1') \ge (j_1^{i'})_{k_2} && , k_2 = s_1 + 1,..., s_2,\\
&H_{k_3}(y/t, i_1') = (L_1^{L'})_{k_3} && , k_3 = s_2 + 1,..., s,\\
& && y \ge 0, t > 0.
\end{aligned}$$

and

(P5):

$$\begin{aligned}
&Max && Z_1(y/t) = F(y/t, a_1),\\
&Max && Z_2(y/t) = F(y/t, a_2)\quad ,\\
&Max && Z_3(y/t) = F(y/t, a_3)\quad ,\\
&Max && Z_4(y/t) = F(y/t, a_1')\quad ,\\
&Max && Z_5(y/t) = F(y/t, a_3')\quad ,\\
&s.t.\\
&G(y/t, b_1') \le 1,\\
&H_{k_1}(y/t, c_3') \le (d_3^{r'})_{k_1}, k_1 = 1,..., s_1,\\
&H_{k_2}(y/t, e_3') \ge (j_3')_{k_2}, K_2 = s_1 + 1,...., s_2,\\
&H_{k_3}(y/t, j_3') = (l_3^{r'})_{k_2}, K_3 = s_2 + 1,...., s,\\
&y \ge 0, t > 0
\end{aligned}$$

Step 2: Solve the models $P_i, i = 1,2,3,4,5,$ as an individual objective functions under the given constraints.

Step 3: Find optimal points of all the sub-problems and let the total solution set be $X = \bigcup_{i=1}^{5} X_i$

Step 4: Find the value of each objective function $Z_i(y/t), i = 1,2,3,4,5$ at each point obtained in step 3.

Step 5: Find upper and lower bounds U and L respectively for objective function,

$L = \min\{Z_i(y/t) : x \in X, i = 1,2,\dots 5\}$ and $U = \max\{Z_i(y/t) : x \in X, i = 1,2,\dots 5\}$

Step 6: Then IMONLFPP (8) is equivalent to the following fuzzy model using Zimmermann's technique [23]:

Find x

$$\text{s.t.} \quad Z(x) \gtrsim U \tag{10}$$

$$\begin{aligned}
&H_{k_1}(x, C) \lesssim D_{k_1}, && k_1 = 1,\dots,s_1,\\
&H_{k_2}(x, E) \gtrsim J_{k_2}, && k_2 = s_1 + 1,\dots,s_2,\\
&H_{k_3}(x, I) \approx L_{k_3}, && k_3 = s_2 + 1,\dots,s,\\
& x_r \ge 0, r = 1,2,\dots,n.
\end{aligned}$$

where $\lesssim$, $\gtrsim$ and $\approx$ are fuzzy inequalities and fuzzy equality respectively. Here the meaning of fuzzy is that some tolerance is allowed in rigid equality and inequalities which is prescribed by the DM.

Since the objective is to be maximized, the DM's satisfaction level increases as the objective value reaches towards upper bound. Let μ_U denotes the degree of attainability of the upper bound U of the objective function Z (x) and the minimum acceptable level of the objective value be L by the DM.

Step 7: Take the membership functions as:

$$\mu_U(Z(x)) = \begin{cases} 0 & , \text{ if } Z(x) < L , \\ \dfrac{(Z(x))^t - L^t}{U^t - L^t} & , \text{ if } L \le Z(x) \le U, \\ 1 & , \text{ if } Z(x) > U. \end{cases} \quad (11)$$

Let μ_D, μ_j *and* μ_L respectively represent the degree of attainability of available constraints and are defined by:

For $(k_1 = 1,2,\ldots,s_1)$,

$$\mu_D(H_{k_1}(x,C)) = \begin{cases} 1 & , \text{ if } H_{k_1}(x,C) < d_{k_1} , \\ \dfrac{(d^r_{k_1})^t - (H_{k_1}(x,C))^t}{(d^r_{k_1})^t - (d_{k_1})^t} & , \text{ if } d_{k_1} \le H_{k_1}(x,C) \le d^r_{k_1}, \\ 0 & , \text{ if } H_{k_1}(x,C) > d^r_{k_1}. \end{cases} \quad (12)$$

For $(k_2 = s_1 + 1,\ldots,s_2)$,

$$\mu_J(H_{k_2}(x,E)) = \begin{cases} 0 & , \text{ if } H_{k_2}(x,E) < j_{k_2} , \\ \dfrac{(H_{k_2}(x,E))^t - (j^L_{k_2})^t}{(j_{k_2})^t - (j^L_{k_2})^t} & , \text{ if } j^L_{k_2} \le H_{k_2}(x,E) \le j_{k_2}, \\ 1 & , \text{ if } H_{k_2}(x,E) > j_{k_2} . \end{cases} \quad (13)$$

For $(k_3 = s_2 + 1,\ldots,s)$,

$$\mu_L(H_{k_3}(x,I)) = \begin{cases} 0 & , \text{ if } H_{k_3}(x,I) < l_{k_3} , \\ \dfrac{(H_{k_3}(x,I))^t - (l^L_{k_3})^t}{(l_{k_3})^t - (l^l_{k_3})^t} & , \text{ if } l^l_{k_3} \le H_{k_3}(x,I) \le l_{k_3}, \\ \dfrac{(l^r_{k_3})^t - (H_{k_3}(x,I))^t}{(l^r_{k_3})^t - (l_{k_3})^t} & , \text{ if } l_{k_3} \le H_{k_3}(x,I) \le l^r_{k_3}, \\ 0 & , \text{ if } H_{k_3}(x,I) > l_{k_3} . \end{cases} \quad (14)$$

where t > 0 is prescribed by the DM.

IFNLFPP may be described as how to make a reasonable plan so that the DM is most satisfied with fuzzy objective as well as fuzzy constraints. That is there should be highest degree of balance between fuzzy objective and Fuzzy constraints. Let

$\lambda = \min\{\mu_U(Z(x)), \mu_D(H_{k_1}(x,C)), k_1 = 1,2,\ldots,s_1, \mu_J(H_{k_2}(x,E)), k_2 = s_1+1,\ldots,s_2, \mu_L(H_{k_3}(x,I)), k_3 = s_2+1,\ldots,s\}$, where λ is the overall satisfaction level for the DM.

Step 8: Ask the DM to select t ,then transform model (10) into the crisp model which can easily be solved by suitable crisp NLPP methods as follows:

$$\begin{aligned} &Max \quad \lambda \\ &s.t. \quad \mu_U(Z(x)) \ge \lambda, \\ &\mu_D(H_{k_1}(x,C)) \ge \lambda, \quad k_1 = 1,2,\ldots,s_1 , \\ &\mu_J(H_{k_2}(x,E)) \ge \lambda, \quad k_2 = s_1+1,\ldots,s_2 , \\ &\mu_L(H_{k_3}(x,I)) \ge \lambda, \quad k_3 = s_2+1,\ldots,s , \\ &x_r \ge 0 , \quad r = 1,2,\ldots,n . \end{aligned} \quad (15)$$

or:

$$\begin{aligned} &Max \quad \lambda \\ &s.t. \quad (Z(x))^t - t^t \ge \lambda\ (U^t - L^t) \\ &(d^r_{k_1})^t - (H_{k_1}(x,C))^t \ge \lambda \left((d^r_{k_1})^t - (d_{k_1})^t\right), k_1 = 1,2,\ldots,s_1, \\ &(H_{k_2}(x,E))^t - (j^l_{k_2})^t \ge \lambda \left((j_{k_2})^t - (j^L_{k_2})^t\right), k_2 = s_1+1,\ldots,s_2, \\ &(H_{k_2}(x,I))^t - (l^l_{k_3})^t \ge \lambda \left((l_{k_3})^t - (l^L_{k_3})^t\right), k_3 = s_2+1,\ldots,s , \\ &(l^r_{k_3})^t - (H_{k_3}(x,I))^t \ge \lambda \left((l^r_{k_3})^t - (l_{k_3})^t\right), k_3 = s_2+1,\ldots,s_2, \\ &x_r \ge 0 \ , r = 1,2,\ldots,n \end{aligned} \quad (16)$$

The above mentioned algorithm is summarized as follows:

Main Algorithm of the Interactive Intuitionistic Fuzzy Non-linear Fractional Programming Problem.

Based on the above discussions, the interactive algorithm is developed as follows, which derives the satisficing solution for the DM ensuring α - Pareto optimality.

Algorithm (2):

Step 1: Elicit a membership function (4) and (5) from the DM for each of the fuzzy numbers in the objective function and the constraints of the problem (6).

Step 2: Ask the DM to select the initial value of $\alpha, 0 < \alpha < 1.$

Step3: Construct intuitionstic non-fuzzy α -non-linear fractional programming problem (α -INLFPP) as in (7) and convert it into IMONLFPP as in (8).

Step 4: Use Charnes and Cooper's transformation [19-20] to convert IMONLFPP into IMONLPP as in (9).

Step 5: Construct model (9) as following five sub-problems MONLPPs according to TIFNs as indicate in step 1 in Algorithm (1)

Step 6: Solve the models Pi, i = 1, 2, 3, 4, 5, as an individual objective NLPP and let the total solution set be $X = \bigcup_{i=1}^{5} X_i$ for $P_i, i = 1,2,3,4,5.$

Step 7: Find upper and lower bound U and L respectively for objective function, and Let $L = \min\{Z_i(y/t); x \in X, i = 1,2,\ldots,5\}$ and $U = \max\{Z_i(y/t); x \in X, i = 1,2,\ldots,5\}$.

Step 8: IMONLFPP (8) is equivalent to the fuzzy model by using Zimmerman's technique as in (10).

Step 9: Elicit a membership function (11), (12), (13) and(14) respectively from the DM. Model (10) can be transformed into the crisp model (16) which can easily be solved by suitable crisp NLPP methods**.**

Step 10: If the DM is satisfied with current solution and the current values of the objective functions, stop. Otherwise ask the DM to update the degree α and return to step (3).

Numerical Example

Let us consider the following IFNLFPP as:

$$\begin{aligned} Max \quad & Z(\tilde{x}^I) = \frac{\tilde{8}\,x_1^2 - \tilde{7}\,x_2^2 + \tilde{9}}{\tilde{8}\,x_1^2 + \tilde{9}\,x_2^2 + \tilde{6}} \quad , \\ s.t. \quad & \tilde{4}x_1 + \tilde{3}\,x_2 \le \tilde{28} \quad , \qquad \text{(A)} \\ & \tilde{5}x_1 + \tilde{3}\,x_2 \le \tilde{20} \quad , \\ & x_1, x_2 \ge 0 \end{aligned}$$

Suppose that DM determines $\alpha = 0.5 \in [0,1]$. The membership function (4) and (5) is used to convert an IFN_s of the above problem (A) into its intuitionistic non-fuzzy numbers (α -IFN_s) refer to problem (7).

Let the IFN_s and α -IFN_s are given by the following listed in the table below:

IFN_s	**α -IFN_s**
$\tilde{8}^I$ = (7,8,9;6,8,10)	8 = (7.5,8,8.5; 7,8,9)
$\tilde{9}^I$ = (8,9,10;8,9,11)	9 = (8.5,9,9.5;8.5, 9, 10)
$\tilde{4}^I$ = (3,4,5;2,4,6)	4 = (3.5,4,4.5;3,4, 5)
$\tilde{5}^I$ = (4,5,6;3,5,7)	5 = (4.5,5,5.5;4,5, 6)
$\tilde{20}^I$ = (18,20,22;16,20,24)	20 = (19, 20, 21; 18 20,22)
IFN_s	**α -IFN_s**
$\tilde{7}^I$ = (6,7,8;5,7,b)	7 = (6.5,7,7.5;6,7,8)
$\tilde{6}^I$ = (4,6,8;4,6,8)	6 = (5,6,7;5,6,7)
$\tilde{3}^I$ = (2,3,4;1.5,3,4.5)	3 = (2.5, 3,3.5; 2.25, 3,3.75)
$\tilde{28}^I$ =(25, 28,30; 24, 28,32)	28 = (26.5, 28, 29; 26,28,30)

Problem (A) is equivalent to the following IMONLFPP(8) as follows:

$$\begin{aligned} Max \quad & Z_1(x) = \frac{7.5x_1^2 - 6.5x_2^2 + 8.5}{8.5x_1^2 + 9.5x_2^2 + 7}, \\ Max \quad & Z_2(x) = \frac{8x_1^2 - 7x_2^2 + 9}{8x_1^2 + 9x_2^2 + 6}, \\ Max \quad & Z_3(x) = \frac{8.5x_1^2 - 7.5x_2^2 + 9.5}{7.5x_1^2 + 8.5x_2^2 + 5}, \\ Max \quad & Z_4(x) = \frac{7x_1^2 - 6x_2^2 + 8.5}{9x_1^2 + 10x_2^2 + 7}, \\ Max \quad & Z_5(x) = \frac{9x_1^2 - 8.5x_2^2 + 10}{7x_1^2 + 8.5x_2^2 + 5} \quad , \qquad \text{(B)} \\ s.t. \quad & 3.5x_1 + 2.5x_2 \le 26.5 \quad , \\ & 4\,x_1 + 3\,x_2 \le 28, \\ & 4.5x_1 + 3.5x_2 \le 29, \end{aligned}$$

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 10 (2018) pp. 8116-8125

$$
\begin{aligned}
&3x_1 + 2.25x_2 \le 26\,,\\
&5x_1 + 3.75x_2 \le 30,\\
&4.5x_1 + 2.5x_2 \le 19,\\
&5x_1 + 3x_2 \le 20\,,\\
&5.5x_1 + 3.5x_2 \le 21,\\
&4x_1 + 2.25x_2 \le 18\,,\\
&6x_1 + 3.75x_2 \le 22\,,\\
&\qquad x_1\,,\; x_2 \ \ge 0.
\end{aligned}
$$

Use the transformation of Charnes and Cooper's [19-20], problem (B) is equivalent to the following IMONLPP (9) as follows:

$$
\begin{aligned}
&Max && Z_1(y/t) = 7.5y_1^2 - 6.5y_2^2 + 8.5\,t^2,\\
&Max && Z_2(y/t) = 8y_1^2 - 7y_2^2 + 9\,t^2,\\
&Max && Z_3(y/t) = 8.5y_1^2 - 7.5y_2^2 + 9.5\,t^2,\\
&Max && Z_4(y/t) = 7y_1^2 - 6y_2^2 + 8.5\,t^2,\\
&Max && Z_5(y/t) = 9y_1^2 - 8.5y_2^2 + 10\,t^2,\\
&s.t. && 8.5y_1^2 + 9.5y_2^2 + 7t^2 \le 1,\\
& && 8y_1^2 + 9y_2^2 + 6\,t^2 \le 1,\\
& && 7.5y_1^2 + 8.5y_2^2 + 5\,t^2 \le 1,\\
& && 9y_1^2 + 10y_2^2 + 7\,t^2 \le 1,\\
& && 7y_1^2 + 8.5y_2^2 + 5\,t^2 \le 1,\\
& && 3.5y_1 + 2.5y_2 - 26.5\,t \le 0,\\
& && 4y_1 + 3y_2 - 28\,t \quad \le 0,\\
& && 4.5y_1 + 3.5y_2 - 29\,t \le 0,\\
& && 3y_1 + 2.25y_2 - 26\,t \le 0,\\
& && 5y_1 + 3.75y_2 - 30\,t \le 0,\\
& && 4.5y_1 + 2.5y_2 - 19\,t \le 0,\\
& && 5y_1 + 3y_2 - 20\,t \le 0,\\
& && 5.5y_1 + 3.5y_2 - 21\,t \le 0,\\
& && 4y_1 + 2.25y_2 - 18\,t \le 0,\\
& && 6y_1 + 3.75y_2 - 22\,t \le 0,\\
& && y_1, y_2 \ge 0 \quad , \quad t > 0.
\end{aligned}
\tag{C}
$$

Problem (C) can be transformed into the following five sub-problems $MONLPP_s$ as follows:

P_1:

$$
\begin{aligned}
&Max && Z_1(y,t) = 7.5y_1^2 - 6.5y_2^2 + 8.5\,t^2,\\
&Max && Z_2(y,t) = 8y_1^2 - 7y_2^2 + 9\,t^2 \quad ,\\
&Max && Z_3(y,t) = 8.5y_1^2 - 7.5y_2^2 + 9.5\,t^2,\\
&Max && Z_4(y,t) = 7y_1^2 - 6y_2^2 + 8.5\,t^2 \quad ,\\
&Max && Z_5(y,t) = 9y_1^2 - 8.5y_2^2 + 10\,t^2 \quad ,\\
&s.t. && 8.5y_1^2 + 9.5y_2^2 + 7t^2 \le 1 \quad ,\\
& && 3.5y_1 + 2.5y_2 - 26.5\,t \le 0,\\
& && 4.5y_1 + 2.5y_2 - 19\,t \le 0 \quad ,\\
& && y_1, y_2 \ge 0, t > 0 \quad .
\end{aligned}
$$

P_2:

$$
\begin{aligned}
&Max && Z_1(y,t) = 7.5y_1^2 - 6.5y_2^2 + 8.5\,t^2,\\
&Max && Z_2(y,t) = 8y_1^2 - 7y_2^2 + 9\,t^2 \quad ,\\
&Max && Z_3(y,t) = 8.5y_1^2 - 7.5y_2^2 + 9.5\,t^2,\\
&Max && Z_4(y,t) = 7y_1^2 - 6y_2^2 + 8.5\,t^2 \quad ,\\
&Max && Z_5(y,t) = 9y_1^2 - 8.5y_2^2 + 10\,t^2 \quad ,\\
&s.t. && 8y_1^2 + 9y_2^2 + 6t^2 \le 1 \quad ,\\
& && 4y_1 + 3y_2 - 28t \le 0 \quad ,\\
& && 5y_1 + 3y_2 - 20\,t \le 0 \quad ,\\
& && y_1, y_2 \ge 0, t > 0 \quad .
\end{aligned}
$$

P_3:

$$
\begin{aligned}
&Max && Z_1(y,t) = 7.5y_1^2 - 6.5y_2^2 + 8.5\,t^2,\\
&Max && Z_2(y,t) = 8y_1^2 - 7y_2^2 + 9\,t^2 \quad ,\\
&Max && Z_3(y,t) = 8.5y_1^2 - 7.5y_2^2 + 9.5\,t^2,\\
&Max && Z_4(y,t) = 7y_1^2 - 6y_2^2 + 8.5\,t^2 \quad ,\\
&Max && Z_5(y,t) = 9y_1^2 - 8.5y_2^2 + 10\,t^2 \quad ,\\
&s.t. && 7.5y_1^2 + 8.5y_2^2 + 5t^2 \le 1 \quad ,\\
& && 4.5y_1 + 3.5y_2 - 29\,t \le 0 \quad ,\\
& && 5.5y_1 + 3.5y_2 - 21\,t \le 0 \quad ,\\
& && y_1, y_2 \ge 0, t > 0 \quad .
\end{aligned}
$$

P_4:

$$
\begin{aligned}
&Max && Z_1(y,t) = 7.5y_1^2 - 6.5y_2^2 + 8.5\,t^2,\\
&Max && Z_2(y,t) = 8y_1^2 - 7y_2^2 + 9\,t^2,\\
&Max && Z_3(y,t) = 8.5y_1^2 - 7.5y_2^2 + 9.5\,t^2,\\
&Max && Z_4(y,t) = 7y_1^2 - 6y_2^2 + 8.5\,t^2,\\
&Max && Z_5(y,t) = 9y_1^2 - 8.5y_2^2 + 10\,t^2
\end{aligned}
$$

$$s.t.\quad 9y_1^2 + 10y_2^2 + 7t^2 \le 1 \quad ,$$
$$3y_1 + 2.25y_2 - 26t \le 0 \quad ,$$
$$4y_1 + 2.25y_2 - 18\,t \le 0 \quad ,$$
$$y_1, y_2 \ge 0, t > 0 \quad .$$

P5:

$$Max \quad Z_1(y,t) = 7.5y_1^2 - 6.5y_2^2 + 8.5\,t^2,$$
$$Max \quad Z_2(y,t) = 8y_1^2 - 7y_2^2 + 9\,t^2 \quad ,$$
$$Max \quad Z_3(y,t) = 8.5y_1^2 - 7.5y_2^2 + 9.5\,t^2,$$
$$Max \quad Z_4(y,t) = 7y_1^2 - 6y_2^2 + 8.5\,t^2 \quad ,$$
$$Max \quad Z_5(y,t) = 9y_1^2 - 8.5y_2^2 + 10\,t^2 \quad ,$$
$$s.t.$$
$$7y_1^2 + 8.5y_2^2 + 5t^2 \le 1 \quad ,$$
$$5y_1 + 3.75y_2 - 30\,t \le 0 \quad ,$$
$$6y_1 + 3.75y_2 - 22\,t \le 0 \quad ,$$
$$y_1, y_2 \ge 0, t > 0 \quad .$$

Solve the models P_i, i = 1, 2, 3, 4, 5 as single objective NLPP. The lower and upper bounds L and U respectively for objective functions are L = 1.214286 and U = 2.000001.

IMONLFPP (B) is equivalent to the following fuzzy model (10) as:

$$Find \quad x$$
$$s.t. \quad \frac{9x_1^2 - 8.5x_2^2 + 10}{7x_1^2 + 8.5x_2^2 + 5} \gtrsim 2.000001, \qquad (D)$$
$$4x_1 + 3x_2 \lesssim 28 \quad ,$$
$$5x_1 + 3x_2 \lesssim 20 \quad ,$$
$$x = (x_1, x_2) \ge 0 \quad .$$

Further, utilizing the membership functions in (11) – (14), model (D) is equivalent to the following crisp model as:

$$Max \quad \lambda$$
$$s.t. \quad \left(\frac{9x_1^2 - 8.5x_2^2 + 10}{7x_1^2 + 8.5x_2^2 + 5}\right)^t - (1.214286)^t \ge \left[(2.000001)^t - (1.214286)^t\right],$$
$$4x_1 + 3x_2 \le 28 \quad ,$$
$$5x_1 + 3x_2 \le 20 \quad ,$$
$$x = (x_1, x_2) \ge 0 \quad ,$$

Taking t = 2 and solving by LINGO, the solution is $(x_1, x_2) = (0,0), \tilde{Z}^I(x) = (1.2143, 1.5, 1.9; 1.2143, 1.5, 2)$ with satisfaction level $\lambda = 1$.

CONCLUSION

In this paper, an interactive method for solving the IFNLFPP where the coefficients of the objective function and the constraints are taken TIFNs is proposed based on α - cut analysis which specified by the DM. In the proposed method, IFNLFPP is transformed to IMONLFPP and the resultant problem is converted to NLPP by using fuzzy mathematical programming approach. The proposed methodology will be very helpful for decision making problems in planning, scheduling and manufacturing systems have uncertainty and hesitation. In future the algorithm can be modified to solve goal intuitionistic fuzzy method for bi-level multi-objective non-linear fractional programming problems.

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