

# A Wavelet Analysis for Identifying Simulated Anomalies Superimposed to Real Signals

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## Abstract

When intense working conditions (e.g. production on continuous shifts) do not permit plant inactivity, conjugating both the continuous request of productivity and the constant demand of high performance, in terms of qualitative standards, becomes one of the most important factor in determining the profit of a business. The aim of this paper is to propose a method based both on wavelet and discriminant analysis for the identification of simulated anomalies superimposed to real signals carried out on mechanical equipment.

**Keywords:** wavelet analysis, anomalies finding, discriminant analysis, rotation system.

## INTRODUCTION

Actually the analysis performed by wavelet represents an advanced technique of windowing. It concurs to use long intervals in the case the information must be excreted both from high and low frequency signal. A wavelet is, like suggests the name, a small wave; many physical phenomena show a structure like wavelet. From a methodological point of view, the wavelet technique provides a multiscale analysis of the signal as a sum of orthogonal signals corresponding to different time scales hierarchically organized [1]. The development of wavelets is fairly recent in applied mathematics, but wavelets have already had a remarkable impact [2]. The most common application of wavelets is in signal processing [3]. A signal is a sequence of numerical measurements, typically obtained electronically (e.g. ECG, broadcasting, seismogram, etc.). Signals are typically

contaminated by random noise and an important part of signal processing is accounting for this noise [4]. If signal processing is to be done in “real time”, i.e., if the signals are treated as they are observed (data logging), it is important that fast and reliable algorithms are implemented [5]. The following methodology was successfully applied in several fields, such as robotics [6-9], mechanics [10-11], advanced signal processing [12-16].

## MATERIALS AND METHODS

A wavelet estimator of is given by [17-18]:

$$\hat{f} = \sum_k \hat{a}_{j_0k} \varphi_{j_0k}(x) + \sum_{j=j_0}^{j_l} \sum_k \hat{b}_{jk} \psi_{jk}(x) \quad (1)$$

where

$$\varphi_{j,k}(x) = 2^{j/2} \cdot \phi(2^j x - k) \quad k \in \mathbb{Z}$$

$$\phi(x) = \begin{cases} 1, & x \in (0,1] \\ 0, & x \notin (0,1] \end{cases},$$

$$\psi_{j,k}(x) = 2^{j/2} \cdot \psi(2^j x - k)$$

$$\psi(x) = \begin{cases} -1, & x \in \left[0, \frac{1}{2}\right] \\ 1, & x \in \left(\frac{1}{2}, 1\right] \end{cases}$$

$$\hat{a}_{j_0k} = \frac{1}{n} \sum_{i=1}^n \varphi_{j_0k}(X_i)$$

$$\hat{b}_{jk} = \frac{1}{n} \sum_{i=1}^n \psi_{jk}(X_i)$$

The definition of this estimator is based on the Parseval Theorem. In fact, according to this result, any  $h \in L_2(\mathbb{R})$  can be represented as a convergent series

$$h(x) = \sum_k a_{j_0,k} \cdot \varphi_{j_0,k}(x) + \sum_{j=j_0}^l \sum_k b_{j,k} \cdot \psi_{j,k}(x) \quad (2)$$

where

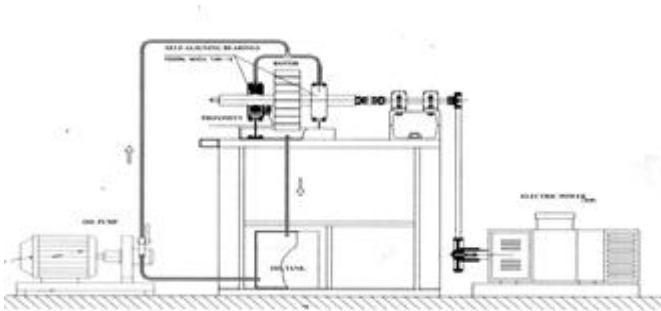
$$a_{j_0,k} = \int_{-\infty}^{+\infty} h(x) \cdot \frac{1}{\sqrt{2^{j_0}}} \psi\left(\frac{x-k}{2^{j_0}}\right) dx$$

and

$$b_{j,k} = \int_{-\infty}^{+\infty} h(x) \cdot \frac{1}{\sqrt{2^j}} \varphi\left(\frac{x-k}{2^j}\right) dx$$

Moreover, the systems of functions  $\{\{\varphi_{j_0,k}\}, \{\psi_{j,k}\}, k \in \mathbb{Z}, j = 0, 1, 2, \dots\}$  is a complete orthonormal system in  $L_2(\mathbb{R})$ : it is called Haar wavelet system. The functions  $\varphi$  and  $\psi$  are called, respectively, father and mother wavelet [19]. Recall that, many authors [17] propose to use  $j_0 = 0$  and  $j_1 = \text{Int}(\log_2 N)$ .

In Figure 1 is showed the mechanical equipment used for generating the signals.



**Figure 1:** A view of mechanical equipment

The motion to rotor is transmitted by an electrical power with a ratio 1:2. The rotor is assembled on two self-aligning bearings lubricated by an external pump. Two transducers were assembled on perpendicular axes in order to acquire both vertical and horizontal components of motion. The anomalies were obtained by hitting with a hammer the rotor during its rotation. The rotor was statically unbalanced in order to generate a basic noisy signal (**BNS**) to which the anomalies were superimposed (**H-BNS**).

The Matlab, LabView and SPSS packages were used to perform, respectively, the wavelet and discriminant analysis. The database was composed of 50 numerical sequence (BNS), corresponding to an acquisition of 5s long and 8000 points, obtained during a rotation at 3375 rpm. In the same conditions, as above mentioned many additional signals (H-BNS) were obtained by hammering the rotor (2 hits) within 5s

at different randomize instant. The normalization was not necessary because of any scale changing occurred. All the signals were decomposed by wavelet analysis into 6 levels. It was performed by the Haar wavelet function. For each signal at each level, 12 coefficients were calculated: 6 entropic values (defined later) and 6 standard deviations (SD). For evaluating the impact caused from the anomalies superimposed to the signal, a parameter (entropy) was defined as follows [20]:

given a set

$$s := \{x_i, i \in \{1, 2, \dots, n\}\}$$

let us consider a function

$$c : x_i \in s \rightarrow c(x_i) \in \mathbb{R}$$

The entropy  $H(c)$  of  $c$  is defined as follows:

$$H(c) := - \sum_{c(x_i) \neq m} \frac{1}{s} \cdot \frac{c(x_i) - m}{M - m} \cdot \ln\left(\frac{1}{s} \cdot \frac{c(x_i) - m}{M - m}\right) \quad (3)$$

where

$$S = \frac{c(x_i) - m}{M - m}$$

$$M := \max \{c(x_i), i \in \{1, 2, \dots, n\}\}$$

and

$$m := \min \{c(x_i), i \in \{1, 2, \dots, n\}\}.$$

The entropy measures the best ratio between the maximum dynamic showed by anomalies and the smallest uniformity of signal. Given  $|s| = n$ , the entropy, as before defined, reaches its maximum value at  $\ln(n)$  iff, for any  $i \in s$ ,  $c(x_i) = \text{const}$ . Finally  $H(c) = 0$  iff  $\{i \in \{1, 2, \dots, n\} : c(x_i) = S \text{ and } j \in \{1, 2, \dots, n\} - \{i\}, c(x_j) = 0\}$ .

Finally a discriminant analysis was carried out on the two classes of signal. The TABLEWilks method was used for selecting the test set asses and the discriminant function, and also for choosing the discriminant variables. The objective was the choosing of the wavelet decomposition level capable of performing the best identification between the two classes of signal: BNS and H-BNS.

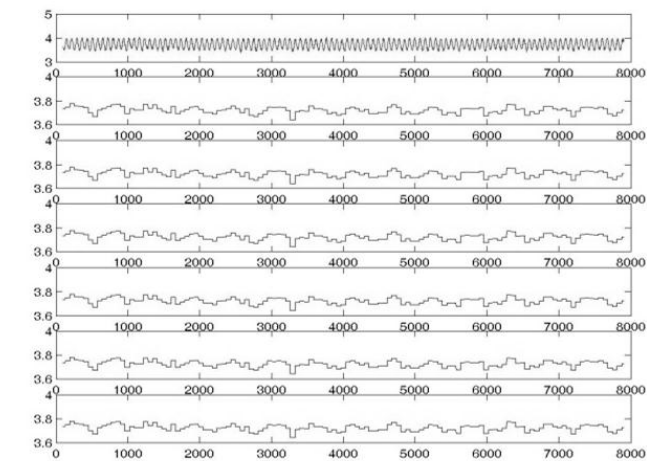
At that level a parametric statistical analysis was applied to the signal for acquiring mean, standard deviation (SD) and the value of entropy showed by the signal points exceeding 1.50 SD. The value of 1.50 SD represents the mean of the SD calculated on BNS distribution.

The proposed method was tested by utilizing some signal external to database (5 BNS and 5 H-BNS) according to the following protocol:

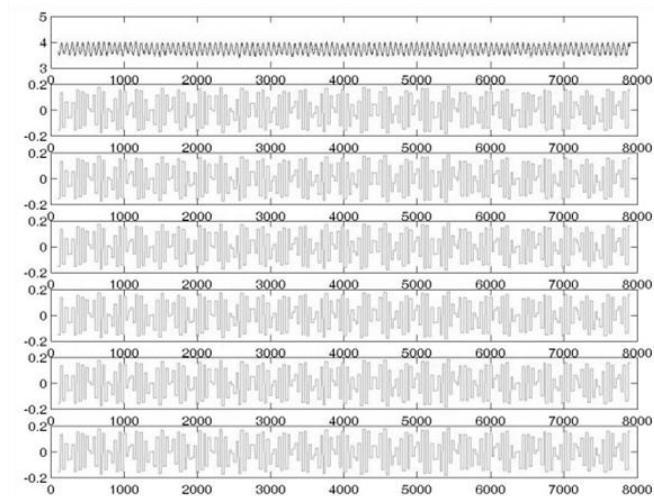
- wavelet decomposition by Haar function;
- entropic calculation of signal exceeding 1.50 SD;
- allocation of signal to BNS or H-BNS class

**RESULTS**

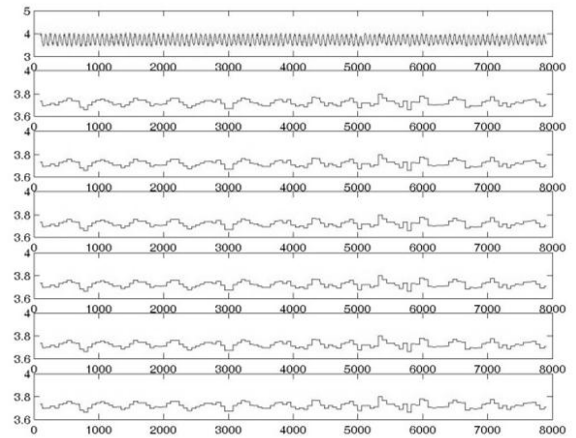
Figure 2a.b and Figure 3a.b show a wavelet decomposition of BNS and H-BNS signal. The identification of the specific characteristics of each class of signal, based only on the graphical comparison of decompositions, is very difficult indeed. It is due to the contemporary presence of a long and complex signal affected by a basic noise and the superimposed anomalies.



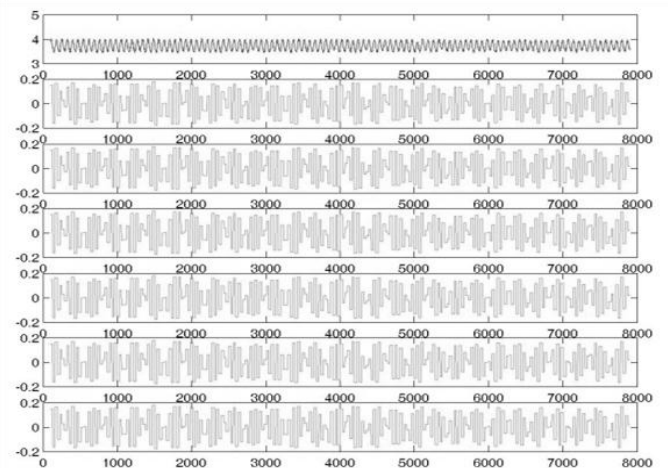
**Figure 2a:** A wavelet decomposition (low frequency) of BNS signal



**Figure 2b:** A wavelet decomposition (high frequency) of BNS signal



**Figure 3a:** A wavelet decomposition (low frequency) of H-BNS signal



**Figure 3b:** A wavelet decomposition (high frequency) of H-BNS signal

The results of discriminant analysis are illustrated in Table 1. Approximately 87 % of signals are well classified. The stepwise analysis is showed in Tab. 2a.b.c. The best level of decomposition is represented by the 6th, the eingvalue is 1.136 and the canonical correlation is 0.729. The Wilks Lambda is 0.468 and the significance is 0.0001.

**Table 1:** Classification results performed by discriminant analysis

**Classification results<sup>b,c</sup>**

		estimated class		Total
		1	2	
Source	1	18	2	20
	2	2	10	12
%	1	90.0	10.0	100.0
	2	16.7	83.3	100.0

b. 87.5% of cases are correctly classified.

c. 81.3% of cases are correctly cross-validated .

**Table 2a:** The stepwise analysis results

variables in the analysis				
Step		Tolerance	F to remove	Wilks Lambda
1	SE1	1.000	15.775	
2	SE1	.968	16.228	.838
	E5	.968	6.354	.655
3	SE1	.964	14.223	.706
	E5	.004	4.827	.549
	SE5	.004	4.154	.538

**Table 2b:** The stepwise analysis results

Eingvalue				
Function	Eingvalue	% of variance	% cumulated	Canonical correlation
1	1.136	100.0	100.0	.729

**Table 2c:** The stepwise analysis results

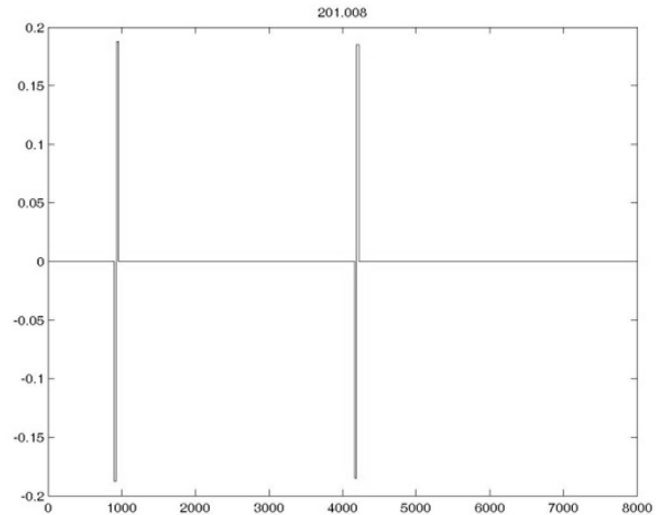
Wilks Lambda				
Function test	Wilks Lambda	Chi-square	df	Sig.
1	.468	21.632	3	.000

Following, on the basis of the above results, for each signal was calculated the entropy for that part of the signal exceeding 1.50 SD at the 6th level; the results are presented in Table 3. It is easy in distinguishing the evident difference which exists between the two classes of signal (BNS and H-BNS).

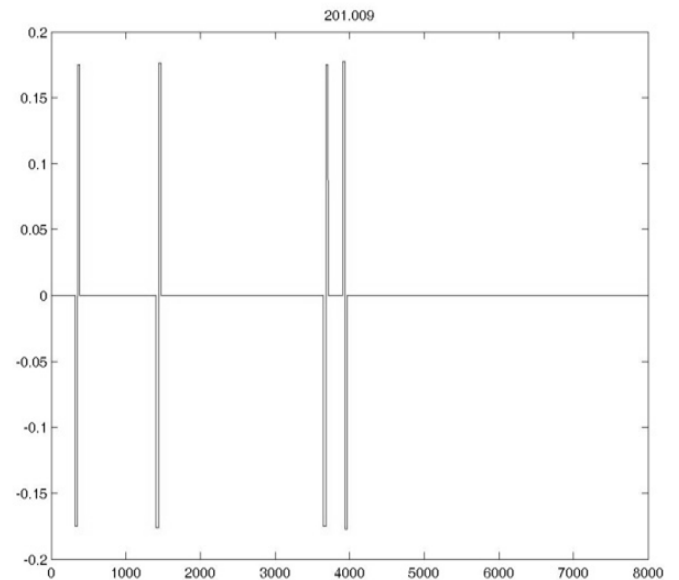
**Table 3:** Statistical analysis

	Statistical analysis	
	CLASS	
	BNS	H-BNS
mean	6.247	12.428
sd	6.299	6.212

The final test, consisting on employment of external signals, showed a 100% of correct identification. The graphical result is showed in the figures reported below (Figure 4 and Figure 5) with the entropic level.



**Figure 4:** Part of a BNS signal exceeding 1.5 SD, H=7.467



**Figure 5:** Part of an H-BNS signal exceeding 1.5 SD, H=19.153

#### 4 CONCLUSIONS

This study demonstrates an application of a new system of analysis based on the decomposition of complex signal performed by wavelet.

It was focused a methodology based both on wavelet and discriminant analysis, which can concur in determining the identification of anomalies superimposed on real signals. Usually many parts of the original signal, in a common data logging, are lost, mainly due to the high-pass and/or low-pass filtering process.

In the future the objective is the preparation of a database for diagnostics and identification of the causes which determine mechanical damages, performed by a fuzzy logic. This could be essential when automated processing is required.

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