Transient Heat Measurement in a Wall Using Thermoelectric Heat Flux Meter: A Mathematical Model

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Abstract
Heat flow through building envelopes is translated to air conditioning load which is equivalent to the amount of heat that need to be removed to maintain the set point temperature in a room. This paper proposes a mathematical model on new method that enable online measurement of transient heat flow through wall using thermoelectric heat flux meter. To predict the magnitude of heat flow at various locations along the thickness of a wall, a finite difference heat conduction model was developed. The temperature on external surface of the wall was varied in sinusoidal, while the node next to internal surface was maintained at 25 °C. The properties of thermoelectric module were extracted from the manufacturer data sheet, with estimated accuracy of 3% for the average temperature of 30 °C to 50 °C. Since the potential difference across the module is proportional to the heat flow; necessity to measure the module surfaces temperatures could be avoided. The smallest magnitude of heat flow that could be detected by thermoelectric module would be determined by the resolution of voltmeter. Across the thickness of the wall, the magnitude of heat flow decreases from its external to internal surfaces. Findings in this paper are relevant to the improvement on control of air conditioning system, where advanced information of heat flow is provided to its control algorithm. Through this method, the efficiency of air conditioning system could be improved significantly.

Keywords: thermoelectric module; heat flow; building envelop.

INTRODUCTION
This paper proposes a method to measure transient heat flow through building envelopes such as wall and roof. Information on transient heat flow is very important in the prediction of cooling load variation in a building. Through this strategy, the air conditioning system could get advance information on the cooling demand of a room. Furthermore, this data could be useful in the sizing of air conditioner for rooms in buildings. However, the location of heat flux sensor may affect the accuracy of the cooling load prediction. For instance, the temperatures across the thickness of wall are not uniform, and keep changing throughout the day. This requires a detail study on the effect of sensor’s location on the measured heat flow.

A thermoelectric module is made of type P and type N semiconductor materials, located between two layers of ceramic. Thermoelectric module has the capability to control the temperature of a body and measure the corresponding heat flow from/into that body [1]. The developed method was used to measure the adsorption kinetic of water vapour onto silica gel [2]. Other than it use as temperature controller and heat flux meter, thermoelectric module also can be used to generate electricity from waste heat [3]. A high potential difference across thermoelectric module could provide some advantages in heat flow measurement in building’s wall.

Cooling load in a building determined the size of air conditioning system. Recent advancement in high rise building requires a new estimation method where the cooling load is decreases by 25% [4]. The use of inverse black box modelling could be applied to determine cooling load for an office building [5]. A properly optimized office has a reduced cooling load, provided some refurbishment is taken to improve thermal performance of the building [6]. Prediction of cooling load could be used in the detection of any malfunction in a building, such as the breakdown of cooling system. Application of deep learning system provides an opportunity to implement transient cooling load prediction, based on the advance algorithm [7]. Another aspect on cooling load estimation is on the occupancy pattern, which has a significant impact on the transient cooling load [8].

Energy conservation in building has become one of the focuses in new building development. Since buildings use up to 45% of total energy consumption [6], it would be beneficial to increase the energy performance of buildings. Measures to improve energy efficiency in buildings are available in [9]. However, new challenges such as urban heat island particularly in street canyon [10] may require an advance method to determine the cooling load. To improve the prediction of cooling load, we proposed a new transient measurement method of heat flow in building’s wall, by adopting commercially available thermoelectric module.

In this paper, transient heat flow through building’s wall was modelled using finite difference heat transfer scheme. Then,
the properties of thermoelectric module were extracted from the manufacturer data sheet. Based on this information, the expected performance of thermoelectric heat flux meter in measuring heat flow inside building’s wall was predicted.

METHODOLOGY

This section concerns the finite difference scheme of transient heat conduction within a wall (thermal conductivity = 0.121 Wm⁻¹K⁻¹, density=593 kgm⁻³, specific heat capacity = 2512 kJkg⁻¹K⁻¹) exposed to sinusoidal temperature variation on one of its surface. Then, the procedures to extract thermoelectric properties from manufacturer data sheet and heat flow measurement are presented.

Transient heat conduction through wall

The temperature of external surface (T₁) (node 1 in Figure 1) was set to a sinusoidal temperature variation (at time = p), as in Equation 1:

\[ T_1(p) = (\alpha \sin(\omega p) + 30) \]  

where \( \omega \) is the angular velocity with the period of 24 hours, and \( \alpha \) is the amplitude (set to 15 °C (0800 hrs to 2000 hrs) and 7 °C (2000 hrs to 0800 hrs)). This was to mimic the external temperature variation of a wall throughout a day. The internal surface of the wall (node M) was exposed to node M+1, which was maintained at 25 °C, according to the recommendation in ASHRAE Standard 55 for thermal comfort. The thickness of the wall was set to 0.1 m and distance between nodes was 0.002 m.

Figure 1: Nodes across the thickness of wall

A one-dimensional transient heat transfer within a slab/wall with isotropic thermal properties is presented in Equation 2

\[ \frac{1}{\kappa} \frac{\partial^2 T}{\partial t^2} = \frac{\partial T}{\partial x^2} \]  

\( T \) is the temperature, \( t \) is time, \( x \) is distance and \( \kappa \) is thermal diffusivity.

Equation 2 was expanded to yield Equation 3, which gives the temperature at node 2, the node adjacent to external surface:

\[ T_2^p = F_0(T_2^{p-1} + T_1^{p-1}) + (1 - 2F_0)T_2^{p-1} \]  

where \( F_0 \) is the Fourier number and the superscript \( p \) means the calculation of temperature at the time step = \( p \).

The temperatures of internal nodes (node 3 to node M-1) for the time step \( p \) was obtained from

\[ T_m^p = F_0(T_m^{p+1} + T_{m-1}^{p-1}) + (1 - 2F_0)T_m^{p-1} \]  

For the node M, the temperature was found using Equation 5:

\[ T_M^p = F_0(T_{M+1}^{p+1} + T_{M-1}^{p-1}) + (1 - 2F_0)T_M^{p-1} \]  

The heat transfer between nodes (from node m to node m+1) was obtained using Equation 6

\[ Q_{m\rightarrow m+1}^p = k\alpha \frac{T_m(p) - T_{m+1}(p)}{x_{m+1}(p) - x_m(p)} \]  

where \( Q_{m\rightarrow m+1} \) is the heat conducted from node \( m \) to node \( m+1 \) at the time step = \( p \).

Mathematical model of thermoelectric heat flux meter

The heat flow at the hot (\( Q_H \)) and cold (\( Q_C \)) surfaces (Figure 2) of thermoelectric heat flux meter are presented as in Equations 7 and 8:

\[ Q_H = \alpha I T_H - \frac{1}{2} R \kappa (T_H - T_C) \]  

\[ Q_C = \alpha I T_C + \frac{1}{2} R \kappa (T_H - T_C) \]  

where \( \alpha \) is the Seebeck coefficient, \( I \) is the electrical current generated by thermoelectric module, \( T_H \) is the hot surface temperature, \( T_C \) is the cold surface temperature, \( R \) is the module electrical resistance and \( \kappa \) is the thermal conductance of the module.

Figure 2: Energy balance for a thermoelectric heat flux meter

The difference between \( Q_H \) and \( Q_C \) is the electrical power produced by the thermoelectric module, represented by potential difference (V) and current (I) in Equation 9

\[ Q_H - Q_C = VI \]  

For an open circuit, there is no electrical current flowing in the circuit, hence \( I=0 \). By use this information to solve...
Equation 7, it yields Equation 10 which enabled the thermal conductance (κ) to be determined:

$$\kappa = \frac{Q_H}{(T_H - T_C)}$$  \hspace{1cm} (10)

The potential difference across the module divided by Seebeck coefficient gives the temperature difference between the module’s surfaces (Equation 11). However, accurate measurement of surface temperature is very difficult and may lead to inaccurate inference of heat flow. Seebeck coefficient could be determined using Equation 11 and data in [11].

$$\frac{V}{\alpha} = (T_H - T_C)$$ \hspace{1cm} (11)

By combining Equations 10 and 11, the necessity to measure the surface temperatures may be omitted and the $Q_H$ can be determined using Equation 11.

$$Q_H = \frac{V}{\alpha} \kappa$$ \hspace{1cm} (12)

Data on thermoelectric module is reproduced in Table 1 [11].

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot side temperature (°C)</td>
<td>300</td>
</tr>
<tr>
<td>Cold side temperature (°C)</td>
<td>30</td>
</tr>
<tr>
<td>Open circuit voltage (V)</td>
<td>14.4</td>
</tr>
<tr>
<td>Heat flow across the module (W)</td>
<td>~415</td>
</tr>
<tr>
<td>Heat flow density (W cm$^{-2}$)</td>
<td>~13.2</td>
</tr>
</tbody>
</table>

RESULTS

This section presents the result on transient temperature variations within the wall, corresponding heat flow and the estimated potential difference created across the thermoelectric module.

Temperature variation within the wall

Figure 3 shows the variation of temperature across the thickness of 0.1 m wood wall. The nodes closer to external surface has higher temperature fluctuation. Although the temperature at external surface reaches 45°C, the temperature at the node adjacent to internal surface was only increased to 33 °C, with the time lag of 8 hours. At night, the external wall temperature was dropped to 23 °C while the node adjacent to inner surface only dropped to 29 °C. The differences in temperature between adjacent nodes are decreases as the nodes are far from the external surface. This may occur due to the temperature imposed on the node M+1 was maintained at 25 °C.

Figure 4 shows the rate of heat flow within the wall. The highest heat transfer rate was between external surface and its adjacent node (node 2). In this simulation, the cross sectional area for heat transfer was 0.056 m x 0.056 m, identical to the
size of one thermoelectric heat flux meter. The rate of heat conduction during the day has the peak at 0.15 W, while the heat flow with the magnitude of -0.1W was recorded at night. The negative sign means the heat is flowing in the outwards direction (from internal to external nodes), indicates cooling is taking place inside the wall. The results obtained from this simulation provide essential information on the most appropriate location of thermoelectric heat flux meter.

**Potential difference across thermoelectric module**

Analysis on the graphs and data (reproduced as Table 1) that are available in [11], the Seebeck coefficient was found to be 0.0533 VK⁻¹ and the thermal conductance was 0.53 WK⁻¹. Based on comparison with the performance curve given in the manufacturer data sheet [11], for the average module temperature at 50 °C, these Seebeck coefficient and thermal conductance values could be used with the accuracy to within 3 %.

Figure 5 shows the expected magnitude of potential difference produce by thermoelectric module at various positions across the thickness of wall. The magnitude of heat flow in the wall was in the range of 0.2 Watt to – 0.15 Watt. The expected potential difference created by one thermoelectric heat flux meter is in the range of 0.02 V to - 0.015V. If a multimeter can measure potential difference with the resolution of 0.001 V, it could detect the minimum heat flow of 0.01 Watt.

**DISCUSSION**

To increase the potential difference across the thermoelectric heat flux meter, several thermoelectric modules could be connected in series. For instance, two identical thermoelectric modules would increase the potential difference to the range from 0.04 V to -0.03 V. This method could improve the accuracy of the measured heat flow across the modules. Furthermore, the necessity in having voltage amplifier could be avoided which would simplify this heat measurement system.

It is very unlikely for a wall to have uniform heat flow due to its anisotropic properties. For example, the thermal conductivity of a wood wall could vary significantly across its width. Measurement of heat flow using a single thermoelectric module only indicates the magnitude of local heat flow for the area that is covered by the module. The magnitude of heat flow at other part of the wall may differ from this measured value. Use of multiple thermoelectric modules would provide the average heat flow through a wall which leads to a more accurate prediction of total heat flow.

The properties of thermoelectric heat flux meter that was used in this paper was assumed constant, although the manufacturer data has some non-linearity on voltage output if the module average temperature is varied by 150 °C. For a narrow range of average temperature (30 °C to 50 °C), it is acceptable to assume the thermoelectric heat flux meter has constant properties since its properties have low dependency on temperature. We estimated the error in heat flux estimation would be less than 3%, by comparing the obtained heat flow calculated in this work with data available in [11].

In future, an automatic data logging system could be developed. A simple calculation shows a 16 bit data logger (with ± 10 V logging capacity) would enable the potential difference of ~0.0003V to be measured, which represents the heat flow ~ 0.003Watt. This automated data logger could be connected to air conditioning system in a building, as advanced information to its control system on the likely heat load of a particular space. Through this technique, a more efficient and effective air conditioning system could be developed.

**CONCLUSIONS**

A method to measure transient heat flow through a wall using a thermoelectric heat flux meter has been proposed. Based on the finite difference heat transfer scheme, the position of thermoelectric heat flux meter has a significant effect on the magnitude of measured heat flow. The properties of thermoelectric module extracted from manufacturer data sheet indicate the inferred heat flow has a fair accuracy. The lowest magnitude of heat flow that could be detected using this proposed method may be limited by the smallest potential difference that could be measured by the voltmeter. One of the possible applications of this proposed heat flow measurement
is on the advanced prediction of heat load in a space, which could optimize the performance of air conditioning systems.

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REFERENCES


NOMENCLATURE

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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>Fo</td>
<td>Fourier number</td>
<td>-</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity of wall</td>
<td>Wm⁻¹K⁻¹</td>
</tr>
<tr>
<td>R</td>
<td>Electrical resistance of module</td>
<td>Ohm</td>
</tr>
<tr>
<td>K</td>
<td>Thermal conductance of module</td>
<td>WK⁻¹</td>
</tr>
<tr>
<td>Q</td>
<td>heat flow</td>
<td>W</td>
</tr>
<tr>
<td>x</td>
<td>distance between nodes</td>
<td>m</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
<td>K</td>
</tr>
<tr>
<td>qg</td>
<td>volumetric heat generation</td>
<td>Wm⁻³</td>
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Greek letters

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<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>α</td>
<td>Seebeck coefficient</td>
<td>V K⁻¹</td>
</tr>
<tr>
<td>κ</td>
<td>thermal diffusivity</td>
<td>m²s⁻¹</td>
</tr>
<tr>
<td>ρ</td>
<td>density of wall</td>
<td>kgm⁻³</td>
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Subscripts

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<thead>
<tr>
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<tbody>
<tr>
<td>∞</td>
<td>air</td>
</tr>
<tr>
<td>M</td>
<td>total number of nodes within the wall</td>
</tr>
<tr>
<td>m</td>
<td>node number</td>
</tr>
<tr>
<td>H</td>
<td>hot surface of thermoelectric module</td>
</tr>
<tr>
<td>C</td>
<td>cold surface of thermoelectric module</td>
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Superscripts

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<tbody>
<tr>
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