Numerical Computations on Sediment Transport Models Base on Threshold Sediment Motions of Shield’s Graphic due to Simulation of the Groyne Placement Analysis

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Abstract

The numerical model of sediment transport modelling is the computer program was constructed composed of 2-D flow by Navier-Stoke equation, sediment transport equations and bad deformations equations. In solving of sediment transport and bad deformations equation, required threshold of sediment that is the critical stress of motion parameters of each subgrade gradations, but the critical stress motion parameters was calculated from searched on the Shield's diagram, that’s always consideration of each process computations, while in the computer program, implementation of sediment transport done repeatedly so that the usage of Shield’s diagram in the computer program will be difficulty because to applied of Shield’s diagram can be do manually.

Therefore, to usage of graphs on the computer program mush be abandoned and needs an effort to modify the Shield's diagram into mathematical equations.

This paper describes of manually process of the Shield's diagram become a mathematical equation that can be applied easily in computer programming.

Keywords: modify the Shield's Graph, Sediment Transport Models

INTRODUCTION

Programming on Sediment Transport Models build from the numerical computations of flow equation and sediment transport equation including bed deformations equations. The sediment transport equation needed the threshold of sediment motions. The threshold of sediment motion is critical stress motion at which sediment particle begin to move, the condition threshold of sediment motion is usually expressed in terms of a critical shear stress or threshold shear stress (τ_c). In the threshold of sediment computations was need the critical stress of motion parameters (τ_c) was calculated and searched from the Shield's graph.

In the calculation of sediment transport parameters are usually represented by sediment grain d50, which is the mean diameter of the existing sediment gradation, there is assume one graded class. To be get accurate of the calculation will be made the parameter sediment grain became some classes gradation, so that each of class gradation will be calculated the critical stress of sediment’s motion and bed deformations. This calculation is repeated on the next item for the upper class gradation to the last class and be repeated on any time. So the accumulation of sedimentation can be calculated from the sum of the results calculation of all class gradations.

In any calculation of each class sediment transport, requires the threshold of sediment motions that’s to be consideration of the Shield’s graph, however in the computer program to calculation of sediment transport was done repeatedly, so that the usage of the Shield’s graph in the computer programing will be difficult, therefore to support the Numerical Model programming sediment transport needed of modified Shield diagrams into mathematical equations.

This paper describes of modified Shield diagrams into mathematical equations that’s be supported the computer programing of sediment transport modelling.

Madsen et al. (1976) on Tuomo Karvonen, (2002), giving converted the Shields diagram in to the relationship between the critical Shields parameter and the sediment fluid parameter.

THE RESEARCHES HAS BEEN DONE

Researches on Transport Sediment Programming

On the explained above, sediment transport program, is a program to build of numerical computation of the 2-D Horizontal flow equation, the transport sediment equations and the bad deformations equation be computations together and repeated. On issues concern on sediment transport program, has been studied by Suharjoko (2012) and Suharjoko (2014), and Jungseok, 2005 to get determination of the good distance between the groyne. Heereveld (2006), conducted a study on the submersible groyne aimed to reducing the speed of the water flow at the bottom and increase in the upper stream (fairway). Brandimarte et al, (2006) and Prohaska (2006), study of groyne functions as erosion control, while
Zhang (2007) study on flow problems and the changes process on the river bed around the groyne, Armani (2010), to analyze the problem of scouring and depositions in around a groyne system. Kuhnle et al, 1999, conducted experiments on groyne immersed (submerged spur dikes), and Zhang, Mizutani and Nakagawa (2011), investigating the influence of groyne size against of the sedimentation on bed topography around groyne. Duan, 2005, to predict bed-load sediment transport around the channel.

The studies mentioned above have been applied to simulate the flow pattern and to get an idea of the distribution of sediment concentration and the concentration on around the groyne.

Researches of the Shield’s Graph Applications on numerical modeling

The sediment transport equation needed the threshold of sediment motions. The threshold of sediment motion is critical stress motion strength at which sediment movement begins, the condition for incipient movement is usually expressed in terms of a critical shear stress or threshold shear stress, which will denote by \( \tau_{cr} \). In the threshold of sediment computations was need the critical stress of motion parameters \( \tau_{cr} \) was calculated and searched from the Shield’s graph. In the calculation of sediment transport parameters are usually represented by sediment grain \( d_{50} \), which is the mean diameter of the existing sediment gradation, there is assume one graded class. To be get accurate of the calculation will be made the parameter sediment grain became some classes gradation so that, each of class gradation will be calculated the critical stress of sediment’s motion and then can be calculated the bed deformations. This calculation is repeated on the next item for the uper class gradation to the last class, so the accumulation of sedimentation can be calculated from the sum of the results calculation of all class gradations.

The sediment transport equation needed the threshold of sediment motions. The threshold of sediment motion is critical stress motion strength at which sediment movement begins, the condition for incipient movement is usually expressed in terms of a critical shear stress or threshold shear stress, which I will denote by \( \tau_{cr} \). In the threshold of sediment computations was need the critical stress of motion parameters \( \tau_{cr} \) was calculated and searched from the Shield’s graph.

Guo (1990) on Guo 2002, proposed a mathematical expression to get wide applications of numerical modeling, Haschenburger and Wilcock, 2003, the widely applications Shields diagram of unisize sediments, be represented on mathematical expression.

Madsen et al. (1976) on Tuomo Karvonen 2002, converted the Shields diagram in to the matematical expression of relationship between the critical Shields parameter \( \theta \) and the sediment-fluid parameter \( S_r \).

Modifications of the Shield’s graph to mathematical equation has done Tuomo Karvonen , 2002, is set in forth other Equation as follows

\[
\log(\theta) = 0.00223x^5 - 0.06043x^4 + 0.20307x^3 + 0.05425x^2 - 0.63639x - 1.0316
\]

Where \( x = \log(S_r) \).

However, this equation still shown error value of Critical Shields parameter at Shield Number \( (S_r) \) value more than 600. Show that Modifications of the Shield’s graph to mathematical equation must be ubdate.

![Figure 1: Modifications of the Shield’s graph has done Tuomo Karvonen, 2002.](image-url)
METHODOLOGY AND SEDIMENT TRANSPORT MODELS EQUATIONS

Method to Developing Mathematical Model on Sediment Transport

Programming on Sediment Transport Models build from the numerical computations of flow equation by the Navier-Stokes and sediment transport equation including bed deformations equations. The flow equation by the Navier-Stokes that meets from the mass conservation equations and momentum equations of water motion. The mass conservation equations derived from conservation of mass water motion and momentum equations derived from Newton’s second law. The sediment transport equation needed the threshold of sediment motions. The threshold of sediment motion is critical stress motion strength at which sediment movement begins, the condition for incipient movement is usually expressed in terms of a critical shear stress or threshold shear stress, which I will denote by $\tau_{\sigma}$.

In the threshold of sediment computations was need the critical stress of motion parameters ($\tau_{\sigma}$) was calculated and searched from the Shield’s graph. In the calculation of sediment transport parameters are usually represented by sediment grain $d_{50}$, which is the mean diameter of the existing sediment gradation, there is assume one graded class.

The Sediment Transport Models Equations

a) Suspended Sediment Diffusion Equations

Wexier (1992) on Signh 2005 developed an analytical solution for two-dimensional diffusion equation.

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = k_C \left[ \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right] - \lambda C$$

$C = C_0$, $x = 0$ and $y_1 < y < y_2$

$C = 0$, $x = 0$ and $y < y_1$ or $y > y_2$

$$\frac{\partial C}{\partial y} = 0, y = 0$$

$$\frac{\partial C}{\partial y} = 0, y = W$$

$$\frac{\partial C}{\partial y} = 0, X = L$$

b) Settling velocity

Liu 2001, The settling velocity of suspended sediment $w_s$ gives

$$w_s = \frac{1}{2.8} \left[ \frac{36v}{d_n} \right]^2 + 7.5(p - 1)gd_n - \frac{36v}{d_n}$$

Where, $d_n$ is Normal diameter, $p$ is sediment concentration and $g$ is sediment concentration.

The Migniot (1989) on Signh 2005, The settling velocity cohesive sediment equations:

$$w_c = \frac{250}{d^2} w_s$$

Where :

$$w_c = \text{settling velocities of cohesive sediment flocs}$$

$$w_s = \text{settling velocities of single cohesive sediment Stoke Law is used to calculate the single cohesive sediment particle}$$

$$\omega_s = \frac{gd^2}{18\mu} (\rho_s - \rho)$$

c) Settlement of Suspended sediments

Dyer 1986 on widagdo 1998, introduce the equation to calculate the rate of settled sediment as follow,

$$\frac{dm}{dt} = pw_s \left(1 - \frac{\tau}{\tau_{\sigma}}\right)$$

Where $m$ is the mass sediment settle to be deposited, $p$ is the sediment, $\tau$ and $\tau_{\sigma}$ are the shear stress and critical shear stress for settlement respectively.

$$\tau = \gamma RS$$

for wide channels hydraulic radius $R$ can be taken as the depth of flow $h$.

Indri’s formula on Signh 2005, proposed the following formula for critical shear stress for incipient motion for sediment particle are:

$$\tau_c = 13.3d \left(\frac{s - 1}{M}\right) + 12.16 \text{ if } d < 1.0\text{mm}$$

$$\tau_c = 54.85d \left(\frac{s - 1}{M}\right) - 74.48 \text{ if } d > 1.0\text{mm}$$

Where $\tau_c$ = Critical shear stress in gm/m2, $d$ = mean diameter of sediment in mm, $M$ = uniformity coefficient

d) Threshold of Sediment

Computation in solving the sediment transport and bed deformations required threshold of sediment particle on each subgrade gradations, which is the critical stress of motion parameters $\tau_{\sigma}$. However, the critical stress of motion parameters was calculated by searched from the Shield’s diagram. Computations on computer program in being usage of graphs should will be abandoned, therefore needs to be done modify of the graph into mathematical equations. Consider the flow on the cohesive particles, the forces acting on the bad particle are shown in Figure 2.
Figure 2: Forces acting on the particles of sediment. (Liu 2001)

If the critical friction velocities $u_{c,c}$, indicating the situation in which the grain particles begin to move, then the drag force is equal to the friction force, and with simplification become the following equation.

$$\frac{u_{c,c}^2}{(s-1)gd} = \frac{f}{\alpha^2 C_d + f\alpha^2 C_L} \frac{4}{3\alpha^2}$$

Sediment particles begin to move if:

- $u_c > u_{c,c}$ critical friction velocity $u_{c,c}$, or
- $\tau_b > \tau_{b,c}$ critical bottom shear stress $\rho u_{c,c}^2$, or
- $\theta > \theta_c$ critical Shields parameter $\theta = \frac{u_{c,c}^2}{(s-1)gd}$

(Madsen 1976 on Liu 2001), to know the critical condition of sediment particles threshold can be calculated by Shields diagram as shown in the following figure, that is graph of relationship between critical Shield parameter with sediment fluid parameters is sediment fluid parameters can be calculated $S_c = \frac{d}{4\nu} \sqrt{(s-1)gd}$ and Critical Shield parameters follow the equation $\rho = \frac{u_{c,c}^2}{(s-1)gd}$, whose value can be determined from the Shield's graph of Figure 3 below.

Figure 3: Shields graph, the relationship between Sediment Fluid Parameter $S_c$ with Critical Shields Parameter
e) Bed Elevation Change

Singh, 2005, the bed elevation changes due to sediment erosion and sediment deposition may be represented as the following equation:

\[
(1 - p_m^1) \left( \frac{C_z b}{\alpha L_s} \right) = \alpha \omega_s (C - C_s) + (q_b^* - q_{b*})/L_s
\]

\( C^* \) and \( q_{b*} \) is a conditions that must be known in advance and can be approached \( C_s = p_b C^* \), \( q_{b*} = p_b q_b^* \). \( p_b \) is the gradation of bed material, \( C^* \) is the potential transport capacity of suspended-load, \( q_{b*} \) is the potential transport capacity of bed-load.

DISCUSSION

Get the Digital Data of Shield’s Graph

To change the graph into a mathematical equation, early times be done to reading of the Shield’s graph of the relationship between critical Shield parameters with sediment fluid parameters into digital data, the reading result is as following Table 1.

Then the data which is the relationship between \( S_* \) and \( \theta c \) to be change into the relationship between \( \text{Log}(S_*) \), with \( \theta c \) then do regression analysis.

Table 1: The relationship of parameters \( S_* \), \( \text{Log}(S_*) \) and \( \theta c \)

<table>
<thead>
<tr>
<th>( S )</th>
<th>( \text{Log}(S) )</th>
<th>( \theta c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>1.5</td>
<td>0.1761</td>
<td>0.0720</td>
</tr>
<tr>
<td>2</td>
<td>0.3010</td>
<td>0.0620</td>
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<tr>
<td>4</td>
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<td>0.0440</td>
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<td>7</td>
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<td>10</td>
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</tr>
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<td>0.0320</td>
</tr>
<tr>
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</tr>
<tr>
<td>50</td>
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<td>2.7782</td>
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<tr>
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</tr>
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</table>

REGRESSION ANALYSIS

To simplify of Regression Analysis, the \( \text{Log}(S_*) \) parameter is replaced by parameter \( x \), then from these two parameters the regression equation is searched. The regression analysis has resulted the modifications Shield’s graph into the mathematical equation in a relationship between \( \theta c \) and \( x \) as follows;

\[
\theta c = -0.0108 x^3 + 0.066 x^2 - 0.1117 x + 0.0899,
\]

where \( x = \text{Log}(S_*) \).

The Standard deviations yield \( R=0.9993 \).

So that the develop of mathematical equation modifications Shield’s graph, can be seen the comparison of the Shield’s graph by Karvon 2002 with the Shield’s graph by Suharjoko 2017, can be seen in Figure 5 below.
The Computation Application of Sediment Transport Model to Simulation of the Groyne Placement

After obtaining the mathematical formulation of the shield’s graph, the analysis of the sediment threshold and the bad deformation analysis will be easy, even the analysis can be improved in accuracy by performing an analysis based on the grading class. The modified mathematical equations of Shield Graphs greatly assist the process of calculating sediment transport programs to simulate the suspended distribution and bad deformation. The following were shown a simulation result of the process computations sediment transport programs to simulate the suspended distribution and bad deformation that's were analysis be considering of each class gradation suspended sediment.

The characteristics of gradation suspended sediments as shown in Figure 6 below

<table>
<thead>
<tr>
<th>No</th>
<th>% class</th>
<th>d (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>2.15</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>1.40</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>0.80</td>
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</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.15</td>
</tr>
</tbody>
</table>

In the implementation of the simulation, considering the important parameters are river width, angle bench, radius bench, current velocity, water depth, groyne length, groyne position and suspended sediment were resulted of the sedimentation volume were be occurred.

The above simulation results can be explained as follows; by taking the example of groyne layout case, that is on channel width 20 m with turn angle 60 º, turn radius 40 m, groyne position in center of arc and length of groyne equal to 1/5 width of channel. upstream flow velocity = 1.4 m / sec, produced as can be seen in figure below, the perspective of water faces occurring, Figure 7; The vektor velocities of water flow velocity, Figure 8; distribution of suspension sediment concentration and Fig. 9; sediment deposition (bad deformations) around the groyne.

Thus, the modified mathematical formulation of the Shield graph, makes it easy for the Numerical Computations On Sediment Transport model.
Figure 8: The vector velocities of water flow around the groyne field occurring simulation result to the case of groyne size L / B = 1/5 at the angle bench $\beta = 60^\circ$, radius bench 40 m, upstream velocity $u = 1.4$ m/sec.

**Figure 8:** Distribution of Suspension Sediment Concentration
CONCLUSION

Computations in solving the sediment transport and bad deformations required threshold of sediment particle on each subgrade gradations, which is the critical stress of motion parameters $\tau_c$. However, the critical stress of motion parameters was calculated by searched from the Shield's diagram. Computations on computer program in being usage of graphs should will be abandoned, therefore needs to be done modify of the graph into mathematical equations.

To get better then made effort modification with regression analysis than yielded the modification of Shield's graph obtained equation of Critical Shields parameter $\theta_c$ as the following Equation

$$\theta_c = -0.0108x^3 + 0.066x^2 - 0.1117x + 0.0899,$$

where $x = \log(S_*)$.

The Standard deviations yield R=0.9993

The modified mathematical formulation of the Shield graph, makes it easy for the Numerical Computations On Sediment Transport model

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