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Comparison of Hydrology Component Spatial Correlation Model (Continuous Model) and Markov Chain (Case Study of Cikapundung Watershed, Saguling Reservoir and Cipanunjang Reservoir)

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Abstract

Water allocation in the watershed is a concern in developing countries where there are limited water resources and greater demands with more parties. Past data on water flow amount is the only information available to estimate future water delivery. One of the strategies used is to determine the exact number of water discharge based on past recorded data. Future prediction data is essential for sustainable management of water resources. The objective of this research is to construct a hydrological model to estimate or anticipate future water discharge using the Discrete Chain Markov model where the results are then compared with continuous discharge rain model by implementing multiple linear regression method. Both components (rain and discharge) are then modified into several alternatives in 3 different research sites to analyze and compare the combination of the results with the highest correlation value. The inflow discharge data in Cipanunjang used in this study is the data retrieved from 2000-2013; data of Cikapunung Watershed are retrieved from 2003-2012. Meanwhile, data of Saguling are extracted from 1986-2013. In spatial correlation method, QQQQ combination was acquired in Cipanunjang, while the best combination that can be acquired in Cikapundung and Saguling watersheds is $QQ_{t-1}PP$. The correlation values based on the spatial correlation method in Cipanunjang, Cikapundung watershed, and Saguling were 0.882, 0.871 and 0.857, respectively. Meanwhile, the correlation values of Discrete Markov method in the same places were 0.899, 0.718 and 0.782, respectively. These results suggest that this continuous method can be used to predict the amount of water discharge in the future well enough and is effectively used in Indonesia territories.

Keywords: Discrete Markov method, forecast discharge, spatial correlation method

INTRODUCTION

Water resource is a natural resource that can be renewed through hydrology cycle and constitutes spatial and temporal function. Water source can be renewed through hydrology cycle, affected by climate and land conversion that form hydrology regime components (rain and discharge) that are random and stochastic, while on relatively sloping ramp, water disposal from land to sea is a deterministic phenomenon (Arwin, 2009; Marselina, 2016).

Water allotment in watershed is a concern in developing countries where the water resource is limited and the demand is higher with more parties (Read et al., 2014; Marselina et al., 2017). Paimin et al. (2006) and Handayani et al. (2010) state that watershed can be viewed as a system, where the input component is rainfall and the output is production, runoff, erosion, and so on. The input is so varied at each place and time (temporal and spatial) that it will result in varied output behavior too.

Data regarding the flow magnitude in the past is the only information that is available to predict future water administration. Until now, practically and historically it is used define critical periods where the available discharge is only a little or too much and later on the critical periods are used to learn the condition of system existing in the future. One of strategies that can be done is determining fixed price of discharge based on data recorded in the past. However, almost all of hydrology processes are stochastic (uncertain), or at least are combination of deterministic and stochastic processes, therefore each study regarding discharge at watershed has to consider stochastic aspects of future discharge (Arwin, 2009; Nuraeni, 2011).

Knowledge of several rainy periods is required for hydraulic structure design such as protection from flood, drinking water infrastructure, and drainage channel system (Benabdesselam, 2013; Maselina et al., 2017). Relationship between rain and discharge is the basis of correct forecasting for hydraulic project operation and to extend discharge data. To predict future discharge accurately or close to the reality, it requires accuracy of calculation method selection and adequate input data. Mostly used hydrology components underlying discharge forecasting are rainfall and discharge component, because based on the result of study, the two components are the most influential component on the availability of discharge compared to other hydrology components (Arwin, 2002; Nuraeni, 2011; Marselina, 2017).

This study aimed to develop a hydrology model to predict or anticipate water discharge to come using Markov Chain Discrete model and the result compared to rain discharge continuous model using multiple linear regression method. The two components (rain and discharge) were then modified into some alternatives in 3 different research locations to observe the combination and compared to the highest correlation score.

HYDROLOGY PROCESS AND MODEL

Process is the illustration of phenomena that change continuously against time. Because all of hydrology phenomena change according to time, it is called as hydrology process. If the variable change during the process is accompanied by the law of certainty that the process does not depend on opportunity, it will be called as deterministic process. (Besag, 2006). Deterministic process is also a process that does not change due to time. However, if variable change is an opportunity factor, the process will be called stochastic of probabilistic (Haan, 2006; Nuraeni, 2011).

Generally stochastic process if viewed as a process that depends on time, while probabilistic process is a process that does not depend on time. Actually, hydrology process has deterministic and stochastic components. The level of each component determines whether the process can be completed in deterministic or stochastic way.

Hydrology model tries to illustrate real physics processes of hydrology cycle by simulating hydrology events that occur, for example, by making transformation of a series of rainfall input into hydrograph of flow resulted from it. The models are portrayed as deterministic based on the physics and conceptual characteristics. The other classification of hydrology models is stochastic model that tries to reveal statistic behaviors of a series of hydrology time without considering real events. Stochastic method provides a method to predict the probability of dry year sequences during the desired period in the future.

Compared to discharge data collection, rainfall data collection is generally easier. If the available daily rainfall data are too long, although the daily discharge data row is short, the daily discharge data row can be ranged along the daily rainfall data row. It can be taken by simulation using mathematical model. The method can even predict in the future in terms of monthly discharge data row based on monthly discharge data row in the past (Arwin, 2009; Nuraeni, 2011).

Characteristics and order of flow in the past gives sign for future flow. If the flow this year is small, although uncertain, the flow in the next year will be smaller than the median. Similarly, big flow tends to follow big flows. Therefore, the history of a flow gives valuable information regarding flow that can occur in the future. Model to regenerate has to use the information although at the same time random component has to be inserted to illustrate the inability to predict future flow sequence.

RESEARCH LOCATION

The study was conducted in 3 different places, namely:

- Cipanunjang Reservoir Inlet. It is located in Cisangkuy watershed, Bandung Regency, West Java Province, Indonesia. The data used are data of rainfall and discharge retrieved from 2000-2013.
- Maribaya Discharge Post, Cikapundung Watershed in Western of Bandung Regency and Bandung Municipality, West Java, Indonesia. The data used are rainfall and debit data retrieved from 2003-2012.
- Saguling Reservoir Inlet. It is located in Citarum River Watershed, Western of Bandung Regency, West Java Province, Indonesia. The data used are data of rainfall and discharge retrieved from 1986-2013.

Rainfall and discharge data in this study are taken from the posts recording rainfall and discharge around 3 sites where this study was conducted.

METHOD

Data obtained are not always complete so it requires linear regression calculation to estimate blank data (Abatzoglou, 2009).

The correlation of rainfall-river discharge is a basis of correct forecasting for water resource development project operation and river flow discharge data expansion. Based on hydrometeorology observation at a watershed, especially rainfall and water discharge, we can investigate the correlation between variable in space and time. Furthermore, study of events between rainfall-discharge was done based on the linkages between two random variables to predict uncertainty of future discharge through discharge estimation model. This study reviewed discharge estimation model using continuous model and Markov stochastic model (Arwin, 2009; Corzita, 2014).

a. Continuous Model

Data of river discharge measurement is frequently incomplete. One of the methods used to complete them is by multiple linear regression using spatial correlation F (x, y, z, t) main components of hydrology (P and Q) (Arwin, 2002; Marselina, 2017)

The continuous model uses multiple linear regression correlation method (Arwin, 2002; Marselina, 2016) developed based on correlation between two random variables, namely rainfall observation station data (P) and discharge observation station data (Q). Model with the highest determination coefficient value (R^2) is selected as the best model to develop discharge data (Figure 1).

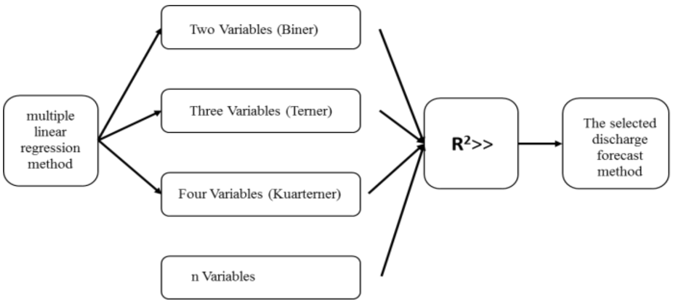

Figure 1: Continuous Model (Arwin, 2002)

Multiple regression is done using enter method, it means that all of the variables are included in analysis. Multiple regression equation is formulated as the following (Arwin, 2009; Corzita, 2014):

$Y = a + b_1X_1 + b_2X_2 + \dots + b_nX_n + E$

Where:

Y = dependent variable value;

a = constants namely the value Y if X=0;

b_n = n^{th} regression coefficient;

X_n = n^{th} independent variable value;

E = error.

Correlation test is done after obtaining regression equation to determine whether the correlation between dependent variable and independent variable is strong enough (correlation coefficient = R). Value of R ranges between -1 and 1. A negative sign denotes an indirect correlation or negative correlation; while a positive sign denotes a direct correlation or positive correlation. For R = 0, it should be interpreted that there is no linear relationship between the variables.

b. Model Markov

Markov chain is a mathematic technique commonly used to do modelling of various systems and processes. The technique can be used to estimate future changes in dynamic variables based on changes of the dynamic variables in the past. The model reads:

"For each time t, when event is K_t and all of previous events is $K_{t(j)}$, ..., $K_{t(j-n)}$ occurring from known process, the probability of future event $K_{t(j)}$ only depends on event $K_{t(j-1)}$ and does not depend on previous events namely $K_{t(j-2)}$, $K_{t(j-3)}$, … , $K_{t(j-n)}$."

The events above are in chain. Therefore, the theory is known as Markov chain. So, Markov chain describes movements of some variables in a period in the future based on movements of the variables in the present. Mathematically Markov model can be written (Descombes dan Berthod, 2006; Nuraeni 2011; Marselina,2013) as the following:

$$K_{t(j)} = P \times K_{t(j-1)}$$

Where:

$K_{t(j)}$ = opportunity of event in $t_{(j)}$

P = transitional probability

$t_{(j)}$ = j^{th} time.

In Markov model, water discharge amount is simplified by dividing it into 3 classes. Based on class division of water discharge, it tries to search memoir/historic recording water discharge event cascading so that on each month, stochastic matrix that divided water discharge into 3 classes can be made. While the stages of Markov chain processing of discharges are as the following (Descombes and Berthod, 2006; Nuraeni, 2011; Marselina, 2017):

- Water discharge class division stage. In this study, the investigated process was the first order of three classes. Therefore, the discharge amount was divided into three classes as the following:
  1. Dry discharge (stated by 0)
  2. Normal discharge (stated by 1)
  3. Wet discharge (stated by 2)

  The determination of class interval for each class division was obtained by dividing probability curve from selected population distribution into 3 equal parts namely 0.333, 0.6667, and 1. The value for each class is the median of each class namely on probability curve 0.1667, 0.5, dan 0.8333.
- The procedure to obtain transitional matrix is as the following:
  - Determining the quantity of class division (n)
  - Inputing historic data into class division in accordance with predetermined class limitation.
  - Finding event probability value j in time t_n occurs if event i occurs in time t_{n-1} with dependency interval of a time unit.
  - Developing transitional matrix with elements of transitional probability calculation in the reviewed time in each row and column of the result.

Discrete Markov model consists of 2 (two) tugs where first tug is the determination of condition, while the second tug is the determination of the quantity. Probability of an event at a certain time is depended/determined by only the previous events. Since the discharge data are stochastic, an approach with Markov model is made by making the transition matrix to explains the probability value (uncertainty) of the occurrence of certain discharge quantities where the number of the whole probabilities is equal to 1 as shown in Table 1. The transition matrix is homogeneous or stochastic matrix for all probability transitions of Pij are fixed and independent of time. Probability of Pij must meet the conditions:

$$\sum_j \text{Pij} = 1 \text{ for all the value of } i;$$

Pij ≥ 0 for all the values of *i* and *j*

Table 1: One Order Transition Matrix (Arwin, 2002)

Discharge Condition at tn-1	Discharge Condition at tn				
	0	**1**	**2**	**…**	**N**
0	P00	P01	P02	…	P0N
1	P10	P11	P12	…	P1N
2	P20	P21	P22	…	P2N
…	…	…	…	…	…
N	PN0	PN1	PN2	…	PNN

RESULTS AND DISCUSSION

Continuous Method

To know which station has the closest relationship in estimating the discharge on the next month, the analysis of the correlation coefficients between the discharges and/or precipitation at each pluviometer station with the following month for each month. The results of this analysis will show 3 stations with the greatest correlation coefficient for each month. Data retrieved from these three pluviometer stations will be used as independent variables in the rain discharge model.

From the comparison results of various linear regression models used, multiple linear regression equations with 4 variables have the largest averages of correlation coefficient. In a variety of linear regression equations with 4 variables, the values of correlation coefficient are always above 0.8 at three sites studied. It shows that there is a considerable positive correlation between the dependent variable and the independent variable being tested. Here is a comparison of correlation coefficient values of multiple linear regression equations in the Cisangkuy watershed:

The regression step is initiated by choosing all variables that will be used as the independent variables of multiple linear regression model. In this step, the selected variables are rainfall data at the surrounding area of pluviometer station. The value of rainfall data recorded for the previous month can also be inputted if needed. The completion of this regression step will be limited to only 3 free variables that make up multiple linear regression equations giving the largest correlation coefficient.

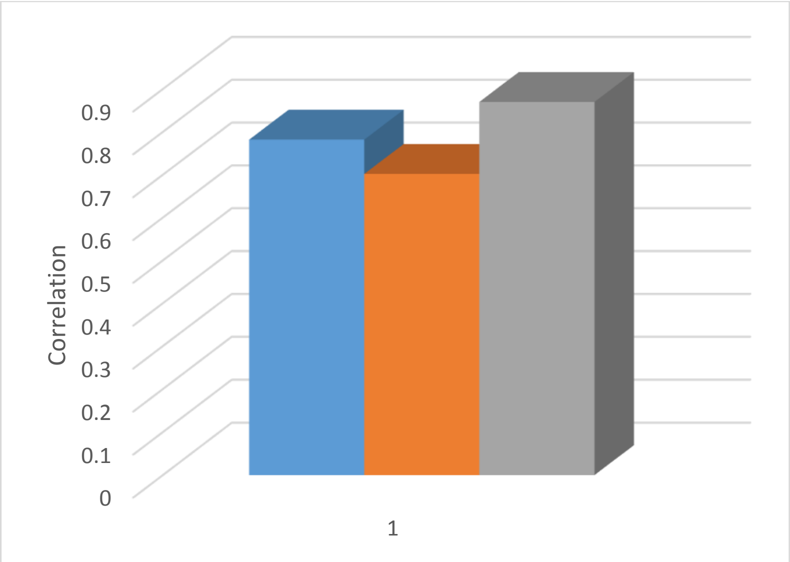

Figure 2: Comparison of the maximum correlation coefficient of 2, 3, and 4 variables Inlet (Soropotan), Cipanunjang Reservoir, Cisangkuy Watershed

In this process, there is no restriction on the number of independent variables which will determine the multiple linear regression equation; however, the optimum number of independent variables should be looked for. The criterion used to find the number of optimum variable is seen from the increasing coefficient of determination. At each increasing number of the variables, there will be the increasing coefficient of determination; however, at some point, it will not provide further significant increase in the coefficient of determination. It means that the increase in the number of independent variables is not effective anymore. At this point, the best regression equation is selected. In three study sites, regression with 4 variables is the optimum number achieved with the best results. In the forecast calculation of water discharge by using rain spatial correlation method and water discharge in three study sites using used SPPS software. The results of multiple linear regression equation in three study sites can be found in the tables and figures below:

Table 2: Results of monthly regression Cipanunjang Resevoir Inlet, Cisangkuy Watershed

Month	Equation
January	Qjan = 0.889+(0.655)Q1-(0.141)Q2+(0.014)Q3
February	Qfeb = (0,845)Q1+(0,341)Q2+(0,341)Q3 -0.999
March	Qmarch = (2.028)Q1-(0,592)Q2+(0,088)Q3-5.521
April	Qapr = (1.248)Q1-(0,340)Q2+(0,036)Q3-1.778
May	Qmay = 1.695+(0.105)Q1+(0.421)Q2-(0.087)Q3
June	Qjun =(0,387)Q1+(0,646)Q2-(0,044)Q3-0.21
July	Qjul = 0.135-(0.008)Q1+(0.701)Q2+(0.019)Q3
August	Qaug = 0.715-(0.425)Q1+(2.115)Q2-(0.086)Q3
September	Qsep = (0,733)Q1+(2.733)Q2-(0,285)Q3-0.412
October	Qoct =(0,796)Q1+(1.072)Q2-(0,029)Q3-0.609
November	Qnov = (1.394)Q1+(0,184)Q2-(0,035)Q3-1.045
December	Qdes = 0.239+(1.030)Q1+(0.649)Q2-(0.185)Q3

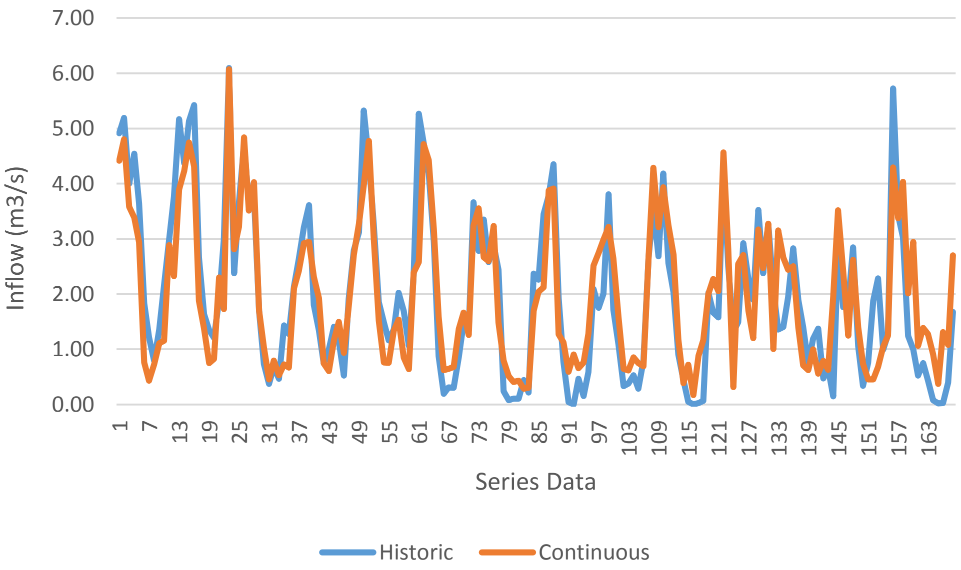

Figure 3: Compariosn of discharge history with debit seizure of continuous model at Cipanunjang Reservoir Inlet, Cisangkuy Watershed

Table 3: Results of monthly regression at Cikapundung Watershed

Month	**Equation**
January	Q Jan = 1.342 + 0.0033.P4(Jan) - 0.0001.P6(Jan) + 0.44.QM(Des)
February	Q Feb = -0.471 + 0.007.P3(Feb) + 0.0003.P5(Feb) + 0.805.QM(Jan)
March	Q Mar = 0.276 + 0.001.P1(Mar) + 0.004.P3(Mar) + 0.692.QM(Feb)
April	Q Apr = -0.764 + 0.004.P3(Apr) + 0.004.P5(Apr) + 0.852.QM(Mar)
May	Q Mei = -0.174 + 0.004.P5(Mei) + 0.002.P6(Mei) + 0.671.QM(Apr)
June	Q Jun = -0.149 + 0.001.P1(Jun) + 0.004.P5(Jun) + 0.703.QM(Mei)
July	Q Jul = 0.086 + 0.004.P3(Jul) + 0.0004.P5(Jul) + 0.745.QM(Jun)
August	Q Agst = 0.211 + 0.0004.P1(Agst) + 0.002.P5(Agst) + 0.731.QM(Jul)
September	Q Sept = 0.209 + 0.004.P5(Sep) - 0.001.P6(Sep) + 0.754.QM(Ags)
October	Q Okt = 0.043 + 0.003.P1(Okt) + 0.001.P3(Okt) + 0.859.QM(Sep)
November	Q Nov = -0.048 + 0.003.P1(Nov) + 0.002.P3(Nov) + 0.923.QM(Okt)
December	Q Des = 0.184 + 0.001.P1(Des) + 0.003.P3(Des) + 0.780.QM(Nov)

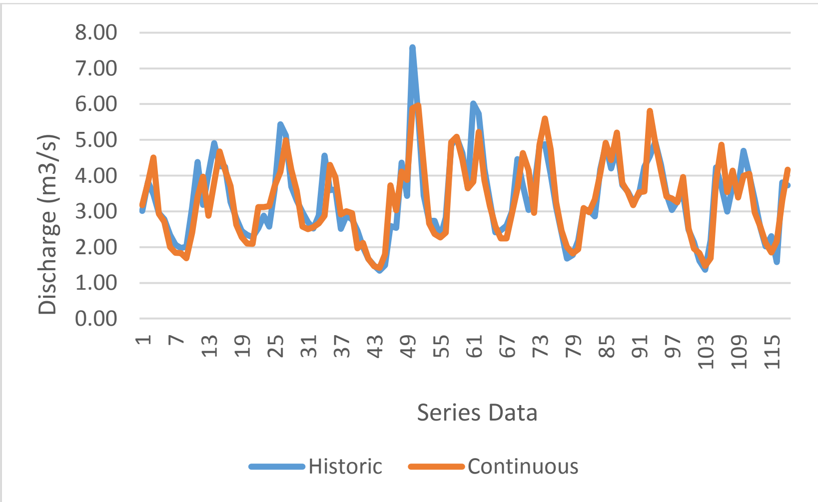

Figure 4: Compariosn of discharge history with debit seizure of continuous model at Cikapundung Watershed

At the first location, Cipanunjang Reservoir Inlet, Cisangkuy Watershed use QQQQ component (Q1, Q2 and Q3 are discharge data retrieved from 3 posts before Inlet) and generates a correlation value of 0.882 which means a positive relationship or big enough correlation. This also means that there is a large influence of the discharge prior to the study site on the discharge at the Cipanunjang Reservoir inlet. The effect size is determined by the coefficient of determination where R2 = 0 .78 or 78%. This means that either increasing or decreasing discharge of 78% can be explained by the linear relationship based on the aforementioned equation. The rest is determined by other circumstances.

At the second location, Maribaya Discharge Forecast Post, Cisangkuy Watershed uses components of QQt-1PP (Qt-1 is the discharge data on Maribaya discharge forecast post retrieved from the previous month, and P is the rainfall data around the location) and generates a correlation value of 0871 which means a positive relationship or a significant relationship. The effect size is determined by the determination coefficient of R^2 = 0.76 or 76%. This means that the increase or decrease of 76% discharge can be explained by the linear relationship based on the aforementioned equation. The rest is determined by other circumstances.

Table 4: Results of monthly regression at Saguling Reservoir Inlet, Citarum Watershed

Month	Equation
January	Qjan = (0,0621)P2+(0,3148)P10+(0,2141)Qdes + 16,449
February	Qfeb = (0,3020)P4+(0,2080)P6+(0,4451)Qjan -3,014
March	Qmarch = (0,1566)P6+(0,2885)P10+(0,3172)Qfeb + 0,8306
April	Qapr = (0,0987)P2+(0,0978)P7+(0,3328)Qmar + 64,625
May	Qmay = (0)P2+(0)P3+(1)Qapr+ 0
June	Qjun =(0,2013)P3+(0,1742)P4+(0,2585)Qmay + 9,4927
July	Qjul = (0,1746)P2+(0,0842)P10+(0,0688)Qjune + 23,7672
August	Qaug =(0,1030)P2+(0,1659)P8+(0,0764)Qjuly + 6,494
September	Qsep = (0,0555)P1+(0,2558)P2+(0,2411)Qaug + 8,9508
October	Qoct =(0,2108)P4+(0,0828)P7+(0,4556)Qsept + 3,4011
November	Qnov = (0,2108)P10+(0,0828)P4+(0,4556)Qoct - 7,5567
December	Qdes =(0,3598)P10-(0,004)P2+(0,2700)Qnovember + 12,6486

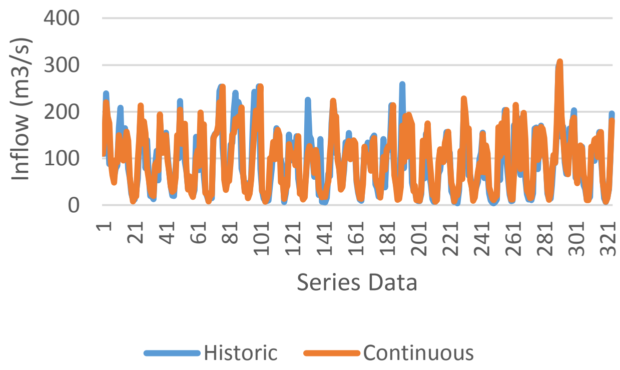

Figure 5: Comparison of discharge history with debit seizure of continuous model at Saguling Reservoir Inlet, Citarum Watershed

At the third location, Saguling Reservoir Inlet, Citarum Watershed uses components of QQt-1PP (2 rainfall data from the nearest station and one discharge data retrieved from the previous month as explanatory) and generates a correlation value of 0.857 which means a positive relationship or quite big correlation. The effect size is determined by the determination coefficient of R^2 = 0.73 or 73%. This means that the increase or decrease of 73% discharge can be explained by the linear relationship based on the aforementioned equation. The rest is determined by other circumstances.

Water Discharge Discrete Chain Markov Model

After distributing the water discharge into three classes, three-classes one-order transition matrix can be formed. Discharge forecast with Markov discrete model is a forecast of future discharge by classifying it into 3 classes, namely: dry, normal, and wet which are sorted starting from the smallest to the largest discharge data. Discharge which will be classified comes from the input discharge of the monthly history. After acknowledging the discharge interval for each class and the average value of each discharge class (normal-dry-wet), historic flow data are transformed into classes 0-1-2.

Afterwards, using monthly transition matrix, the condition of the next month is forecasted (wet-normal-dry). After that, it will obtain the forecasted value of discharge in which its quantity corresponds to the average value of discharge in each class (wet-normal-dry) obtained before. Below is comparison of discharge history with discharge of 3-classes Markov model in each study site:

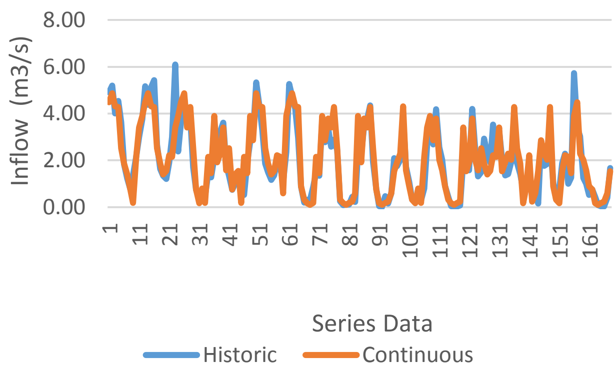

Figure 6: Calibration of discharge forecast in Markov Chain model with input discharge history at Cipanunjang Reservoir, Cisangkuy Watershed (1986-2013)

In Cipanunjang Reservoir Inflow, Cisangkuy Watershed, the result of correlation between discharge history and continuous discharge shows the value of 0.899 which means a positive relationship or quite big correlation. The effect size is determined by the determination coefficient of $R^2 = 0.8$ or 80%. This means that the increase or decrease of 80% discharge can be predicted using Markov Chain Model. The rest is determined by other circumstances.

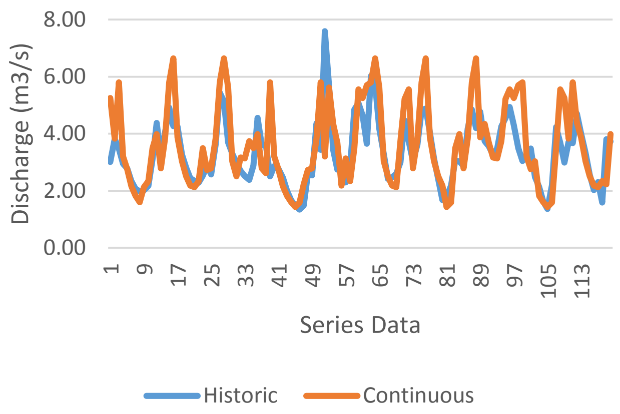

Figure 7: Calibration of discharge forecast in Markov Chain model with input discharge history at Cikapundung Watershed (1986-2013)

In Cikapundung Watershed, the result of correlation between discharge history and continuous discharge shows the value of 0.718 which means a positive relationship or big enough correlation. The effect size is determined by the determination coefficient of $R^2 = 0.52$ or 52%. This means that the increase or decrease of 52% discharge can be predicted using Markov Chain Model. The rest is determined by other circumstances.

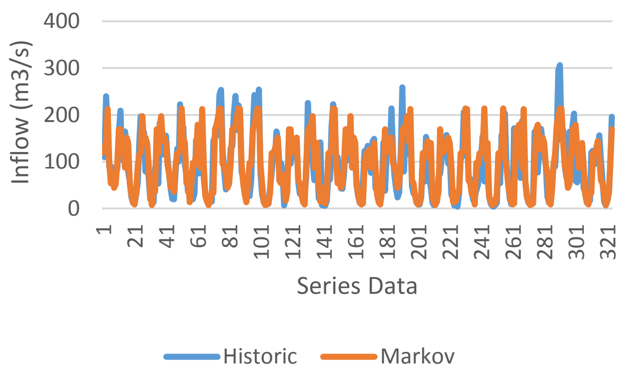

Figure 8: Calibration of discharge forecast in Markov Chain model with input discharge history at Saguling Reservoir (1986-2013)

In Saguling Reservoir, Citarum Watershed, the result of correlation between discharge history and continuous discharge shows the value of 0.782 which means a positive relationship or big enough correlation; however, it is not bigger than the result in continuous method. The effect size is determined by the determination coefficient of $R^2 = 0.61$ or 61%. This means that the increase or decrease of 61% discharge can be predicted using Markov Chain Model. The rest is determined by other circumstances.

CONCLUSION

The rain-discharge spatial correlation method (continuous method) and the Markov discrete method are effective methods to be used in forecasting the future discharge. The simulation results showed that the correlation value of discharge forecasts at Saguling, Cipanunjang and Cikapundung Reservoirs which are located in West Java, Indonesia with Monsoon rain type by using continuous method tends to be constant in these three locations in which its correlation value is always above 0.8; meanwhile, Markov method showed different results in each location with the interval between 0.718 – 0.899.

The correlation of discharge history forecast and discharge of continuous method in the Cipanunjang Reservoir, Cisangkuy Watershed is 0.882; meanwhile, discrete Markov method shows 0.899. Cikapundung Watershed discharge showed that the correlation using continuous method was 0.871 and the correlation using discrete Markov method was 0.718. Saguling Reservoir showed that the correlation using continuous method was 0.857 which was bigger than the correlation using discrete Markov method, i.e. 0.782. This suggests that this continuous method can be used to predict the amount of water discharge in the future well enough and generally it is effectively used in Indonesia territories.

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