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Pricing Multi-asset Equity Options Driven by a Multidimensional Variance Gamma Process Under Nonlinear Dependence Structures

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Abstract

In finance, dependence structure between assets is of great importance. For example, pricing options involving many assets, one must make preassumption about the dependence structure between assets or one important issue in risk management is to find out the dependence structure when calculating VaR. The aim of this paper is to explore the dynamic properties of a multidimensional Variance Gamma process, which has non Gaussian marginal features and non linear dependence structure. We use copula functions to specify the dependence structure of underlying assets. We study the effect of different choices for the dependence functions to the prices of a set of multi-asset equity options. The analysis is conducted using 5-dimensional baskets that consist of Jakarta Stock Exchange Composite Index (IHSG) and four other Asian Indices, Hang Seng, Nikkei, KOSPI, Straits Times Index (STI) and a standard payoff functions for multi-asset options. The results show that the different choices of dependence structure do not give significantly different option prices.

Keywords: Multidimensional Variance Gamma, Multi-assets Option Pricing, Nonlinear Dependence Structures, non Gaussian marginal.

INTRODUCTION

The most current issues in mathematical finance is the extension of the risk-neutral concept to the multidimensional case, that is a financial instrument constructed by more than two underlying assets, known as multivariate options. At least there are five deferent types of multivariate options, these are, the best or worst performer of a basket of underlying assets, an option on the difference between the prices of underlying assets, or an option on the maximum or minimum of the underlying assets.

The difficulty with the Black-Scholes model is the assumption that the marginal is lognormally distributed. This assumption has been argued by many researchers or practitioners (see, Carr et al, 1998 or Madan, 1990) and suggested that Black-Scholes model has to develop to be more realistic model to describe the behavior of stock prices. The most powerful and flexible model to describe drawback of Black and Scholes model is the class of Lévy models; see for example Cont and Tankov (2004), Luciano and Semeraro (2008), or Schoutens (2003), for reference of Lévy processes and their application in finance.

One example of Levy process is the variance gamma (VG) process. The univariate variance gamma has been proposed to model stock prices, see for example in Carr et al (1998), Cont and Tankov (2004), and Geman et al (2001). The application of jump process for a single asset model has been initiated by Cont Tankov (2004) and Geman et al (2001). The popularity of Levy processes is due to its ability to capture jumps, skewness and kurtosis observed in the return distribution and risk-neutral return densities (Cont and Tankov, 2004). VG model is the most popular Levy processes in the financial context, proposed firstly by Madan and Seneta in Madan (1990). VG model for stock price models allow for jumps, excess kurtosis and skewness and, as a result, are more suitable for modelling stock price behavior than the Black and Scholes model.

Another difficulty of pricing multidimensional options come from the fact that there is no closed form solutions for the formulas. The key point in evaluating multivariate options is the determination of dependence between the underlying assets (Luciano and Semeraro, (2008). For example, when pricing basket options, one needs to estimate the dependence structure from the historical time series of asset returns and the risk-neutral marginals to price the option. Therefore, one needs to be able to separate the dependence structure from the margins (Cherubini et al, 2004).

Understanding the dependence structure among multivariate assets is an important key to price multi-asset derivatives or managing risks consist of many financial assets. The standard approach to find the dependence in multivariate distribution is by assuming the distribution is multivariate normals or student-t. This approach is chosen because those distributions are mathematically simple to solve. It has been argued in many literatures, se for example in Cherubini et al (2004) or in Embrechts (2003) that the use of multivariate normal distribution restricts the correlation between assets to be linear as measured by covariances. The real dependence structure between two random variables is often much more complicated.

In this paper the limitation of the multidimensional Brownian motion model is extended. The assumption that the dependence structures linearly correlated are extended to nonlinear dependence. We use copula functions discussed in Embrechts et al (2003) and Nelsen (2006) to specify the dependence structure of underlying assets. From a practical standpoint, however, these models can easily become difficult to handle and calibrate, especially for truly multidimensional products like the ones traded on the markets. The approach that we apply here refers to Luciano and Semeraro (2008) and the correlation matrix is calibrated from the time series data of 5-dimensional baskets of Asian shares and a standard payoff functions for multi-asset options.

The purposes of the paper is to study the effect of different choices for the dependence functions to the prices of a set of multi-asset equity options, and to present a case study of the pricing of multi-asset options. The analysis is conducted using 5-dimensional baskets that consist of Jakarta Stock Exchange Composite Index (IHSG) and four other Asian Indexes, Hang Seng, Nikkei, KOSPI, Straits Times Index (STI) and a standard payoff functions for multi-asset options.

CONSTRUCTION OF MULTIVARIATE VG PROCESS

In one-dimensional problem, variance gamma model has been successfully to show optimal performance when modelling the skewness and kurtosis observed from financial time series data of financial market. The model for dynamic price of stock returns is given by

$$S_t = S_0 e^{\mu t + \omega t + X(t)}$$

where $X(t)$ is a VG process, μ is the drift of the stock price, and ωt is a parameter used to ensure the martingale property of the discounted stock price process, that is

$$\mathbb{E}[S_t] = S_0 e^{-\mu t}$$

The parameter ωt is also known as the compensation term or additive adjustment with a value of

$$\omega t = \ln\left(\frac{t}{\nu} - \theta t - \frac{\sigma^2 t}{2\nu}\right) \tag{1}$$

The value of function (1) is chosen such that it makes the discounted process $e^{-\mu t}$ into a martingale by adjusting it into the correct mean.

To construct a multivariate VG, we choose the most popular Gamma process $\{G(t), t \geq 0\}$ with parameters θ and κ, which has the probability distribution function (pdf)

$$f(x;\theta,\kappa) = \frac{1}{\theta^{-\kappa}\Gamma(\kappa)} x^{\kappa-1} e^{-x\theta}. \tag{2}$$

The mean and variance of the gamma process are θ/κ and $\kappa\theta^2$. If we suppose that X_t is a process with a time increment Δt, then the pdf (2) can be written as

$$f(x;\theta\Delta t,\kappa) = \frac{1}{\theta^{-\kappa\Delta t}\Gamma(\kappa\Delta t)} x^{\kappa\Delta t-1} e^{-x\theta}, \tag{3}$$

and the characteristic function of (3) is given by $\phi(x;\theta\Delta t,\kappa) = (1 - i\omega/\kappa)^{-\kappa\Delta t}$. By setting the mean rate to t, $\mathbb{E}(X_t) = t$ and the variance to $\kappa = t/\nu$ and $\theta = 1/\kappa$. This setting remains valid when the subordinator is replaced by another subordinator.

The multivariate price process is presented as an exponential of the d-dimensional VG process X_t. The dynamic of the univariate marginal are given by

$$S_i(t) = S_i(0) e^{rt - \omega_i t + X_i^{VG}(t)}$$

where $X_i^{VG}(t)$ is a variance gamma process defined as follows:

$$X_i^{VG}(t) = \theta_i G(t) + \sigma_i W_i\big(G(t)\big), \quad i = 1, \cdots, d \tag{4}$$

where the gamma process $\{G(t), t \geq 0\}$ or process $\{X^{VG}(t), t \geq 0\}$ has parameters $\kappa = 1/\nu$ and $\theta = 1/\nu$. In this case ν is the volatility time change. The parameter ν represents the magnitude of jumps and the magnitude of the tail of the variance gamma process. The variance gamma process (4), has a standard Brownian diffusion, $\{W_i(t)\}$ with diffusion $\sigma > 0$, and drift μ.

In vector notations (4) is presented as

$$X^{VG}(t) = \begin{pmatrix} X_1(t) \\ \vdots \\ X_d(t) \end{pmatrix} = \begin{pmatrix} \theta_1\big(G(t)\big) + \sigma_1 W_1\big(G(t)\big) \\ \vdots \\ \theta_d\big(G(t)\big) + \sigma_d W_d\big(G(t)\big) \end{pmatrix}$$

where $W_i$and W_j are correlated with coefficient correlation ρ_{ij}. In [11], the correlation is given by

$$\rho\big(X_i, X_j\big) = \frac{\mu_i \mu_j \mathrm{Var}[G(t)]}{\mathrm{Var}[X_i(t)]\mathrm{Var}\big[X_j(t)\big]}, \qquad i \neq j$$

where μi is the mean of stock return i, given by

$$\mu_i = r + 1/\nu \log\left(1 - \frac{1}{2}\nu\sigma^2 - \theta_i \nu\right)$$

with the interest rate r.

Instead of using ρ as in Kienitz and Wetterau (2012), one may choose another ρ for describing the dependence structure between underlying assets. There many choices for describing the dependence structure, such as Gaussian copula or t copula from elliptic copula families or Frank, Gumbel, Clayton, etc from archimedean families. Applying Gaussian copula on multi-asset VG proces for describing dependence structure may not realistic. This is mainly because the joint normal distribution does not exhibit tail dependence. Therefore, our goal in this paper is to build more realistic models, incorporating jumps, and non-Gaussian dependence structure.

The question that may arise is what kind of the dependence structure should be chosen in order to all dependence structure between the underlying assets are captured? In other words, how should the dependence structure between the components of VG process $X(t)$ be modelled? To answer this question, ones may refer to Kienitz and Wetterau (2012), or Linders and Stassen (2016) to the modelling of the dependence structure between the components of the VG process X(t). The dependence structure between components of a multivariate pure jump VG process can be reduced to the VG measure, see Chen (2008).

STATISTICAL PROPERTIES OF VG-COPULA

As is stated in the stylized facts of financial returns that the dependence structure between different return series changes depend on the market situations. In normal situations, the prices of assets move in independent ways of each other, but they may fall together in crisis. As reported in Danielson (2011) that the assumption that the multivariate normality and linear correlation is unlikely suitable and joint extreme outcome is more likely to occur. One may use exceedance correlation for treating the presence of nonlinear dependence as discussed in Bedendo et al (2010) or Canela and Padreira (2012). For this reason, it seems reasonable to assume that two asset returns are nonlinearly correlated. Hence, we use copulas to model non-linear dependencies. Copulas provide a means of separating the description of a dependence structure from the marginal distributions. To investigate the effects of different copulas, we model correlations between two jump diffusions using the Archimedean copulas (Gumbel copula, Clayton copula, Frank copula). Copula describes the dependence structure of random variables. Copula binds together the probability distributions of each random variables into their joint probability. Sklar's theorem (1959), discussed in Nelsen (2016) provides the theoretical foundation for the application of copulas.

Copula function can be obtained by the following procedure. Let $F_i(x,y)$ be the cumulative distribution function (cdf) of two dimensional VG process at time t and (X,Y) are random samples with marginal VG and inverse (Quantile) functions (F_X^{-1}, F_Y^{-1}). Let $C(u_x,u_y)$ be the VG copula function, defined as

$$C(u_x,u_y) = F_{XY}\left(F_X^{-1}(u_x), F_{Y(u_y)}^{-1}\right).$$

The density function of the copula is given by

$$\frac{\partial^2 C}{\partial u_x \partial u_y}(u_x,u_y) = \frac{\partial^2 F}{\partial x \partial y}\frac{\partial F_X^{-1}}{\partial u_x}\frac{\partial F_Y^{-1}}{\partial u_y} \qquad (5)$$

Equation (5) can be extended to d-dimensional case.

The VG measure determines the frequency and size of jumps, either it moves down or up, of the stock prices. As the main interest in this case is the large moves/jumps, so the discussions are focused on the tail of the distribution. Now, it is conveniently to work with tail integral of the VG measure and to model dependence between jumps by a VG copula. Lastly, substituting the VG process in the exponential VG model by a d-dimensional VG process with dependence structure given by a VG copula to obtain a d-dimensional VG model.

EMPIRICAL STUDIES

In this section, we analyze the performance of the bivariate variance gamma model on a dataset of five names of Asian stock indexes, HANGSENG, NIKKEI, KOSPI, STI and JKSE are used to see the effect of different choices of copulas on the option price. The descriptive statistics of the data set of daily log-returns recorded during the period of 10 June 2014 to 5 July 2016 are given in Table 1. On Figure 1, one sees the index values of five index which is normalized with respect to JKSE. The index value of NIKKEI dominates the other four index significantly.

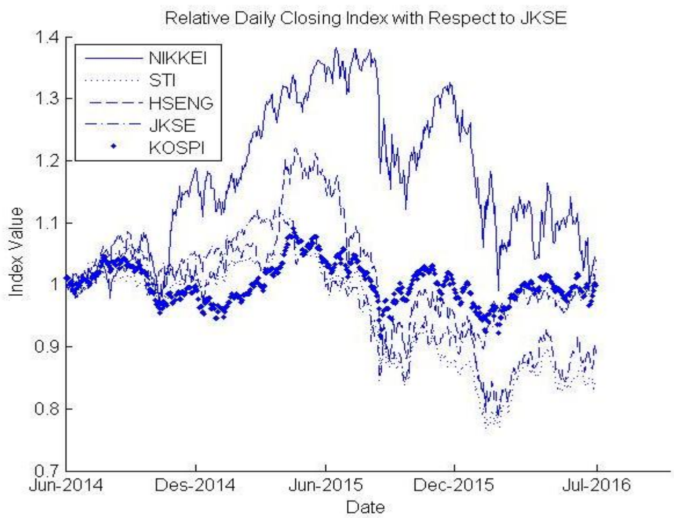

Figure 1: Relative Daily Closing Index with Respect to JKSE recorded from 10 June 2014 to 5 July 2016

Table 1: Log-return Descriptive Statistics

	NIKKEI	STI	HSENG	JKSE	KOSPI
Mean	0.0084	-0.0314	-0.0222	0.0052	-0.0021
Std. Dev.	1.4760	0.8173	1.2111	0.9187	0.7551
Skewness	-0.2096	-0.2281	-0.2297	-0.3790	-0.2348
Kurtosis	7.7036	5.5480	5.4154	5.9155	4.7888
JB Test	1	1	1	1	1
# of Obs	526	526	526	526	526

Figure 2, shows the scatter plots of the daily returns of the stocks JKSE-NIKKEI, JKSE-STI, JKSE-HANGSENG, and JKSE-KOSPI. These plots show a dependence structure between the assets and one can see that JKSE and KOSPI are highly correlated compare to between JKSE and NIKKEI or HANGSENG. This is in accordance with the correlation presented in Table III) which shows that JKSE is more correlated to KOSPI than to NIKKEI. As shown in Table 1, NIKKEI has the fattest tails than the other four indices. Table 1 also shows that none of the indices are distributed like a Gaussian distribution but KOSPI seems like to have normal distribution. Overall, it can be seen from the JB test that all daily log-returns of indices are not normally distributed.

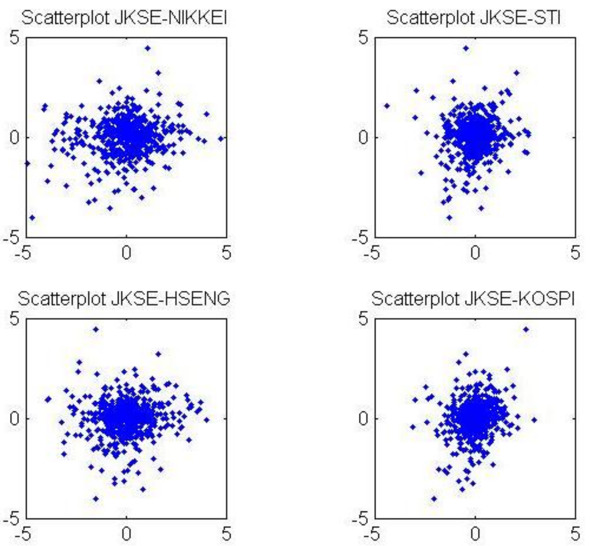

Figure 2: Scatterplot of daily returns.

The four moments of the characteristic functions are as follows:

$$\sigma = \sqrt{\frac{V}{t}}, \quad \nu = \left(\frac{K}{3} - 1\right)t, \quad \theta = \frac{S\sigma t}{3\nu} \tag{6}$$

where V is variance, S is skewness, and K is kurtosis. The initial guess for the parameters is given by Equation (6). The following results are the estimated Variance Gamma parameters for the five indices, shown in Table 2.

Table 2: Variance Gamma Estimation for The Five Indices

	$\hat{\theta}$	$\hat{\nu}$	$\hat{\sigma}$	$\hat{\mu}$
NIKKEI	0.0084	-0.0314	-0.0222	0.0052
STI	1.4760	0.8173	1.2111	0.9187
HSENG	-0.2096	-0.2281	-0.2297	-0.3790
JKSE	7.7036	5.5480	5.4154	5.9155
KOSPI	1	1	1	1

The copulas parameters in this paper are estimated by using *copulafit*() function on Matlab. The results are presented in Table 3 and 4. Table 3 represents the estimated parameters for the Archimedean copulas whereas Table 4 represents the estimated parameters for elliptical Copulas. The results show that the coefficient of dependence among indices are below 0.5, except for STI vs HSENG which is 0.7538 (see Table 3). This indicates that in general they are not strongly dependence to each other.

Table 3: Estimated Parameters on Pairs with Dependence Structure given by Clayton Copula

Index	NIKKEI	STI	HSENG	JKSE	KOSPI
NIKKEI	1	0.2407	0.2534	0.1753	0.4020
STI		1	0.7538	0.1210	0.4928
HSENG			1	0.0827	0.4337
JKSE				1	0.1824
KOSPI					1

Table 4: Estimated Parameters on Pairs with Dependence Structure given by Gaussian Copula

Index	NIKKEI	STI	HSENG	JKSE	KOSPI
NIKKEI	1	0.1594	0.2211	0.1283	0.2778
STI		1	0.4437	0.0819	0.3157
HSENG			1	0.0827	0.4337
JKSE				1	0.1824
KOSPI					1

Table 5: Multi-asset option price with depedence structure given by Clayton Copula

Indices	Basket	Spread	WorstOfCall	BestOfCall
JKSE-NIKKEI	0.1546	0.0843	0.1124	0.1967
JKSE-STI	0.1390	0.0531	0.1124	0.1655
JKSE-HSENG	0.1453	0.0658	0.1124	0.1782
JKSE-KOSPI	0.1428	0.0607	0.1124	0.1731

Table 6: Multi-asset option price with depedence structure given by Gaussian Copula

Indices	Basket	Spread	WorstOfCall	BestOfCall
JKSE-NIKKEI	0.2036	0.1037	0.1517	0.2554
JKSE-STI	0.1865	0.0696	0.1517	0.2213
JKSE-HSENG	0.1938	0.0842	0.1517	0.2359
JKSE-KOSPI	0.1903	0.0772	0.1517	0.2289

CONCLUSION

The aim of this paper is to discuss the influence of the different copula choices on the price of options where the underlying assets consist of more than one asset. Our results show that the different choices of dependence structure do not give significantly different of option prices. In this paper, the use of two copulas is reported, we do not report the results for t, Frank, Gumbel copulas. The influence of t, Frank, Gumbel copulas is not significant on the price of the options, results are summarized in Table 5 and Table 6. In general, the use of Clayton Copula gives a lower price of all options (Basket, Spread, WorstofCall, and BestOfCall) than Gaussian Copula. This is explained by the fact that the Clayton Copula is able to capture a better dependence in the negative tail than in the positive tail of distribution functions.

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