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Mathematical Modelling of Cracking Process in Concrete Pavement Highways

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Abstract

One of the most important ways of improving the design of road with concrete pavement is to investigate the processes that cause their destruction The use of concrete pavement on the roads of Russia is largely constrained by the lack of objective information on the potential of such pavement to serve for up to 50 years and more without capital repairs. The material presented in the article describes the scientific study of the crack formation processes in concrete surface due to the vehicle flow and changing weather conditions. A mathematical modelling has been applied as a research method.

The study proposes a mathematical model of the meteorological environment with ambient temperature and humidity parameters, the intensity of solar radiation in light of clouds, the number of rainfalls based on years of observation at the weather stations of the country. The computational experiment made it possible to assess the functionability of the mathematical model, the modification of the strength of cement concrete and the increase of the thickness of the concrete pavement and to influence the crack formation of the soil elasticity module.

Keywords: crack formation, cement concrete fatigue, single damage, damage measure.

INTRODUCTION

Modern design techniques for the construction structures in most cases are based on comparing alternatives and choosing optimal solutions by comparing the cost of a full life cycle. This means that in defining the basic parameters of the design and selecting the necessary materials, preference should be given to the variants that require a minimum of one-time construction costs and subsequent time-distributed costs for repairs and maintenance during the operation of the facility.

This approach required the creation of a new scientific direction to predict the destruction and development of various structural distresses in the process of operation.

The experience so far accumulated in various areas of knowledge, coupled with the increased ability to store and process large amounts of data based on modern intelligent systems, allows new perspectives to meet traditional challenges, related to prediction of structural destruction processes, on large time segments.

The major complexities of long-term forecasting are obvious and the expected inaccuracies are inevitable, but with respect to the need to take into account differences in repair and maintenance costs when comparing competing options in the design process of the road pavement, the development of forecasting methods is becoming more and more necessary.

Monitoring of the condition of the road pavement structures with cement concrete pavement [1-6] showed that their destruction was occurring gradually, with the passage of time in all layers, including subgrade, the damage accumulated. The mechanism for the development and accumulation of damage to individual layers varies considerably. In subgrade, residual deformations accumulate as a result of excessive tangential stresses. Sand draining layers as a result of silting are losing the ability to effectively convey water away. The layers of the base from such unbound materials as rubble are gradually losing rigidity as a result of the cyclical pressures from the passing vehicles. Overlay layers resulting from multiple bending of the vehicle wheels are susceptible to fatigue cracking. The formation of the tracing rut on the surface of the coating is the result of the gradual accumulation of plastic deformation and abrasion, especially under the influence of studded tires.

It is important to stress that all the processes listed are developed against the backdrop of changing meteorological conditions such as temperature and humidity, solar intensity, rainfall and wind velocity. All of this leads to changes in material properties, which significantly accelerates the development of damage in the constructive layers of road pavement.

FUNDAMENTAL PRINCIPLES OF RESEARCH METHODOLOGY

In the research of such phenomena, from the standpoint of reliability theory [7, 8], there is a growing proliferation of so-called interval cumulative mathematical failure models that describe the quasimonotonic deterioration of quality parameters of the construction in the process of its operation under the influence of the environment and the cyclical loads of the passing vehicles. Given the constant change in the state of the structure over time, the forecasting period is broken down into intervals of such duration, within which the structure's state is not significantly changed.

One of the main schemes for the destruction of concrete pavements is the formation of cracks due to the multiple effects of car wheels combined with the stresses caused by temperature changes of the concrete slabs.

It is assumed that in the contact area of each wheel of the vehicle there may be tension causing a single damage. To estimate the value of this single damage on the pavement, consider the tense-warped state at an arbitrary point with the x, y, z coordinates.

It is known that the stress of a slab is determined by the shape of the curved surface w is the coordinate function [9, 10] and depends on the amount of the vehicle load and the temperature distribution of the slab thickness. The task is greatly complicated by the constant change in the temperature of the surface caused by changes in air temperatures and the intensity of solar radiation. The deformation caused by a car load is done over the temperature deformation of the slab, some of which are implemented in stress due to restrictions on the free warpage and linear movement of the slab.

In order to quantify the damage to the concrete pavement at randomly selected point A, which is caused by the passing of one vehicle, use the results of the cement concrete tests by the cyclical bending loads. The testing of concrete beams of standard sizes shall normally be carried out in cycles with the same intervals between the load cycles.

At the same time, the experimental studies of cement concrete fatigue [11] have two parameters defining the strength parameters from the number of load loops. The cycle options are: stress ratio $\acute{\eta}$, or stress level and cycle characteristic ρ. The stress level of the ή refers to the ratio of maximum stress of the cycle σ_{max} to the tensile strength of R, and the cycle characteristic ρ – is the ratio of maximum stress of the cycle σ_{max} to minimum σ_{min}.

In accordance with the hypothesis of a linear addition of damages known abroad as "Miner rule", it can be accepted that the single damage is a value that is the reverse of the limit number of cycles N for these characteristics of the cycle. The damage measure D is usually defined as the sum of the single damage, in the form of a continuously increasing function in the range of 0 to 1.

The condition of cracks' occurrence is written as:

$$D > 1. \tag{1}$$

Thus, the mechanism for accumulating damages is the process of addition of single damages.

When considering the process of changing the damage measure, it is important to note that when a critical value is reached at some point or area of the damage measure, the slab moves to a new state where there are cracks of limited sizes. In this state, you must change the calculation scheme to reduce the section at the cracking locations.

Modelling of intensely-deformed coating state under influence of temperature changes

The complexity of the task is that the temperature and its distribution by the thickness of the coating are constantly changing in accordance with changes in the meteorological conditions and are functions of time following changes in air temperatures and solar intensity.

On the basis of the analysis of the temperature regime of the concrete coatings and their stress [12], the following types of stress have been allocated:

1. Expansion and compression stresses caused by the limitations of movement on the slab contour and due to abrasion on the slab contact and the base σ_1^{T}.
2. Stresses caused by the total or partial limitation of the warpage slab σ_2^{T}.
3. Internal stresses resulting from the non-linearity of the temperature diagram of the slab thickness σ_3^{T}.

Compression stresses caused by movement restrictions on a slab contour are determined in the simplest way, based on the assumption that there are deformation constraints on the slab ends and, therefore, if the slab temperature increases for a value of ΔT, the amount of stress will be:

$$\sigma_1 = \alpha \cdot E \cdot \Delta T \ , \tag{2}$$

where α – is the coefficient of linear expansion of cement concrete, 1/°C;

E - modulus of elasticity of cement concrete, MN/m^2;

ΔT – differential temperature in the middle section.

The value of the compression stresses on the contour is usually associated with the change in temperature of the middle layer, and the extent to which it differs from the initial temperature, is judged by the tension in the coating.

Another obstacle to the free deformation of concrete coatings when changing linear dimensions is the abrasion between the slab and the base.

When the surface temperatures and the lower surface of the slab are different in the concrete coatings, there is a change in the shape of the slab, called warpage. Obstacles to the free warpage of the slab on the side of the slab's own weight and restrictions in joints result in a part of the warpage deformation being implemented in stress, while the other part remains deformed.

In daylight, the surface of the slab may have a temperature of 10 ... 15 °C higher than the slab lower layer, resulting in compression stresses on the surface and tension stress on the soles. For the first time, for a linear distribution of temperature of the coating thickness, the Westergaard [13] proposed the following formula for determining normal stresses:

$$\sigma_2^T = \frac{\alpha \cdot \mathrm{E} \cdot \left(T_{(o,t)} - T_{(h,t)}\right)}{2 \cdot (1-\mu)}, \tag{3}$$

where $T_{(o,t)}$, $T_{(h,t)}$ are the temperature on the surface and the floor line of the slab, respectively, at time t;

μ – is the Poisson ratio of concrete.

According to the presented equation, the stress is independent of the size of the slab. However, experimental data show that the temperature for only a limited period of time has a linear distribution over the thickness of the slab. Especially during the peak temperature period on the surface, the distribution of its depth is significantly deviating from the straight line.

Tomlinson [14] performed an analysis of the warpage stress change based on the assumption that the surface temperature is changed by the next periodic dependency

$$\mathrm{T}_{(\mathrm{o,t})} = A \cdot \cos\left(\frac{2 \cdot \pi}{t_{day}}\right) \cdot \mathrm{t}, \tag{4}$$

where A is the amplitude of temperature fluctuations on the surface;

t_{day} - duration of the day, hrs.;

t - the current time, hrs.

Bradbury, using the Westergaard's solution for limited sized rectangular slabs, proposed the following equations [15]:

For slab edge

$$\sigma_2^T = \frac{\alpha \cdot \mathrm{E} \cdot \left(T_{(o,t)} - T_{(h,t)}\right)}{2} \cdot C; \tag{5}$$

For the middle of a slab

$$\sigma_2^T = \frac{\alpha \cdot \mathrm{E} \cdot \left(T_{(o,t)} - T_{(h,t)}\right)}{2} \cdot \left(C_{\mathrm{y}} + \mu C_x\right), \tag{6}$$

where C, C_x, C_y – are coefficients depending on the relationship of ℓ/L or b/L; ℓ and b the length and width of the slab, respectively, as defined by Figure 1 [16];

$$\mathrm{L} = 0.6 \cdot \mathrm{h} \cdot \sqrt[3]{\frac{\mathrm{E_b}}{\mathrm{E_o}}}; \tag{7}$$

$\mathrm{E_b}$, $\mathrm{E_o}$ - Modulus of elasticity of concrete and base respectively, MN/m^2.

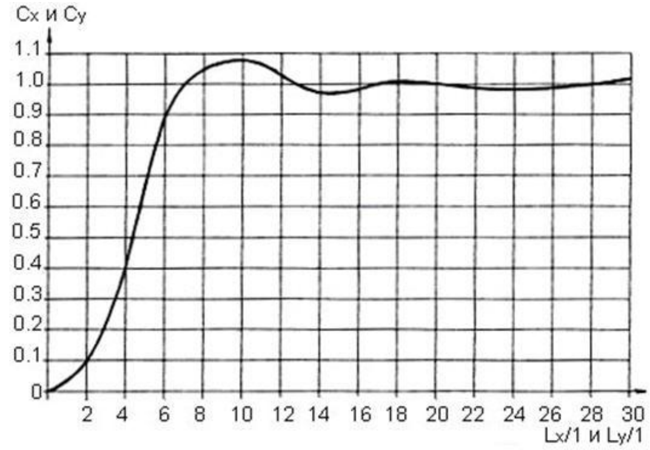

Figure 1: Graph for determining C_x and C_y coefficients.

The integrated solution of the stresses in the slab under the influence of the temperature field is presented in works [17, 18].

In summarized form, the formula for the determination of temperature stresses includes all three components discussed above: σ_1, σ_2 and σ_3.

$$\sigma^T = \alpha \cdot E \cdot T + \frac{1}{2\mathrm{h}} \int_{-h}^{+h} \alpha \cdot E \cdot T \cdot dz + \frac{1}{2\mathrm{h}^3} \int_{-h}^{+h} \alpha \cdot E \cdot T \cdot z \cdot dz, \tag{8}$$

Where z –is the current coordinate of the slab thickness

h – half of slab thickness.

$$\sigma^T = \sigma_1^T + \sigma_2^T + \sigma_3^T. \tag{9}$$

On the basis of a synthesis and analysis of the characteristics of the strained state of the concrete coatings due to the changing solar intensity and ambient temperature, the temperature field of the structure is changed and accordingly, her tense and deformed state.

Modelling of temperature change of cement concrete surface

As noted above, the temperature field of the concrete resulting from atmospheric agents causes temperature stresses, which are capable of exceeding the stresses of the vehicle load. The problem of numerical evaluation of temperature stresses is a major issue of the theory of strength and plays a decisive role in many industries.

The thermoelasticity theory, synthesizing classical elastic theory and conductivity theory, includes the following tasks: the transfer of heat under a transient thermoexchange to an outside environment and the thermoelastic stresses caused by a temperature gradient; the thermal effects caused by the interaction of deformation and temperature fields.

Relaxation of temperature stresses is possible within the daily and moreover seasonal cycles. The temperature stresses can be summed up with the mechanical stresses of the external forces and reduce the durability of the coating.

It is therefore agreed that the cracking and durability of the concrete coating are largely determined by the nature of the distribution of temperature stresses in the structures.

The temperature fields of the multilayered "road pavement - earthwork" system under consideration are defined by known thermal conductivity equations and boundary and initial conditions. The boundary conditions accept the temperature of the surface and the temperature at some depth, where it is constant or varies slightly.

L.I. Goretsky [12] proposed the following ratio defining the surface temperature for practical use:

$$T_P = T_B +_{eq} = T_в + \frac{\rho \cdot \mathrm{I} \cdot \mathrm{k_z} \cdot \mathrm{k_{ob}}}{\mathrm{a_н}}, \quad (10)$$

where T_B - is the air temperature;

T_{eq} - Increasing the temperature of the surface under the solar radiation;

ρ – absorption factor;

I – solar intensity;

$a_н$ - heat transfer coefficient, W/m^2;

k_{ob} – cloud coverage coefficient;

k_z – air dustiness coefficient.

Each of the components of the equation (10) is a function of time, and the process of temperature change on the covering surface can be represented as the sum of the two air temperature change processes and the equivalent temperature change process of additional heating of the coating surface through solar radiation.

Using the models of air temperature change $T_B(t)$ and solar radiation in light of cloud changes OBL(t), we get a mathematical model of the surface temperature change in the coating:

$$T_P(0, \mathrm{t}) = T_B(t) + T_{eq}(\mathrm{t}). \quad (11)$$

A second boundary condition is required to calculate the cement temperature of the thermal-water equation solution. This condition shall allow for the influence of temperature in the deeper layers of the ground to be taken into account. For this purpose, it is proposed to select a temperature at a depth of 3.2 m. At this depth, the daily temperature fluctuations are not affected, and the periodic observations performed at the weather station provide information for analysis and synthesis.

The comparison shows that the temperature is virtually unchanged during the month, and the annual changes with high accuracy can be presented on the basis of statistical processing of weather observation data, fixing the temperature at a depth of 3.2 m.

The temperature change of the cement coating is determined by the solution of the known heat conductivity equation:

$$c \cdot \gamma \cdot \frac{\partial \mathrm{T}}{\partial \mathrm{t}} = \frac{\partial}{\partial z}\left(\lambda \cdot \frac{\partial T}{\partial t}\right), \quad (12)$$

where λ – heat conduction coefficient;

c - coefficient of heat capacity;

γ – density.

For a time-independent constant coefficient of heat capacity, the equation is recorded as:

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}} = a \cdot \frac{\partial \mathrm{T}}{\partial \mathrm{t}}, \quad (13)$$

where a is the temperature conductivity coefficient equals $a = \frac{\lambda}{c \cdot \gamma}$.

A special study requires the question of initial conditions, i.e. the distribution of temperature at the beginning of time. Applying the temperature field at some point in timet0 as the initial condition, we assume that there is a time lag of Δt, during which the model goes into the specified mode.

The duration of this interval will be so that, regardless of the temperature distribution at the time of the t_0, the distribution must be the same at the time t. In some cases, use the term "heat" the model during the Δt time period. However, during Δt, the model must be in normal mode, affected by the specified surface temperature change process. Numerous calculations have shown that this condition corresponds to a period of 4-5 days. This allows you to specify the temperature distribution at the depth of a linear view constraint:

$$\mathrm{T}(0, \mathrm{z}) = T_P(0) - A \cdot \mathrm{z}, \quad (14)$$

where $T(0, z)$ is the temperature of the surface at the starting point in time;

A - coefficient depending on the temperature difference between the surface and the lower boundary;

z is the distance from the surface.

In the final delta, the equation (13) takes the form:

$$\frac{\left(\mathrm{T_{i+1,j}} - T_{i,j}\right)}{\Delta\tau} = \frac{a \cdot \left(\mathrm{T_{i,j-1}} - 2 \cdot T_{i,j} + T_{i,j+1}\right)}{\Delta z^2}, \quad (15)$$

where $T_{i,j}$ is the temperature of the material in layer j at time i;

$\Delta\tau$ – is a time interval step;

Δz is a grid pitch in depth.

With the boundary and initial conditions, you can calculate the temperature at each point in a depth at any point in time. This solution can be easily implemented as a program for calculating the temperature pattern of a road pavement construction at a certain point in time.

Modelling of changing process of temperature deformations and stresses in coating

The length coating of the road is considered to consist of rectangular slabs of thickness h, width B, separated by the expansion joints at a distance of ℓ_p and a compression at a distance of ℓ_c. Under the influence of atmospheric phenomena, there is a continuous process of changing the temperature condition of the coating. With some assumptions, it can be assumed that the temperature is not dependent on the x, y coordinates and is a function of time t and z coordinate. You can read the process as defined if you know the function $T(z,t)$.

Observations and special experiments show that, as a result of the temperature change, the surface of the slab may become distorted as a result of the vertical movements of $W(x, y)$ and change linear dimensions, receiving the increment of $\Delta\ell$. Obstacles to free movement through restrictions at the edges, self-weight of the slab and base abrasion of the slab cause some possible displacements to be delayed and implemented as $\sigma(x, y, z)$ stresses.

It is important to note that not only the value of the stresses are relevant, but also the vertical movement of W and the reduction or increase in the length of the slab $\Delta\ell$. Vertical movements result in areas with missing contact in the base. Reducing the length of the slab results in the opening of joints and the corresponding reduction in the ability to transmit the load to the adjacent slab through the joint.

In order to quantify these phenomena, it is necessary to develop a calculation procedure that will result in the output of stresses value $\sigma(t)$, vertical movements $W(t)$, and the opening of the joint $\Delta\ell(t)$.

The proposed mathematical model of an evaluation of the tense-warped state involves the following procedures:

1. Calculation of the function of thermal probability distribution during construction period $F(T_{cp})$.

2. Calculation of the first temperature criterion for the temperature of the middle layer with the specified level of provision $T(h/2)$ to determine the start of the compression period.

3. Calculation of the second temperature criterion T^*, which defines the beginning of the period of linear movements and the opening of the joint.

4. Calculation of the third temperature criterion $\Delta T_{\mathrm{h}}^{*}$ determining the beginning of the period of vertical deformation and separation of the slab from the base.

Figure 2 shows the data-flow diagram for the calculation the temperature deformations and stresses.

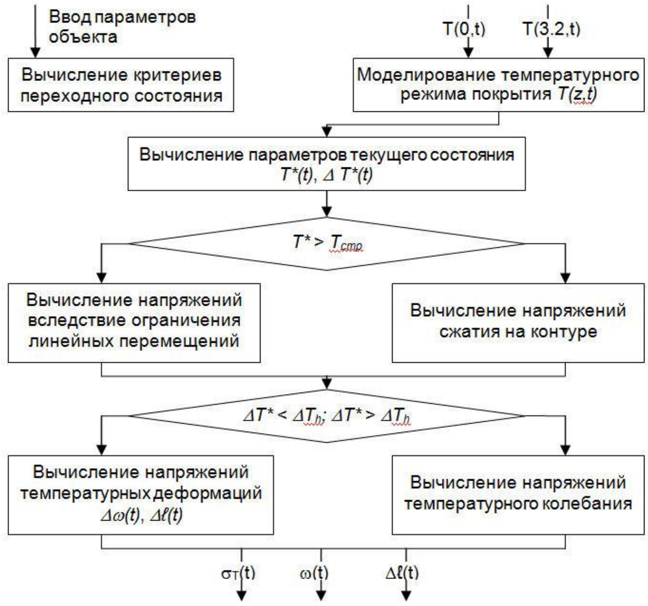

Figure 2: Schematic block diagram of temperature deformations and stress calculations.

5. Function call $T(z,t)$ at the specified points in time t_1, t_2, $t_3, \ldots, t_n$ and conditional check:

$$T\left(\frac{\mathrm{h}}{2}, t\right) > T^*\left(\frac{h}{2}\right). \tag{16}$$

6. When the condition (16) is met, the calculation of the compression stress is performed according to the formula (2).

7. In the case of failure to comply with the condition (16), test the condition:

$$T\left(\frac{\mathrm{h}}{2}, t\right) - T^*\left(\frac{h}{2}\right) < \Delta T_h^*. \tag{17}$$

8. When the condition (17) is met, calculates the stresses arising from the abrasion of the slab on the base.

9. If the condition (17) is not met, the stresses are due to the partial delay of the linear deformation.

10. Conditional check:

$$\left[T(0,\mathrm{t}) - T(h,t)\right] < \Delta T_h^*. \tag{18}$$

11. When the condition (18) is met, the warpage stresses σ_2^T are calculated with due allowance for the total delay.

12. If the condition (18) is not met, the warpage stresses σ_2^T due to the partial delay of vertical movements are calculated by formula (5, 6).

13. Calculation of the vertical movement of *W*.

14. The calculation of the total stress of σ^T is performed as the sum of the components σ_1^T, σ_2^T, σ_3^T.

EXAMPLES OF MATHEMATICAL MODEL APPLICATION FOR PREDICTION OF DAMAGE MEASURE TO CEMENT COATING

As an illustration of the capabilities of the proposed model, a scheme is used where the slab is considered infinite or semi-infinite, the load is uniformly distributed around the area or concentrated force. The base is represented by a linearly deformable half-space, the base warping module is based on the multilayered system in accordance with the methodology described above.

To implement the mathematical models, software was developed to implement the algorithm presented [19].

The complex includes the possibility of entering meteorological parameters for simulating climatic conditions in the form of tables describing: the temperature and humidity of the air, the solar intensity, the cloud coverage, the precipitation and the wind velocity.

The computational experiment was conducted in three successive phases, each of which could be performed independently of the others.

1. Modelling of the meteorological parameters with a record of the daily values of each parameter for the simulation period.

2. Modelling of the water-thermal regime with soil moisture, depth freezing recording for every 24 hours and temperatures at 6 points at the depth of each hour.

3. Modelling of the process of accumulating damage based on traffic flow models and stress-deformed state with the recording of the damage measure every 10 days, month and year.

Each of these steps has a separate meaning and can be used to solve private tasks when assessing the working capacity of the defined road pavement. The software complex provides the ability to vary several dozen parameters and to determine their impact on the intensity of the damage accumulation process.

CONDITIONS AND RESULTS OF COMPUTATIONAL EXPERIMENT

To illustrate the possibilities of the proposed model, several examples of its application are discussed below to address the private challenges encountered in the design, construction and operation of roads.

In order to implement the computational experiment, it was accepted that the facility was located near Moscow and was a category III road, with a carriageway of 7 meters width. Coating from cement concrete with tensile strength of 4 MPa, the distance between the compression joints 5 meters. Base of gravel limestone with elasticity modulus 300 MPA, thickness of 15 cm. Sand with elasticity module 100 MPA, thickness of 30 cm, is used as a drainage layer. The traffic in the first year of operation amounted to 1000 calculated vehicles per day.

The baseline data for meteorological conditions are the actual data of the Moscow State University weather station in four years.

When optimizing the design of road pavement, it is necessary to assess the extent to which the increase in the thickness of the coating will contribute to slowing the accumulation of damage. An indirect answer can be obtained by comparing the damage accumulation processes for roads that differ in the coverage thickness. Figure 3 shows the results of the computational experiment, which examines the design of the road pavement with a coating thickness of 18, 20 and 22 cm. You can see from the picture how much you reduce the developmental intensity of the damage measure by increasing the coverage thickness by 2 cm.

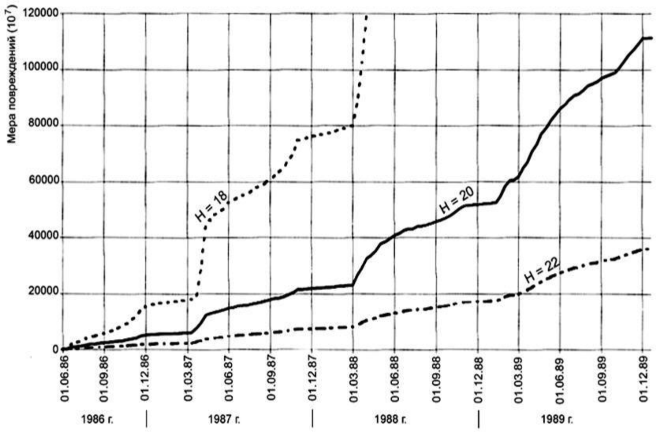

Figure 3: Change of damage measure for structures with varying coating thickness.

In assessing the quality of construction works, it is often required to determine the extent to which the reduction, for example, of the tensile strength of the cement concrete on bending from 4MPa to 3 MPa influences the intensity of the accumulation process. The computational experiment makes it possible to compare the quantitative differences in the development processes of damage for cement with different strength.

During the operation, the road is exposed to atmospheric phenomena constantly changing over time. This results in a change in the state of the road pavement design and the earthwork.

In order to illustrate the impact of this factor, a computational experiment was carried out which determined how the damage measure was modified in the three scenarios of changing the elasticity modulus of the earthwork: the elasticity module does not change and retains the minimum value of 19 MPa during the life of the service, does not change, and retains the value of 200 MPa and is constantly modified to reflect changes in the meteorological conditions in the simulation data.

CONCLUSION

The results of computational experiments point to the possibilities of mathematical modelling to quantify the influence of various factors on the intensity of the formation of cracks on concrete roads. Based on a comparison of the change processes of the damage measure of the road pavement structure with different parameters, the following conclusions are reached:

The increase in the thickness of the cement coating on the 2 cm roughly halves the crack formation process.

Increasing the strength of the cement concrete from 3 MPa to 3.5 MPa three-fold slows down the cracking process.

Increasing the stiffness of the base of concrete coatings in deep frost soil can significantly retard the formation of cracks.

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