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Feedback Queue with Services in Different Stations under Reneging and Vacation Policies

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Abstract

This paper describes a single server queuing system where the units arrive in batches of variable size under Poisson stream. The server provides services in k stations. The service times in each station follows general distribution. The principles of feedback, vacation and reneging are employed in the system. The steady state probability functions that the server is providing service in any service station and that on vacation are derived. The corresponding steady state probabilities are also obtained. The expected number of units in the queue has been obtained for some special cases.

Keywords: Single server, feedback queue, reneging, steady state probability, vacation

INTRODUCTION

An emerging area of queueing theory is the bulk queue, in which arrival and/or departure can happen in batches either fixed or in variable size. Vital applications of this queueing model can be seen implied in many areas like communication system, computer networks, production industry, logistic sector etc.

Queueing models with vacation policies have been studied by many researchers including reneging. In real life, there are some queueing situations when some unit is discouraged by long waits in the queue. The units may be decided to balking or reneging. Balking and reneging have attracted the attention of many researchers and study on queues with behavior of such units has developed extensive amount of literature.

Concept of reneging in queueing systems was introduced by Ancker and Gafarian [1] and Daley [2]. Bae *et al.,* [3] have studied the waiting time of M/G/1 queue with impatient customers. Medhi [4] has explained a single server Poisson arrival queue along with a second optional channel. Altman and Yechaili [5] have analyzed customer impatience in queues with server vacation. Choudhury and Medhi [6] have studied balking and reneging in multi-server Markovian queueing systems.

Vacation policies in queues were extensively studied previously by Baba [7], Doshi [8], Keilson and Servi [9], Madan et al., [10, 11, 12], Borthakur and Choudhury [13, 14, 15]. Ganesan and Sundar Rajan [16] have studied bulk arrival queue with breakdown analysis. Thangaraj and Vanitha [17] have discussed two-stage heterogeneous service, compulsory vacation and random breakdowns. Ganesan and Sundar Rajan [18] have analyzed a queue with heterogeneous services and random breakdowns. Ayyappan and Sathiya [19] have studied three stage heterogeneous service and server vacation of $M^x/G/1$ feedback queueing model as well as two types of random breakdowns and multiple vacations with restricted admissibility [20]. Monita Baruah et al.,[21] have studied bulk input queue which have second optional service along with reneging during vacation.

The present study deals $M^x/(G_1, G_2,\ldots\ldots G_k)/1$ queueing model, where the units arrive in batches of variable size defined by Bailey [22] and once the units enter the initial service process, it must go through k stations of service on first in first out (FIFO) discipline. The service time of this model follows general distribution. The unit follows Bernoulli feedback after completion of k stations of service and if it has not received a quality service, the unit will rejoin at the end of the queue with probability p or leaves forever from the system with probability (1-p). Whenever the system is empty, the server may go on vacation for a random duration. Here, the server adopts multiple vacation policy until at least one batch of arrival present in the system. The vacation periods of the server are distributed according to general distribution. It is assumed that when the server is on vacation the unit may renege from queue and it follows exponential distribution with parameter β.

ASSUMPTIONS AND DESCRIPTIONS OF THE QUEUEING MODEL

This queueing model is described under the assumptions are as follows:

- Units arrive in batches of variable size and follow Poisson stream. Let $\lambda a_\theta dt$ $(\theta = 1,2,\ldots n)$ be the first order probability that a batch of θ units arrive to the system during the small interval *(t, t + dt),* where $0 \le a_\theta \le 1, \sum_{j=1}^{\infty} a_\theta = 1$ and $\lambda > 0$ is the mean arrival rate of batches.

- Each unit undergoes k stations of heterogeneous service provided by a single server on a first in first out (FIFO) discipline.

- The service times of k stations follow different general distributions function $B_i(V)$ and density function $b_i(V)$

- $B_i(V), i = 1,2,\ldots k$ assumes that they have finite moments $E\,(s_i^m)$ for m≥1 and $i = 1,2,\ldots k.$.

- After completion of k stations of service, if any units require repeating its service for any reason or for unsuccessful service, the unit rejoins at the end of the queue. Service time for a feedback unit is independent of its previous service time.

- Let $s_i(x)\,dx$ be the conditional probability of completion of the i^{th} station of service during the time interval, *(x, x + dx)* given that the elapsed time is x, so that

$$s_i(x) = \frac{b_i(x)}{1 - B_i(x)}, \qquad i = 1,2,\ldots k \tag{1}$$

and

$$b_i(x) = s_i(\gamma) e^{-\int_0^\infty s_i(x)dx}, \qquad i = 1,2,\ldots k \tag{2}$$

- The units may renege when the server is on vacation. This reneging follows exponential distribution with parameter β. Thus

$$f(t) = \beta e^{-\beta t} dt, \beta > 0 \tag{3}$$

- Let $\beta\, dt$ be the probability that a customer can renege during a short interval of time *(t, t + dt)*

- After completion of service, if there is no unit waiting in the system, then the server goes for vacation with random duration. On returning from vacation, the server instantly starts servicing depending on the availability of the units. Otherwise, if no unit is waiting in the system, then it goes for vacation again. The server continues to go for vacation until it finds at least one batch in the system. The vacation time follows a general distribution with distribution function *V(x)* and density function *v(x)* Laplace-Stieltjes Transform (LST) *V(t)* and finite moments $E(V^m)$, *m≥1*

- Let $\gamma(x)\, dx$ be the conditional probability of a completion of a vacation during the interval *(x, x + dx)*, given that the elapsed vacation time is x, so that

$$\gamma(x) = \frac{v(x)}{1 - V(x)}, \quad i = 1,2,\ldots k \tag{4}$$

and

$$V(v) = \beta(v) e^{-\int_0^\infty \beta(x)dx} \tag{5}$$

- Inter-arrival times, service times of each station of service and the vacation times are independent.

The steady state probabilities are defined as,

$P_n^{(i)}(x)$: Steady state probability that, the server is providing service in the i^{th} station and there are n (≥0) units in the queue excluding the one being served.

$V_n(x)$: Steady state probability that, the server is on vacation and there are n (≥0) units in the queue.

Formulation of steady state equations

Based on the postulates given as above, the limiting behavior of this queuing process at the stationary point of time can be done by using below mentioned Kolmogorov forward equations

$$\frac{d}{dx} P_n^{(i)}(x) + \big(x + s_i(x)\big) P_n^{(i)}(x) = \lambda \sum_{\theta=1}^{n} a_\theta P_{n-\theta}^{(i)}(x), \quad n \ge 1 \tag{6}$$

$$\frac{d}{dx} P_o^{(i)}(x) + \big(\lambda + s_i(x)\big) P_o^{(i)}(x) = 0, \quad i = 1,2,\ldots k \tag{7}$$

$$\frac{d}{dx} V_n(x) + (\lambda + \gamma(x) + \beta)\, V_n(x) = \lambda \sum_{\theta=1}^{n} a_\theta V_{n-\theta}^{(x)} + \beta V_{n+1}^{(x)} \tag{8}$$

$$\frac{d}{dx} V_o(x) + (\lambda + \gamma(x))\, V_o(x) = 0 \tag{9}$$

The boundary conditions for solving the above equations (6) to (8) are stated as

$$P_n^{(1)}(0) = \int_0^\infty V_{n+1}(x)\gamma(x)dx + p\sum_{i=2}^{k}\int_0^\infty P_n^{(i)}(x)\, s_i(x)d(x) + q\sum_{i=2}^{k}\int_0^\infty P_{n+1}^{(i)}(x)\, s_i(x)d(x), n \ge 0 \tag{10}$$

$$P_n^{(i)}(0) = q\int_0^\infty P_n^{(i-1)}(x) s_{j-1}(x)\, d(x), \quad n = 0,1,2,3 \ldots \ldots, i = 2, 3\ 4, \ldots k \tag{11}$$

$$V_n(o) = 0 \tag{12}$$

$$V_0(o) = \int_0^\infty V_0(x)\gamma(x) + q\sum_{i=2}^{k}\int_0^\infty P_o^{(i)}(x)\, s_i(x)d(x) \tag{13}$$

Probability Generating Function

Let us define the probability generating functions as follows

$$\left.\begin{aligned} P^{(i)}(x,z) &= \sum_{n=0}^{\infty} P_n^{(i)}(x)\, z^n \\ P^{(i)}(z) &= \sum_{n=0}^{\infty} P_n^{(i)}\, z^n \\ V(x,z) &= \sum_{n=0}^{\infty} V_n(x)\, z^n \\ V(z) &= \sum_{n=0}^{\infty} V_n\, z^n, \quad \forall\, |z| \le 1,\ x > 0 \end{aligned}\right\} \qquad (14)$$

Solution of steady state equations

Multiply equations (11) and (8) by z^n and taking summation overall possible values of n and on simplification we get,

$$\frac{d}{dx}P^{(i)}(x,z) + (\lambda - \lambda A(z) + s_i(x))\, P^{(i)}(x,z) = 0 \qquad (15)$$

$$\frac{d}{dx}V(x,z) + \left[\lambda - \lambda A(z) + \gamma(x) + \beta - \frac{\beta}{z}\right]V(x,z) = 0 \qquad (16)$$

We multiply equations (10),(11) and (12) with appropriate powers of z and summing over suitable values of n, and get,

$$zP^{(1)}(0,z) = \int_0^\infty V(x,z)\gamma(x)dx + (q+pz)\sum_{i=2}^{k}\int_0^\infty P^{(i)}(x,z)s_i(x)dx - q\sum_{i=2}^{k}\int_0^\infty P_0^{(i)}(x)s_i(x)dx - \int_0^\infty V_0(x,z)\gamma(x)dx \qquad (17)$$

$$P^{(i)}(0,z) = \int_0^\infty P^{(i-1)}(x,z)s_{i-1}(x)dx,\quad i = 2,3,\dots k \qquad (18)$$

$$V(0,z) = V_0(0) \qquad (19)$$

Now, use the equations (13) and (17) and get,

$$zP^{(1)}(0,z) = \int_0^\infty V(x,z)\gamma(x)dx + (q+pz)\sum_{i=2}^{k}\int_0^\infty P^{(i)}(x,z)s_i(x)dx - V_0(0) \qquad (20)$$

By using the principal of linear differential equation in the equations (15) and (16) which provide,

$$P^{(i)}(x,z) = P^{(i)}(0,z)e^{-(\lambda-\lambda A(z))x - \int_0^\infty s_i(t)dt} \qquad (21)$$

$$V(x,z) = V(0,z)\, e^{-(\lambda-\lambda A(z)+\beta-\frac{\beta}{z})x - \int_0^\infty \gamma(t)dt} \qquad (22)$$

Now, integrate equation (21) with respect to x and get,

$$P^{(i)}(z) = P^{(i)}(0,z)\left[\frac{1-B_i^*(\lambda-\lambda A(z))}{(\lambda-\lambda A(z))}\right] \qquad (23)$$

where,

$$B_i^*(\lambda - \lambda A(z)) = \int_0^\infty e^{-(\lambda-\lambda A(z))x}\, dB_i(x)$$

is the Laplace transform of the service time in i^{th} station. Again, apply simple mathematics and using the expression (2) in the equation (21), which becomes

$$\int_0^\infty P^{(i)}(x,z)s_i(x)dx = P^{(i)}(0,z)B_i^*(\lambda - \lambda A(z)) \qquad (24)$$

In the same manner, the equation (22) is reformulated as

$$V(z) = V(0,z)\left[\frac{1-V^*\left(\lambda-\lambda A(z)+\beta-\frac{\beta}{z}\right)}{\left(\lambda-\lambda A(z)+\beta-\frac{\beta}{z}\right)}\right] \quad (14) \qquad (25)$$

and

$$\int_0^\infty V(x,z)\gamma(x)dx = V(0,z)\, V^*(\lambda - \lambda A(z) + \beta - \frac{\beta}{z}) \qquad (26)$$

where,

$$V^*(\lambda - \lambda A(z) + \beta - \frac{\beta}{z}) = \int_0^\infty e^{-(\lambda-\lambda A(z)+\beta-\frac{\beta}{z})}\, dV(x)$$

Using (18) and (24), we have

$$P^{(i)}(0,z) = \frac{1}{D(z)}\left|\prod_{j=1}^{i-1}\{B_j^*(\lambda - \lambda A(z))\}\right\} \{V^*(\lambda - \lambda A(z) + \beta - \frac{\beta}{z}) - 1\}]V_0(0),\quad i = 1,2,\dots k \qquad (27)$$

where,

$$D(z) = z - (q+pz)\sum_{i=2}^{k}\prod_{j=1}^{i}B_j^*(\lambda - \lambda A(z))$$

By applying (27) in (23) and get the probability generating functions of the i^{th} station of service.

$$P^{(i)}(z) = \frac{1}{D(z)}\left|\left\{\prod_{j=1}^{i}B_j^*(\lambda - \lambda A(z))\right\}\left\{V^*\left(\lambda - \lambda A(z) + \beta - \frac{\beta}{z}\right) - 1\right\}\right| V_0(0)\left[\frac{1-B_i^*(\lambda-\lambda A(z))}{(\lambda-\lambda A(z))}\right], i = 1,2\dots k \qquad (28)$$

Similarly, substitute the equation (19) in the expression (25) and get,

$$V(z) = \left[\frac{1 - V^*\left(\lambda - \lambda A(z) + \beta - \frac{\beta}{z}\right)}{\left(\lambda - \lambda A(z) + \beta - \frac{\beta}{z}\right)}\right]V_0(0) \qquad (29)$$

The probability generating function of the whole system is obtained by using the relations (28) and (29),

$$P(z) = V(z) + z\sum_{i=1}^{k}P^{(i)}(z) \qquad (30)$$

The computation of expected queue size is very difficult since the expression (30) is the sum of products of Laplace Transform of the probability distribution functions. In this juncture, the number of service stations is reduced and some special cases are further discussed.

The steady-state probabilities that the server is providing i^{th} station of service and that the server is on vacation are respectively denoted as $P^{(i)}(1)$ and $V(1)$

On applying z=1 in the expressions (28) and (29), which become indeterminate form. So use L' Hospital's rule and get the required steady-state probabilities after using some mathematical applications,

$$P^{(i)}(1)=[\frac{\lambda E\,(s_i)E(V)}{(q-p-2\lambda\sum_{i=1}^{k-1}E(s_i)-\lambda E(s_k\,)}]V_0(0) \quad (31)$$

and

$$V(1)=E(V)V_0(0) \quad (32)$$

In order to find the value of $V_0(0)$, apply normalizing condition,

$$\sum_{i=1}^{k}P^{(i)}(1)+V(1)=1 \quad (33)$$

Now, use the expressions (31) and (32) in the condition (33) and get

$$V_0(0)=\frac{q-p-2\lambda\sum_{i=1}^{k-1}E(s_i)-\lambda E(s_k)}{E(V)[\,q-P-\lambda\sum_{i=1}^{k-1}E(s_i)]} \quad (34)$$

Special Cases

Case (i): The number of service stations are assumed as k=3

Suppose, the number of service station is taken as three and the equation (30) reduces to

$$P(z)=\left[\frac{\{V^*\left(\lambda-\lambda A(z)+\beta-\frac{\beta}{z}\right)\}B_1^*(\lambda-\lambda A(z))B_2^*(\lambda-\lambda A(z))B_3^*(\lambda-\lambda A(z))}{\lambda\,[\,z-(q+pz)B_1^*(\lambda-\lambda A(z))B_2^*(\lambda-\lambda A(z))-(q+pz)B_1^*(\lambda-\lambda A(z))B_2^*(\lambda-\lambda A(z))B_3^*(\lambda-\lambda A(z))}\right]V_0(0) \quad (35)$$

Differentiating *P(z)* w.r.t. z and letting z=1 and get in determinant form. So L' Hospital's rule is applied and get the following results which lead to expected queue size.

$$N'(1)\;=2q(\lambda A'(1)+\beta)E(V) \quad (36)$$

$$N''(1)=2q\Big[(\lambda A'(1)+\beta)^2E(V^2)+E(V)\Big\{(\lambda A''(1)-2\beta)+2(\lambda A'(1)+\beta)\lambda A'(1)\;\left(E(s_1)+\;E(s_2)+\;\frac{1}{2}E(s_3)\right)\Big\}\Big] \quad (37)$$

$$D'(1)\;=q-p-\lambda A'(1)\{2E(s_1)+E(s_2)\;+E(s_3)\} \quad (38)$$

$$D''(1)=\;-\Big[2p\lambda A'(1)\{E(s_1)+2E(s_2)+E(s_3)\}+2\lambda^2\Big\{A''(1)\big(2E(s_1)\;+E(s_2)\big)+(A'(1))^2\left(2E(s_1^2)E(s_2^2)+(A'(1))^22E(s_1)E(s_2)+E(s_1)E(s_3)\right)\Big\}+\lambda^2\Big\{\big(A''(1)\big)E(s_3)+\big(A'(1)\big)^2(E(s_3^2)))+(1+p)\big(A'(1)\big)^2E(s_2)\,E(s_3)\Big\}\Big] \quad (39)$$

Then

$$L_q=\frac{D'(1)N''(1)-N'(1)D''(1)}{2\left[(D'(1))^2\right]}\quad(\frac{V_0(0)}{\lambda})$$

$$=\Big[q(\lambda A'(1)+\beta)^2E(V^2)\xi_1+qE(V)\Big[\{(\lambda A''(1)-2\beta)+2\lambda A'(1)(\lambda A'(1)+\beta)\xi_2\}\xi_1+(\lambda A'(1)+\beta)\Big\{2p\lambda A'(1)\xi_3+2\lambda^2\left(A''(1)\xi_4\;+(A'(1))^2\xi_5\right)+\lambda^2\xi_6\Big\}\Big]\Big]\,\xi_1^{-2}.\left(\frac{V_0(0)}{\lambda}\right) \quad (40)$$

Where,

$$\xi_1=q-p-\lambda A'(1)\{2E(s_1)+E(s_2)+E(s_3)\}$$

$$\xi_2=E(s_1)+E(s_2)+\frac{1}{2}E(s_3)$$

$$\xi_3=E(s_1)+2E(s_2)+E(s_3)$$

$$\xi_4=2E(s_1)+\;E(s_2)$$

$$\xi_5=2E(s_1^2)+E(s_2^2)+2E(s_1)\,E(s_2)+E(s_1)E(s_3)$$ and

$$\xi_6=A''(1)E(s_3)+\big(A'(1)\big)^2E({s_3}^2)+(1+P)A'(1)^2E(s_2)\,E(s_3)$$

Case (ii): Suppose the vacation times follow Erlang distribution with shape parameter 'a' and scale parameter 'b'. Its mean and variance are respectively a/b and a/b^2.Then, the required expected queue size is given by

$$L_q=\Big[q(\lambda A'(1)+\beta)^2\frac{a}{b^2}(1+a)\xi_1+q\frac{a}{b}\Big[\{\,(\lambda A''(1)-2\beta)+2\lambda A'(1)(\lambda A'(1)+\beta)\xi_2\}\xi_1+(\lambda A'(1)+\beta)\Big\{2p\lambda A'(1)\xi_3+2\lambda^2\left(A''(1)\xi_4\;+(A'(1))^2\xi_5\right)+\lambda^2\xi_6\Big\}\Big]\Big]\,\xi_1^{-2}\left(\frac{V_0(0)}{\lambda}\right) \quad (41)$$

Case (iii): The service times of the three service stations are distributed exponentially with same parameter μ. The vacation times follow general distribution. Then the expected queue size given in (40) is reduced as

$$L_q=\Big[q\mu(\lambda A'(1)+\beta)^2E(V^2)\{(q-p)\mu-5\lambda A'(1)\}+qE(V)\Big[\{(\lambda A''(1)-2\beta)\mu+5\lambda A'(1)(\lambda A'(1)+\beta)\}\{(q-p)\mu-5\lambda A'(1)\}+(\lambda A'(1)+\beta)\Big\{8p\lambda\mu A'(1)+2\lambda^2\left(3\mu A''(1)+9\big(A'(1)\big)^2\right)+\lambda^2(\mu A''(1)+\;(3+p)\big(A'(1)\big)^2)\Big\}]]\{(q-p)\mu-5\lambda A'(1)\}^{-2}\Big](\frac{V_0(0)}{\lambda}) \quad (42)$$

Case (iv): The units are arriving one by one and reneging is not allowed. Then the system is reduced as M/(G_1, G_2, G_3)/1 model and its expected queue size is obtained by using A(z)=z and β=0, as follows

$$L_q=[q\lambda E(V^2)\eta_1+qE(V)[2\lambda\eta_1\xi_2+\{2\lambda p\xi_3+2\lambda^2\xi_5+\lambda^2\eta_2\}]]\eta_1^{-1}V_0(0) \quad (43)$$

where,

$$\eta_1=q-p-\lambda\{2(E(s_1)+E(s_2))+E(s_3)\}$$

and

$$\eta_2=E(s_3^2)+(1+p)E(s_2)\,E(s_3)$$

CONCLUSION

In this paper, the steady state results of a single server queue with some concepts, namely, feedback, vacation, reneging and batch arrival are mathematically derived. The expected queue sizes in different situations are discussed. Suppose one may introduce Erlang arrival and/or bulk service, then the researcher may get different types of valuable results. This queuing model and derived results can be used in communication networks and industries.

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