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Performance Analysis of the LMS Adaptive Algorithm for Adaptive Beamforming

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Abstract

The demand for higher capacity wireless communication networks has motivated research in the techniques of adaptive beamforming using smart antennas. The technique is to radiate narrow beams in a desired direction and to suppress interferences. Antenna array beamforming is a fundamental technique for directional signal transmission and reception[1].The LMS algorithm has been identified as a suitable technique that optimises the SNR of the desired signal in a particular direction. The proposed adaptive beamforming scheme uses an array of antennas to realise maximum reception in a specified direction. This is achieved by adjusting the weights of each of the antennas with changing signal environment. This paper analyses the performance of the LMS adaptive algorithm in terms of the convergence rate of the Mean Square Error (MSE), also the effect of some antenna array elements like the number of elements and the spacing between elements, based on the Direction of Arrival (DOA), but also the special case when the interference is correlated to the desired user. The simulation results show that increasing the number of antenna elements results in narrower beams reduces the correlation of the interference DOA on the desired user direction, and the optimum value of separation distance between elements is half wavelength for which the MSE gives an optimum error in a particular direction. Also, increasing the SNR of the interference over the desired signal affects the convergence performance of the algorithm. The convergence of the LMS algorithm remains unchanged even the initial weight vector changes. The results are simulated using MATLAB.

Keywords: Beamforming, LMS, Mean Square Error, number of element, Direction of Arrival, elements spacing, SNR, correlated signal

INTRODUCTION

The need for wider coverage area, improved capacity and higher transmission quality for wireless communication systems is rising as the number of mobile users increase. This has motivated research in the techniques of adaptive beamforming using smart antennas as a potential solution in the improvement of the quality of service [2][3]. The common adaptive algorithms that have been investigated for beamforming in mobile communications include LMS algorithm [4][5]. Digital beamformers are a means for separating a desired signal from interfering signals [4]. Michal Vavrda describes opportunities and constraints for application digital beamforming techniques and adaptive beamforming techniques in wireless communications. His work defines the process of digital beamforming and the process of adaptive beamforming. Digital beamforming (DBF) is defined as a combination of antenna technology and digital technology, where the antenna converts spatiotemporal signals into strictly temporal signals, thereby making them available to a wide variety of signal processing techniques. In the work by Mallaparapu et al [6], the good attraction of the LMS algorithm is its low computational complexity. In addition, beamforming for smart antenna can be used to increase the channel bandwidth and capacity and at the same time minimize the channel interference in wireless communication. Smart antennas systems, have two main functions: Direction Of Arrival (DOA) and adaptive beamforming [7]. In this literature, antenna array with adaptive beam forming technique is used to achieve the high capacity, wider coverage and efficient spectrum utilization, by using the smart signal processing algorithm such as Least Mean Square algorithm (LMS). Zhou Yuanjian and Yang Xiaohui [8], proposed a new adaptive beamforming algorithm by improving on a projection of a gradient vector to a Uniform Linear Array (ULA). Its performance is compare to the conventional Least Mean Square (LMS) algorithm. The signal processing in smart antenna system mainly focuses on the Direction of Arrival (DOA) estimation and the development of the adaptive beamforming algorithm. Also in A. Senapati and K. Ghatak [3] performance of the LMS adaptive beamforming schemes for smart antennas, depend on the value of step size parameter used in the algorithm.

ADAPTIVE BEAMFORMING ALGORITHMS CLASSIFICATION

Adaptive Beamforming algorithms can be classified into two categories which are non-blind adaptive algorithms and blind adaptive algorithms. In blind adaptive algorithms, training signal *d(t)* is not used where as in non-blind algorithm signal *d(t)*, is known to both the transmitter and receiver during the training period [6]. They both rely on statistical knowledge about the transmitted signal in order to converge to a solution[9]. Typical non-blind algorithms used are least mean square (LMS), and Normalized Least Mean Square (NLMS).

Blind adaptive algorithms

an adaptive algorithm that does not require a training sequence, or other special knowledge of the environment, is known as a blind adaptive algorithm. The use of a blind algorithm can potentially eliminate the need for training sequence, thereby increasing the available data rate[10]. However, blind algorithms have some drawback relative to conventional training sequence-based algorithm. First, blind algorithms cannot in general be guaranteed to converge to the desired solution, unlike the case when a known training sequence is used. Furthermore, blind adaptive algorithms generally converge more slowly.

Non-Blind adaptive algorithm

Non-blind adaptive algorithms need statistical knowledge of the transmitted signal (training sequence, *d(k)*) in order to converge to a weight solution. This is typically accomplished through the use of a pilot training sequence sent over the channel to the receiver to help identify the desired user[10]. Therefore, during the transmission of the training sequence, no communication in the channel can take place. This dramatically reduces the spectral efficiency of any communications system.

LEAST MEAN SQUARE ALGORITHM

In the LMS algorithm, the computation of the weight vector is based on Minimum Squared Error (MSE)[11]. The minimum mean squared error (MMSE) algorithm minimizes the error with respect to a reference signal *d(t)*.

The actual output is: $y(n) = w^H(n)x(n)$ (1)

where *H* is the Hermitian operator (conjugate transpose). From figure (4.1) belw, the error can then be written as:

$$e(n) = d(n) - y(n) \quad (2)$$

Where *d(n)* is the desired output from the antenna array.

The MMSE finds the weights *w* that minimize the average power in the error signal, the difference between the reference signal and the output signal obtained using equation (1).The mean-squared error (MSE) is then given by the following equation: $\mathrm{W_{MMSE}} = \arg\min \mathrm{E}\{|\mathrm{e(t)}|^2\}$ (3)

Where

$$\mathrm{E}\{|\mathrm{e(t)}|^2\} = \mathrm{E}\{|W^H\mathrm{X(t)} - \mathrm{d(t)}|^2\},$$

$$= \mathrm{E}\{W^H X X^H W - W^H X d^* - X^H W d + dd^*\},$$

$$= W^H R W - W^H r_{xd} - r_{xd}^H W + dd^*, \quad (4)$$

Where

$$\mathrm{r_{xd}} = \mathrm{E}\{Xd^*\} \quad (5)$$

To find the minimum of this functional, we take its derivative with respect to w^H (we have seen before that we can treat *w* and w^H as independent variables).

$$\frac{\partial \mathrm{E}\{|e(t)|^2\}}{\partial \mathrm{W^H}} = \mathrm{RW} - \mathrm{r}_{xd} = 0,$$

$$\Rightarrow \mathrm{W_{MMSE}} = w_{opt} = R_{xx}^{-1} r \quad (6)$$

This solution is also commonly known as the Wiener filter.

The LMS algorithm is non-blind type algorithm, so it uses a reference signal [4]. It is a search algorithm in which a simplification of the gradient vector computation is made possible by appropriately modifying the objective function[12]. it has a low computation complexity[2]. The optimum Wiener solution in equation (6), requires the calculation of the inverse of the correlation matrix *R* and this results in a high computational complexity. The Least mean square algorithm is a gradient based quadratic approach[13]. Gradient algorithms assume an established quadratic performance surface which is a function of the array weights, the performance surface *J (W)* is in the shape of an elliptic parabola having one minimum[14].

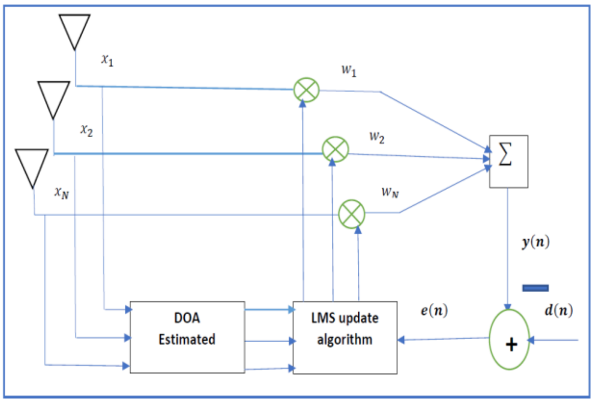

Figure. 3.1: LMS adaptive Array System

In Fig. 3.1 the outputs of the individual sensors are linearly combined after being scaled with corresponding weights optimizing the antenna array to have maximum gain in the direction of desired signal and nulls in the direction of interferers. The linear combination of input vector *x(n)* and weight vector ***w****(n)* is the output of uniform linear antenna *y(n)* at any time *n* and it is given by equation (1).

$$y(n) = w^H(n)x(n)$$

Where

$$w^H(n) = [w_1,\ w_2,\cdots\cdots w_N]^H \quad (7)$$

And

$$x(n) = [x_1,\ x_2, \cdots\cdots\cdots, x_N] \quad (8)$$

The error signal $e(n)$ is given by equation (2).

The LMS algorithm avoids matrix inverse operation and uses the instantaneous gradient vector $\nabla J(n)$ for weight vector up gradation. From the method of Steepest descent, the weight vector w(n+1) at time (n + 1) can be written as

$$w(n+1) = w(n) + \frac{1}{2}\mu[-\nabla J(n)] \quad (9)$$

Where J(n)=E[|e(n)|2] is the mean square error (MSE) cost function and μ is the step size parameter which control the convergence rate. It value lies between 0 and 1.

To calculate instantaneous gradient vector $\nabla J(n)$, auto correlation matrix R and cross-correlation vector r is needed. This can be calculated by following equations

$$\nabla J(n) = -2r(n) + 2R(n)w(n) \quad (10)$$

Where

$$R(n) = x(n)u^H(n) \quad (11)$$

And

$$r(n) = d^*(n)x(n) \quad (12)$$

With $d^*(n)$ the complex conjugate of the desire signal

By putting values from (10), (11), (12) in (9) the weight vector is found to be

$$w(n+1) = w(n) + \mu[r(n) - R(n)w(n)]$$

$$= w(n) + \mu x(n)[d^*(n) - x^H(n)w(n)]$$

$$w(n+1) = w(n) + \mu x(n)e^*(n) \quad (13)$$

Where $e^*(n)$ is the conjugate of the error signal.

The convergence of the LMS algorithm in Equation (13) is directly proportional to the step-size parameter μ. If the step-size is too small, the convergence is slow which will results in the overdamped case. If the convergence is slower than the changing angles of arrival, it is possible that the adaptive array cannot acquire the signal of interest fast enough to track the changing signal. If the step-size is too large, the LMS algorithm will overshoot the optimum weights of interest. This is called the underdamped case. If attempted convergence is too fast, the weights will oscillate about the optimum weights but will not accurately track the solution desired. It is therefore imperative to choose a step-size in a range that insures convergence. Therefore, it is better to select the step-size value μ within bounded conditions as defined in equation (14) below:

$$0 < \mu < \frac{1}{\lambda_{max}(R_{XX})} \quad (14)$$

Where, $\lambda_{max}(R_{XX})$is the largest Eigen value of the autocorrelation matrix R.

In order to avoid the underdamped case, the step-size actual value should range within: $0 < \mu < 1$. In most cases LMS for adaptive beamforming is suitable for small value of step-size[3].

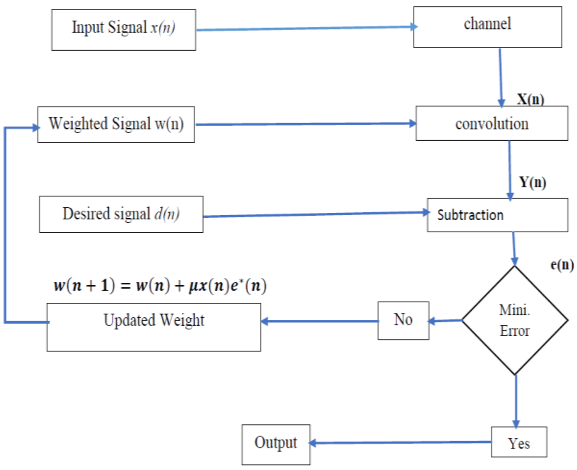

Figure 3.2: Flow chart of LMS Algorithm for weight updating

Array Factor (AF)

The array factor is a function of the weights, positions, and steering vector used in the antenna array or phased array. This factor quantifies the effect of combining radiating elements in an array without the element specific radiation pattern taken into account.

The radiation pattern of antenna array is given by the product of array factor and element factor. If we assume all elements radiates in all direction equally, the radiation pattern is equal to the array factor. Assuming far field conditions such that $r >> d$, we can derive the array factor as follows:

$$AF = \sum_{n=1}^{N} e^{j(n-1)(kd\sin\theta+\delta)} = \sum_{n=1}^{N} e^{j(n-1)\psi} \quad (15)$$

Where $\psi = kd\sin\theta + \delta$,

$$k = \frac{2\pi f_0}{\lambda}$$

where δ is the phase shift from element to elements,

d is the spacing between antenna elements

SIMULATIONS RESULTS AND DISCUSSIONS

The following simulations are done considering the desired user arriving at angle 30° and interference at angle -50°. The spacing between the individual element is half wavelength (0.5λ) and the Signal to Noise Ratio is 5dB. The array factor for

8 elements antenna arrays is computed.

General case: Plotting the polar Array Factor, the rectangular array factor, the MSE, Magnitude of array weights, and the tracking output signal

Figure (1.a) shows polar array factor plot for different angle of arrival. Figure (1.b) shows rectangular array factor plot for different angle of arrival. The uniform linear array for 8 elements is computed considering the desired user at DOA of 30°, and the interference DOA at -50°.

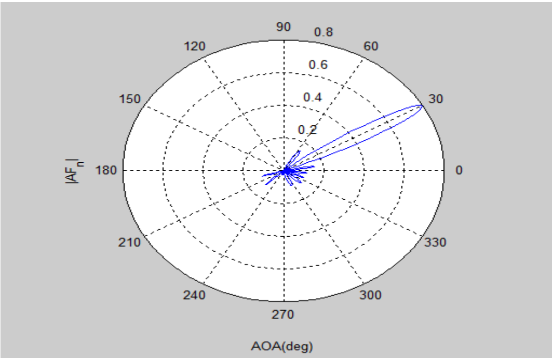

Figure 1.a: Polar array factor plot when desired user is 30° and the interferer is -50°

The desired user is pointing at 30° on the polar array factor plot.

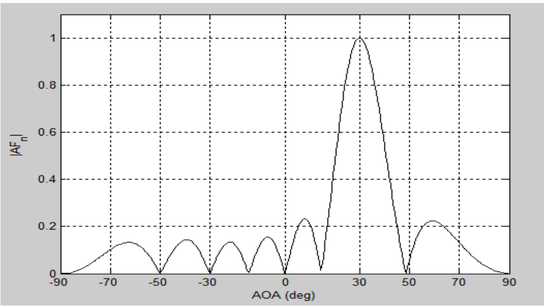

Figure 1.b: Array factor plot when desired user is 30° and the interferer is -50°

The array factor for 8 elements antenna arrays is computed and Figure 1.b shows the normalized array factor plots and how the LMS algorithm places deep nulls in the direction of interfering signals and maximum in the direction of the desired signal. The beam of the desired signal is pointing at the desired direction angle 30°on the array factor plot from figure (1.b).

The optimum complex weights in the case for which the algorithm converges is as follows.

for the N = 8 ULA are:

w1 = 1

w2 = 0.039372+0.96988j

w3 = -1.0193+0.035316j

w4 = -0.0013838-0.98314j

w5 = 0.98221-0.050948j

w6 = 0.018406+1.0196j

w7 = -0.97039+0.012486j

w8 = -0.052968-0.999j

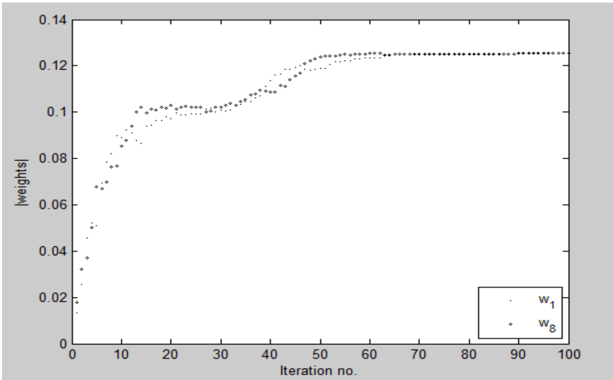

Figure 1.c: Magnitude of array weights when the desired user is 30° and the interferer is -50°

The magnitude of the array weights shows a convergence after 60 iterations.

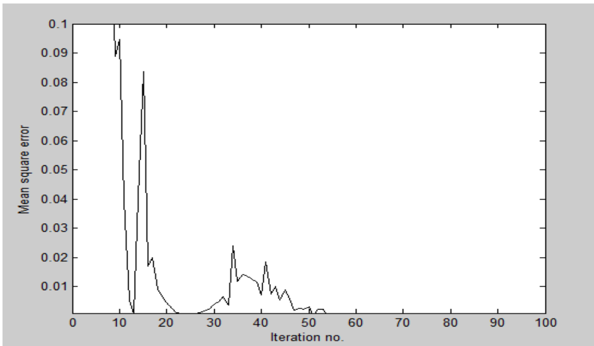

Figure 1.d: Mean square error when the desired user is 30° and the interferer is -50°

The LMS error plot in Figure (1.d) shows that the MSE decreases with iterations and converges after 60 iterations. In this case the LMS error is satisfactory nearly 0.001 at around 100 iterations.

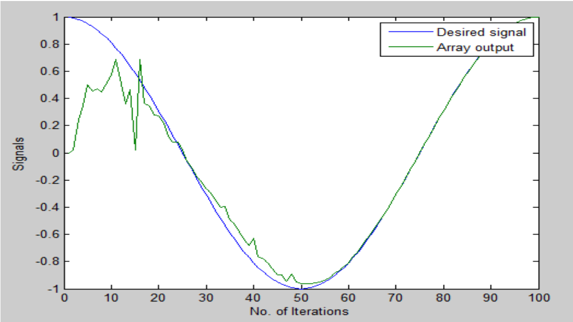

Figure 1.e: Acquisition and tracking of desired signal and actual array output

Figure (1.e) shows the graph of signal versus number of iterations. The array output was able to acquire and to track the desired signal after 60 iterations. If the characteristics of the signal rapidly changing, the LMS algorithm may not allow tracking of the desired signal in a satisfactory manner[11].

Special onecase: effect of number of elements on Array Factor and Mean Square Error

The effect of number of elements on the Array Factor and the MSE is studied considering the desired user arriving at angle 30° and the interferer at angle -50° for different number of elements such as N = 8 and 16 with the SNR 5dB. Figure (2.a) shows the array factor plot of LMS algorithm when number of antenna array element is 8 and 16 desired user is arriving at angle 30° and the interferer is at an angle of -50°.

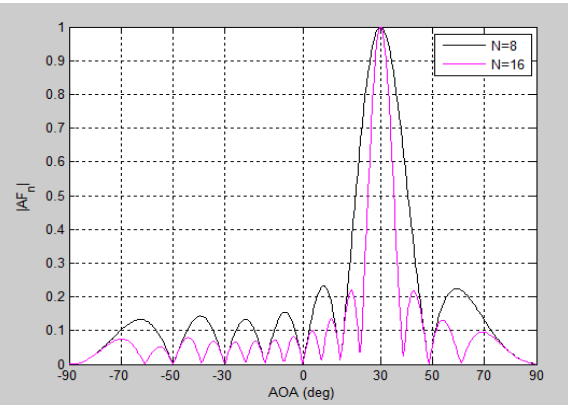

Figure 2.a: Effect of number of array element on Array Factor.

The simulation result shows that as the antenna array element goes on increasing from 8 (black), to 16 (red) the beam width of the desired angle becomes narrow. Also, there is a multiplication of the number of side lobes but the level of these side lobes is low as compared to those generated by small number of elements as shown on table 1 below.

Table 1: Values of desired angle of arrival for the array factor at different number of elements.

Number of elements (N)	N=8	N=16
DOA (desired)	30°	30°
Side Lobe Level	0.1322	0.07421

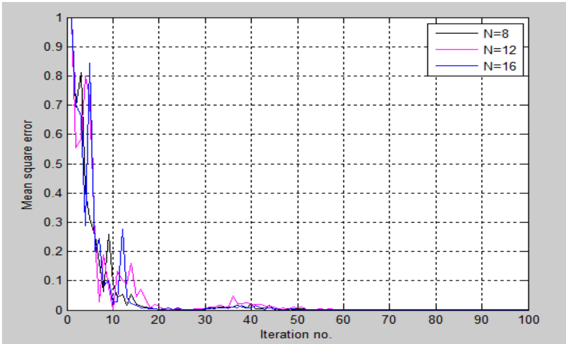

Figure 2.b: Effect of number of array element on Mean square error

The increasing number of antenna array element produce an increase in system noise, and overall MSE tends to be almost the same for the given values of antenna element as shown in figure 2.b.

Effect of interelement spacing on Array Factor and Mean Square Error

Figure (3.c) shows the effect of interelement spacing on the Array Factor and the Mean Square Error. The simulation is considered for the desired user arriving at angle 30° and the interferer at -50° for different spacing such as d = λ/4, λ/2, and λ. the Signal to Noise Ratio is 5dB.

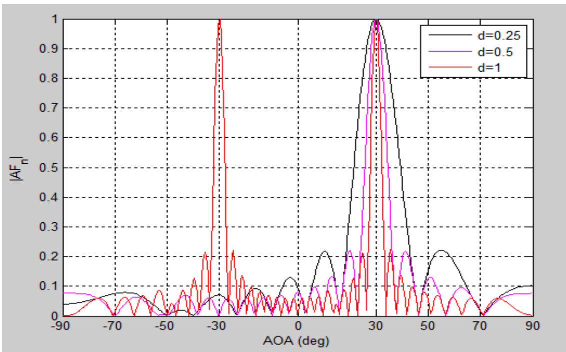

Figure 3.c: Effect of interelement spacing on Array Factor.

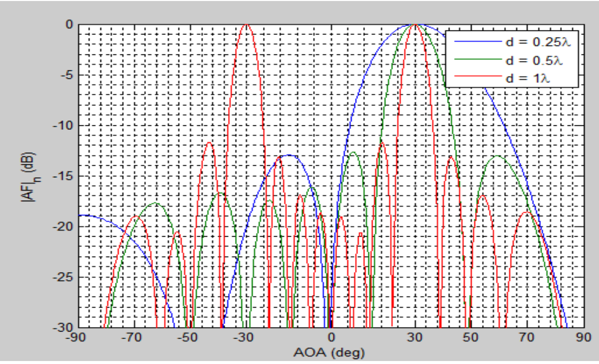

Figure 3.d: Effect of interelement spacing on Array Factor in dB

The simulation result shows that increasing the spacing between array element produces narrow beams, but this also increases the number of side lobes. For a spacing equal to the wavelength, granting lobe of the same side lobe level as the desired user angle is created as shown on figure(3.d). This new main-beam causes errors in the received signal due to interferences and wasted power. It is also observed that when distance between array element is half wave length or lesser than that, then granting lobes are avoided. Thus the optimum spacing distance between the array elements is half the wavelength. The results are presented in table 2 below.

Table 2: Values of desired angle and granting lobes at different elements spacing.

Element spacing (d*λ)	d=0.25	d =0.5	d =1
Desired DOA(degree)	30°	30°	30°
Granting Lobe (degree)	-	-	-30°
Side Lobe Level (dB)	0 dB	0 Db	0 dB

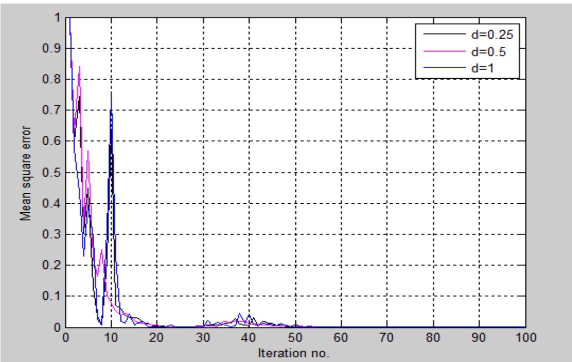

Figure 3.e: Effect of inter element separation on Mean square error.

From the graph of MSE it is observed that the error level increases by increasing the displacement between the antenna elements. specially for d=λ the error increases after 10 iterations which is not suitable for the performance of the algorithm, meanwhile for d=λ\2 the error decreases nearly to zero as shown in Figure (3.e) thus, spacing between array elements equal to half wavelength gives an optimum error in a particular iteration [15], this is well established in theory.

Effect of different SNR on the desired signal and the interference signal

The effect of different SNR for the desired signal and the interference signal on the performance of the LMS adaptive Beamforming algorithm are given in figures 4.a), 4.b), 4.c), 4.d), 4.e), and 4.f). These figures are plotted using the same array parameters such as spacing between arrays (0.5λ), and number of array (N=8).

The desired signal arriving at angle of 30°, and the interference signal at angle -50°.

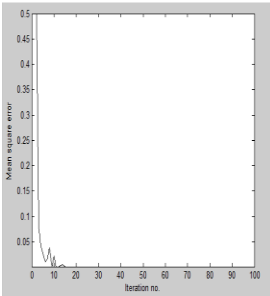

Figure 4.b): Iteration number versus Mean Square Error when desired signal is at 0dB and interferer signal is at 0dB

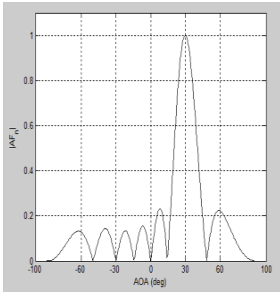

Figure 4.a): Array Factor versus Direction of Arrival when desired signal is at 0dB and interferer signal is at 0dB

When desired signal is at 0dB, and interference is also at 0dB, the reception of the desired angle is not affected. The antenna is able to place the nulls at the undesired direction. The minimum error decreases faster to reach a value of 0.002086 and the mean square error converges after 12 iterations.

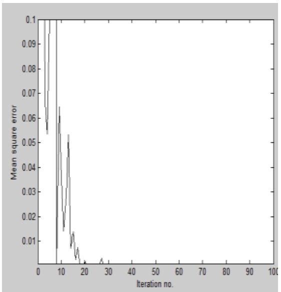

Figure 4.d): Iteration number versus Mean Square Error when desired signal is at 0dB and interferer signal is at 5dB

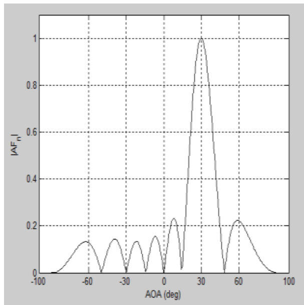

Figure 4.c): Array Factor versus Angle of Arrival when desired signal is at 0dB and interferer signal is at 5dB

When the desired signal is at 0dB and interference is at 5dB, the antenna is still able to place the null at the undesired direction and to direct the desired beam at the desired angle of arrival. Thus, the reception of the signal is not affected. The minimum error has increased of 39.50 %, and the mean square error converges after 28 iterations. Thus, the performance convergence of the LMS algorithm has been affected as compared to the case of figure (4.b) when the signal to noise ratio of both desired signal and undesired signal were at 0dB. This is due to the increase of the interference signal over the desired signal.

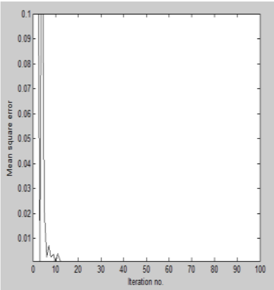

Figure 4.f): Iteration number versus Mean Square Error when desired signal is at 5dB and interferer signal is at 0dB

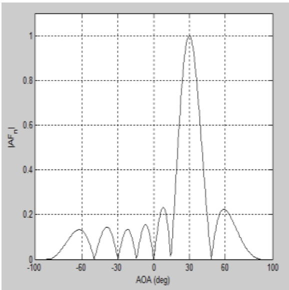

Figure 4.e): Array Factor versus Angle of Arrival when desired signal is at 5dB and interferer signal is at 0dB.

When the desired signal (S_d) is at 5dB and interference signal (S_i)is reduced to 0dB, the reception of the desired user is not affected by the interference. When the signal to noise ratio of the desired signal is increased to 5dB, the minimum error tends to zero and the mean square error converges after 11 iterations. Table 3 below presents the results of this experiments:

Table 3: Values of desired and interferer Direction of Arrival at different SNRs.

SNR S_d (dB)	SNR S_i (dB)	DOA (θ_d°)	DOA (θ_i°)	Mean Square Error	Iteration number
0 dB	0 dB	30°	-50°	0.002086	12
0 dB	5 dB	30°	-50°	0.002915	28
5 dB	0 dB	30°	-50°	0	11

For all the three cases, the antenna was able to place the nulls in the direction of interference and the reception of the signal was not so much affected. The major noticed was when the interference was increased to 5 dB and the desired user reduced to 0dB where the MSE was not satisfactory as the minimum error was high (figure4.d) with a slow convergence of the algorithm (after 28 iterations), as compared to the case when the desired signal was increased to 5 dB SNR and the interference decreased to 0db, the MSE was satisfactory as the error in the signal was almost zero (figure 4.f), and the convergence was rapid (after 11 iterations) as shown on table 3. Thus the LMS algorithm has better performance in terms of convergence rate of the MSE and in term of minimum error. This is due to the fact that the increase of the interference signal results in the increase of the error which has affected the desired user signal and so the performance convergence of the LMS algorithm.

Effect of initial weight vector on the convergence of the LMS algorithm

The influence of the initial vector weights on the convergence rate was also investigated. The performance is evaluated considering the effects on the convergence of the MSE and the convergence of the magnitude of the weights for the conditions when the initial weights are all zeros, or all ones, or random.

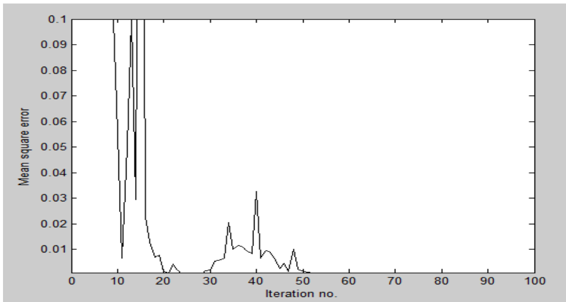

Figure 5 a): Iteration number versus Mean Square Error when initial weights are zeros

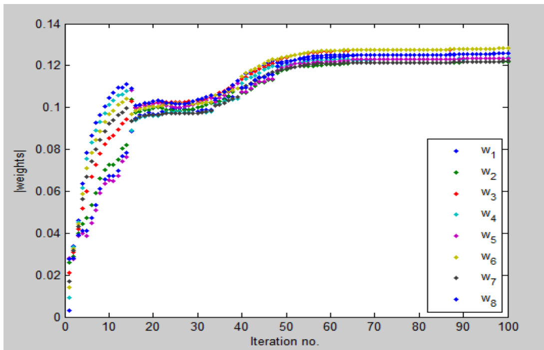

Figure 5 b): iteration number versus weights when initial weights are all zeros

The weights W_A for the N = 8 ULA are:

w1 = 1 |w1| = 1

w2 = 0.03899+0.96916j |w2| = 0.96994

w3 = -1.0188+0.035723j |w3| = 1.019426

w4 = -0.0011146-0.98308j |w4| = 0.98308

w5 = 0.98133-0.050797j |w5| = 0.98264

w6 = 0.018773+1.0194j |w6| = 1.01957

w7 = -0.9701+0.01204j |w7| = 0.97017

w8 = -0.053094-0.99814j |w8| = 0.99955

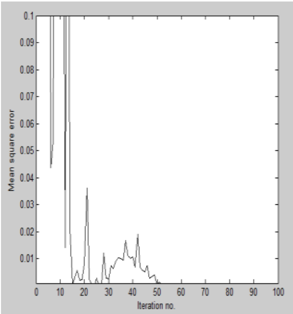

Figure 5 c): Iteration number versus Mean Square Error when initial weights are ones

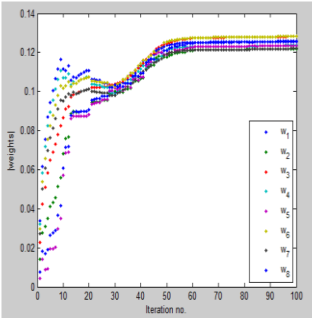

Figure 5 d): iteration number versus weights when initial weights are all ones

The weights W_B for the N = 8 ULA are:

w1 = 1 |w1| = 1

w2 = 0.03857+0.96814j |w2| = 0.96890

w3 = -1.0181+0.036341j |w3| = 1.01874

w4 = -0.00076057-0.98295j |w4| = 0.98295

w5 = 0.98014-0.050709j |w5| = 0.98145

w6 = 0.0193+1.0191j |w6| = 1.01928

w7 = -0.96966+0.011476j |w7| = 0.96972

w8 = -0.053378-0.997j |w8| = 0.99842

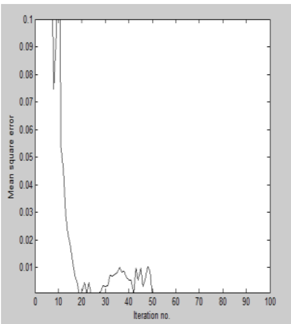

Figure 5 e): Iteration number versus Mean Square Error when initial weights are random

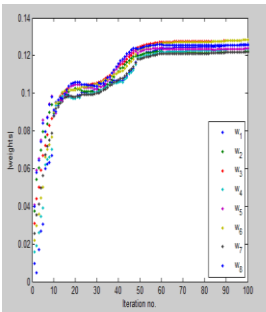

Figure 5 f): iteration number versus weights when initial weights are all random

The weights W_C for the N = 8 ULA are:

w1 = 1 |w1| = 1

w2 = 0.039654+0.97034j |w2| = 0.97114

w3 = -1.0196+0.035065j |w3| = 1.02020

w4 = -0.0015655-0.98318j |w4| = 0.98318

w5 = 0.98278-0.051083j |w5| = 0.98410

w6 = 0.018172+1.0198j |w6| = 1.01996

w7 = -0.97056+0.012794j |w7| = 0.97064

w8 = -0.052917-0.99958j |w8| = 1.00097

Reference to figures after graphs have been given to verify when is the convergence of the LMS algorithm faster for the case when initial weights are all zeros, ones or random. The actual value of each of the weights W_A, W_B, and W_C at iterations 100 have been computed. The results are presented in table (4.1) below:

Table 4.1: Values of the convergence rate when initial weights are all Zeros, Ones or Random

Initial weight w_i	w_i = Zeros	w_i = Ones	w_i = Random
Number of iteration (Convergence MSE)	60	60	60
Number of iteration |weights| (Convergence)	60	60	60

As shown in table (4.1), figure (5 a) and (5 b) are plotted for all initial weight at zeros (i.e. w_i = zeros). The MSE and the magnitude of the weight graph converges after 60 iterations. Figure (5 c) and figure (5 d) are plotted with initial weight are all ones (i.e. w_i=ones). The MSE and the weight graph converges after 60 iterations. Figure (5 e) and figure (5 f) are plotted with initial weight are all random (i.e. w_i=random). The Mean Square Error and the weight graph converges after 60 iterations.

Table 4.2: actual value of the weights W_A, W_B, and W_C

W_i	$\|w_1\|$	$\|w_2\|$	$\|w_3\|$	$\|w_4\|$	$\|w_5\|$	$\|w_6\|$	$\|w_7\|$	$\|w_8\|$
W_A	1	0.96994	1.019426	0.98308	0.98264	1.01957	0.97017	0.99955
W_B	1	0.96890	1.01874	0.98295	0.98145	1.01928	0.96972	0.99842
W_C	1	0.97114	1.02020	0.98318	0.98410	1.01996	0.97064	1.00097

In table (4.2), the actual value of all the weights of the three different cases are presented. It is observed that all the weights are almost the same, there is no different whether the initial weight vector at zeros, ones or random the convergence performance of the LMS algorithm is not affected.

Especific case two: Effect of closed angle of interference on the desired signal

The following experiment is carried out to show the effect of an angle of interference too closed to the desired angle on the reception of the desired signal. The desired angle is maintained at 30°, but the angle of interference is varied from -40° to 50° in order to determine at what angle is the desired signal affected. The experiment is carried out for different angles of interference such as -40°, -30°, -20°, -10°, 0°, 10° 20°, 40°, and 50°. The number of element is equal to 8, and the spacing between elements is half wavelength.

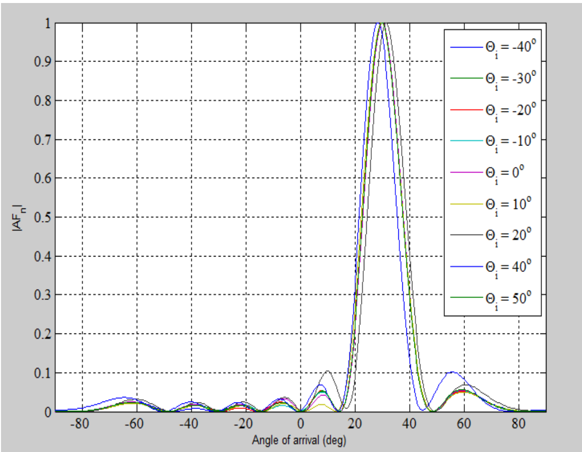

Figure 6 a): Array Factor versus Direction of Arrival when desired angle is at 30° for different interferences: -40°, -30°, -20°, -10°, 0°, 10° 20°, 40°, and 50°

Figure (6.a) shows the behaviour of the array factor when the desired angle is 30° and for different interferences closer to the desired angle. The interference varies from -50° to get closer to the desired angle at -40°, -30°, -20°, -10°, 0°, 10°, 20°, 40° up to 50°. The interference at -50° has been displayed from the graph, its plot result is shown on figure (1b).

It is observed from figure (6.a) that, for the angle of interference such as -50°, -40°, -30°, -20°, -10°, 0°, 10° and 50°, the reception of the signal at the desired direction is not affected. The antenna is able to place the nulls at the undesired direction. But for the angle of interference at 20°and 40°, the reception of the desired beam at the expected direction is affected as it is shown on figure (6.b) below:

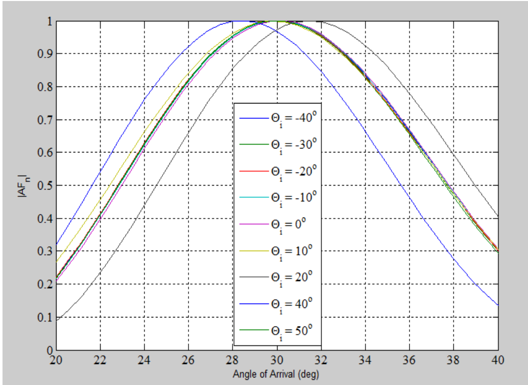

Figure 6 b): Array Factor versus Direction of Arrival when desired angle is at 30° for different interferences: -40°, -30°, -20°, -10°, 0°, 10° 20°, 40°, and 50° (with x axis zoomed)

Figure (6.b) shows the array factor plot versus Direction of Arrival with the X axis zoomed to focus on the achieved direction of the beam. The experimental result from the graph shows that at $\theta_i = 20°$, the desired beam is shifted from 30° to 31° on the right, also at $\theta_i = 40°$ the desired beam is shifted from 30 to 28° on the left. From -50° the reception of the desired signal is not affected up to 10°, but from 20° to 40° it is affected, and as from 50° the reception is still good at new. Thus, for the angle of interference less or equal to 10°, and for the angle of interference higher or equal to 50° the reception of the desired signal is not affected. The results are presented on table (5.1) below:

Table 5.1: values of the angle of arrivals of the signals for different desired and interferer angles

Desired AOA θ_d	Interference AOA θ_i	Experimental results desired AOA θ_e
30°	-50°	30°
30°	-40°	30°
30°	-30°	30°
30°	-20°	30°
30°	-10°	30°
30°	0°	30°
30°	10°	30°
30°	20°	31°
30°	40°	28°
30°	50°	30°

Effect of larger array size on the array factor when angle of interference is 20° and 40°

Figure (6.c) and figure (6.d) show the array factor plot for desired angle at 30° and interferer is at respectively 20° and 40°, when array size varies from N=8, 12 and 16 elements.

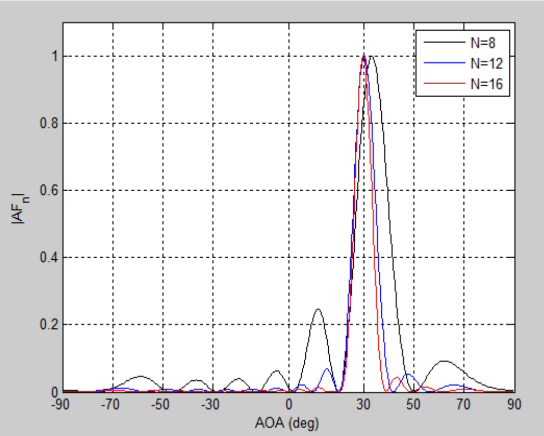

Figure 6.c): array factor plot when desired user is 30°, and interference is 20°, for N=8, 12 and 16

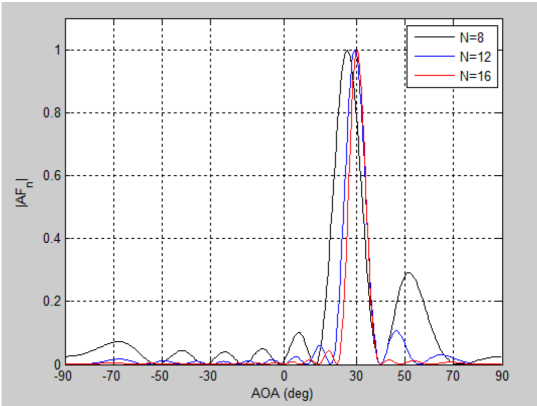

Figure 6.d): array factor plot when desired user is 30°, and interference is 40°, for N=8, 12 and 16

The results are shown in table 5.2 below.

Table 5.2: Values of desired Direction of Arrival for different array size

Desired angle θ_d	Interference angle θ_i	Number of element N	Experimental desired angle θ_e
$\boldsymbol{\theta_d = 30°}$	$\boldsymbol{\theta_i = 20°}$	**N=8**	$\boldsymbol{\theta_e = 31°}$
$\theta_d = 30°$	$\theta_i = 20°$	N=12	$\theta_e = 30°$
$\theta_d = 30°$	$\theta_i = 20°$	N=16	$\theta_e = 30°$
$\boldsymbol{\theta_d = 30°}$	$\boldsymbol{\theta_i = 40°}$	**N=8**	$\boldsymbol{\theta_e = 28°}$
$\theta_d = 30°$	$\theta_i = 40°$	N=12	$\theta_e = 30°$
$\theta_d = 30°$	$\theta_i = 30°$	N=16	$\theta_e = 30°$

Figure (6.c) and (6.d) show the directivity of 3 arrays with 8 (black), 12 (blue) and 16 (red) elements. The element spacing is half wavelength for all the arrays. Note the presence of side lobes next to the main lobes. The side lobes level decreases with the number of elements. The beam width becomes narrow as the number of elements increases. From table (5.2) it is observed that for a number of elements equal to 8 (N=8) when the interference is at 20 degrees the desired signal arrives at an angle of 31 degrees and as the number of elements increases, the desired angle of arrival is adjusted to the desired direction 30 degrees. The same as when the interference is at 40 degrees given a small size of the array (say N=8), the desired signal is deviated at 28 degrees, but as the number of elements increases from 8 to 12 and 16, the desired user beam is adjusted to the desired direction of 30 degrees. Thus, the array directivity increases with the number of elements.

CONCLUSION

The performance of the LMS algorithm is evaluated here for three different cases such as general case and two specific cases. In the general case, the LMS error is satisfactory and the MSE converges. In the first specific case, the increase in the number of elements has resulted in the production of narrower beams, and the MSE gives an optimum error in a particular direction for element spacing equal to half wave lenght. The LMS algorithm performs in a satisfactory way when the desired signal is less affected by the interference in terms of SNR level. In The second specific case, for a better performance of the LMS algorithm for adaptive beamforming, the interference angles should be less or equal to one third of the desired angle (in this case desired at 30° and all the interferences ≤10°) or higher or equal to five third of the desired angle (in this case desired at 30°, and all the interferences ≥50°). The main feature of the LMS algorithm is its low computational complexity. The LMS algorithm can then further be easily implemented in real time.

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