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Perturbation Analysis of 2-Dimensional Boundary Layer Flow of an Inelastic Fluid Using Williamson Model

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Abstract

In this paper, we have discussed steady boundary layer flow of an inelastic fluid encountered in a number of engineering and biomedical applications, using the constitutive equation of the Williamson fluid. The flow is assumed to take place near a stagnation point on an infinite rigid flat surface. Using similarity transformations, the governing partial differential equations have been reduced to a nonlinear boundary value problem. The emphasis of our work in this study is to analyze the conventional perturbation solution vis-à-vis higher order effects. It is established that the higher order terms in the perturbation expansion, usually not considered in perturbation analyses, do influence the flow in the boundary layer region of the inelastic fluid. A quantity of engineering interest, namely, wall shear stress, has also been computed and analyzed.

Keywords: inelastic fluid, Williamson model, boundary layer flow, stagnation point, perturbation analysis, wall shear stress.

INTRODUCTION

It is well known from the vast literature in fluid dynamics that non-Newtonian fluids and their flow behavior have received major attention of a very large body of researchers during the last five to six decades. This includes both experimental and theoretical investigations covering a wide range of aspects. The principal reason for this interest stems from the fact that non-Newtonian flows are encountered in a large number of applications in polymer industry, mineral processing industry, chemical engineering, biomedical engineering – to name a few [1–3]. The study of these fluids does not follow the classical Newtonian fluid hypothesis in that the former class of fluids are distinguished by specific nonlinear relationships between the stress tensor and the rate of deformation tensor. The non-Newtonian fluids are further broadly categorized as inelastic and viscoelastic fluids.

Our interest in this work is with regard to the analysis of a special class of inelastic fluids which finds applications in industry as well as bio-rheology. It is worth mentioning here that in the study of non-Newtonian inelastic fluids, researchers have investigated both dilatant and pseudoplastic behavior of the fluids although the literature is replete with the use of a variety of pseudoplastic fluid models. However, the present authors have carried a number of studies on the boundary layer flows of a special class of dilatant fluids and have brought out the effects of shear thickening phenomena [4–8]. Based on the type of the constitutive equations of these inelastic fluids, the dilatant and pseudoplastic fluids have also been referred to in the literature as shear thickening and shear thinning fluids, respectively. Some of the most commonly encountered inelastic fluids in industries such as food processing, cosmetics, pharmaceuticals, cement, etc., are: highly concentrated suspensions (e.g., cement paste), coal-oil mixture, slurries, coarse coal, red mud suspensions, blood, honey, butter, polyelectrolyte solution, paint, drilling muds, jams, etc.

The study of the boundary layer flows of non-Newtonian fluids, in general, and pseudoplastic fluids, in particular, is of prime interest in industrial applications such as extrusion of polymer sheets, emulsion coated sheets like photographic films, solutions and melts of high molecular weight polymers, etc. In order to account for the anomalous flow features of pseudo-plastic fluids, a number of constitutive equations have been proposed in the literature. The most common among these are power law models, Carreau's model, Cross model, Ellis model, Reiner-Philippoff model, Williamson model, among others. Our interest in this paper is to highlight some peculiar features of Williamson model. In addition to other shear thinning industrial applications, Williamson model has been reported in the literature to represent blood flow features more accurately than the other commonly used models – Casson model, Bingham-plastic model and power-law model. It is also worth remarking here that Williamson had discussed and verified the applicability of pseudoplastic model both analytically and experimentally [9]. It is worth observing that researchers engaged in rheological flows have largely focused more on a variety of other models exhibiting pseudoplastic behavior, for example, power law and generalized power law models. Relatively much less attention has been devoted to the inelastic model proposed by Williamson. However, in recent years, a limited number of theoretical studies have been reported in the literature. In these investigations, researchers have used the basic Williamson model (limiting up to the first order approximation in the constitutive equation) while analyzing a variety of flows including boundary layer flows [10–14].

In continuation of the above works, in the present paper, we have investigated the steady two-dimensional boundary layer flow of the basic Williamson fluid model near a stagnation point over a fixed, impermeable infinite flat surface. As a first step, we have converted our basic fluid dynamical PDEs that govern the boundary layer flow to a nonlinear third order ODE using a set of similarity transformations [14]. In order to solve the resulting boundary value problem, we have first subjected it to a perturbation series solution in terms of a rheological parameter, retaining the terms up to and including the third order. This has resulted in sets of four perturbed third order ODEs, each ODE being subject to a set of three boundary conditions. There arise two-point boundary value problems which have been solved numerically. As stated earlier, the main focus of this study has been to analyze the effects of retaining various orders in the perturbation expansion vis-à-vis the Newtonian counterpart. In other words, we are interested in analyzing the increasing order of non-Newtonian effects in comparison to the Newtonian boundary layer flow.

THE CONSTITUTIVE EQUATION

The constitutive equation of the Williamson model is well-known in literature; and has been employed by several researchers (see, e.g., [14], [16], [17]). Following these works, we give below the constitutive equations of the pseudoplastic flow governing the Williamson model.

The expressions for the Cauchy stress tensor for the Williamson fluid is given by

$$\boldsymbol{S} = -p\,\boldsymbol{I} + \boldsymbol{T} \tag{1}$$

where p, $\boldsymbol{I}$, $\boldsymbol{T}$ are the usual quantities representing the pressure, unit tensor and the deviatoric stress tensor, respectively. For the two-dimensional flow considered in this study, the Williamson fluid is a three-parameter model in which $\boldsymbol{T}$ **is** given by

$$\mathbf{T} = \left[\mu_\infty + \frac{\sqrt{2}\,(\mu_0 - \mu_\infty)}{\sqrt{2} - \Gamma\sqrt{I_2}}\right]\boldsymbol{A}_1 \tag{2}$$

In the above, μ_0 and μ_∞ are the zero shear viscosity and infinite shear viscosity, respectively; Γ is a time constant representing non-Newtonian effects (assumed small), $\boldsymbol{I}_2$ is the well-known second invariant of the rate of deformation tensor, and $\boldsymbol{A}_1$ is the first Rivlin-Erickson tensor [18].

In the present study, we consider a special pseudoplastic fluid for which the infinite shear viscosity μ_∞ is negligible, and $\Gamma\sqrt{I_2} < 1$. Thus, we can expand the right side of Eq (2) in a binomial series to get

$$\boldsymbol{T} = \mu_0\left[1 + \Gamma(0.5\boldsymbol{I_2})^{\frac{1}{2}} + \Gamma^2(0.5\boldsymbol{I_2}) + \Gamma^3(0.5\boldsymbol{I_2})^{\frac{3}{2}} + \cdots\right]\boldsymbol{A_1} \tag{3}$$

Assuming $\Gamma(0.5\boldsymbol{I_2})^{1/2} \ll 1$, we can finally approximate $\boldsymbol{T}$ by

$$\boldsymbol{T} = \mu_0\left[1 + \Gamma(0.5\boldsymbol{I_2})^{\frac{1}{2}}\right]\boldsymbol{A_1} \tag{4}$$

Equation (4) is the first order approximation for $\boldsymbol{T}$ that will be used in our analysis.

BOUNDARY LAYER EQUATIONS

In order to obtain the boundary layer equations for our two-dimensional steady flow, we first consider the equation of continuity and the momentum equations, given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$\rho\left(u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \boldsymbol{T}_{xx}}{\partial x} + \frac{\partial \boldsymbol{T}_{xy}}{\partial y} \tag{6}$$

$$\rho\left(u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial \boldsymbol{T}_{yx}}{\partial x} + \frac{\partial \boldsymbol{T}_{yy}}{\partial y} \tag{7}$$

where ρ is the fluid density, and u, v are the velocity components in the x and y directions, respectively. The stress components $\boldsymbol{T}_{xx}$, $\boldsymbol{T}_{xy}$, $\boldsymbol{T}_{yx}$ and $\boldsymbol{T}_{yy}$ are given by

$$\boldsymbol{T}_{xx} = 2\mu_0(1+s)u_x \tag{8}$$

$$\boldsymbol{T}_{xy} = \mu_0(1+s)(u_y + v_x) \tag{9}$$

$$\boldsymbol{T}_{yx} = \mu_0(1+s)(u_y + v_x) \tag{10}$$

$$\boldsymbol{T}_{yy} = 2\mu_0(1+s)v_y \tag{11}$$

where the subscripts of u and v denote partial derivatives. Furthermore, in Eqs (8)–(10), $s = \Gamma\sqrt{0.5\boldsymbol{I}_2}$.

Employing the usual boundary layer approximations, the boundary layer equations for the two-dimensional steady flow are: the continuity equation (5), the equation $p_y = 0$, and the u-component of the momentum equation

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu_0\,\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\,\Gamma\nu_0\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \quad (12)$$

where ν_0 is the zero shear kinematic viscosity. The above equation will be used in the next section for the analysis of the two-dimensional stagnation point flow of the Williamson fluid.

FLOW NEAR A 2-D STAGNATION POINT

The stagnation point flow corresponds to the flow of a fluid near a stagnation region of a solid boundary – stationary or moving. They have been widely studied in literature due to their applications in engineering and industry. For the stagnation point flow of the Williamson fluid, the boundary conditions are

$$u = v = 0 \quad \text{at } y = 0, \text{ and } u \longrightarrow U(x) \text{ as } y \longrightarrow \infty \quad (13)$$

where $U = U(x) = ax$ is the free stream velocity of the fluid, and a is a constant. Furthermore, the pressure gradient term in the boundary layer equation (12) can be replaced by $U\,(dU/dx)$. We thus re-write Eq (12) as

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = U\,\frac{dU}{dx} + \nu_0\,\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\,\Gamma\nu_0\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \quad (14)$$

In order to solve the above boundary layer equations subject to the conditions (13), we use the similarity transformations

$$\eta = \sqrt{\frac{a}{\nu_0}}\,y,\quad u = Uf'(\eta),\quad v = -\sqrt{a\nu_0}\,f(\eta) \quad (15)$$

The above transformations automatically satisfy the continuity equation. Furthermore, using Eq (15) in Eq (14), we obtain the ODE governing the similarity function f, in the form

$$f''' + ff'' - (f')^2 + 1 + hN\,f''f''' = 0 \quad (16)$$

together with the boundary conditions

$$f(0) = 0,\ f'(0) = 0,\quad f'(\infty) = 1 \quad (17)$$

In Eq (16), primes denote differentiation with respect to $\eta = \sqrt{\frac{a}{\nu_0}}\,y$, and the non-dimensional quantities h and N are defined by

$$h = \sqrt{\frac{a}{\nu_0}}\,x,\quad N = \sqrt{2}\,a\Gamma \quad (18)$$

In order to analyse the qualitative as well as different order effects of the pseudoplastic phenomenon on the stagnation point flow, we assume

$$f(\eta) = f_0(\eta) + Nf_1(\eta) + N^2 f_2(\eta) + N^3 f_3(\eta) + \cdots \quad (19)$$

Using Eq (19) in Eqs (16) and (17), and equating coefficients of different powers of N, we obtain sets of boundary value problems corresponding to the various order terms. Restricting ourselves up to and including terms of order 3, there result four boundary value problems, which are given below:

$$f_0''' + f_0 f_0'' - (f_0')^2 + 1 = 0 \quad (20)$$

$$f_1''' + f_0 f_1'' - 2f_0' f_1' + f_0'' f_1 = -hf_0'' f_0''' \quad (21)$$

$$f_2''' + f_0 f_2'' - 2f_0' f_2' + f_0'' f_2 = (f_1')^2 - f_1 f_1'' - h[f_0''' f_1'' + f_0'' f_1'''] \quad (22)$$

$$f_3''' + f_0 f_3'' - 2f_0' f_3' + f_0'' f_3 = 2f_1' f_2' - f_1 f_2'' - f_1'' f_2 - h[f_0''' f_2'' + f_1'' f_1''' + f_0'' f_2'''] \quad (23)$$

$$f_0(0) = 0,\ f_0'(0) = 0,\quad f_0'(\infty) = 1 \quad (24)$$

$$f_n(0) = 0,\ f_n'(0) = 0,\ f_n'(\infty) = 0,\ (n = 1, 2, 3) \quad (25)$$

Equations (20) – (23) subject to the boundary conditions (24)–(25) have been solved numerically by a suitable shooting method. The results are discussed in the next section.

DISCUSSION

In our work, we have solved the boundary value problem governing the laminar boundary layer flow of a special class of Williamson model fluid near a two–dimensional stagnation point on a flat infinite rigid surface. We have shown that the flow is basically governed by a nonlinear third order ordinary differential equation for the similarity function f, subject to a set of well-posed boundary conditions. The presence of a small-valued non-Newtonian parameter N (< 1) in this equation is a subject of great interest in view of likely engineering applications related to pseudoplastic phenomenon, particularly in chemical engineering and processing industries, where the effect as well as the extent of this pseudoplasticity, however small, plays a vital role in real life applications. It is to be borne in mind that there exists a large number of non-Newtonian inelastic fluids, both pseudoplastic as well as dilatant, which show only small to moderate deviations from the Newtonian fluid counterparts. In view of this, we have used in our investigation a perturbation expansion in terms of this parameter N by retaining one, two or three terms over and above the zeroth order Newtonian term in order to compute various order effects for pseudoplasticity. In other words, our main focus has been to assess whether the higher order terms in the perturbation expansion (e.g., second order and third order) plays significant role in the transport processes within the boundary layer. To this end, our numerical computations have thus been largely confined to analyzing *relative effects* of various order terms assuming N to be a small-valued quantity.

In what follows, we shall now analyze a set of carefully chosen 10 figures which are related to the variation of longitudinal and transverse velocities, represented by the function f and its first derivative, respectively. Although we have two non-dimensional parameters N and h, the latter parameter has been kept constant in all but two graphs (Figs 1–8); its effect, in fact, has only been considered in the Figs 9 and 10 where we have let N to be fixed while varying h.

In the Figs 1 and 2, we have shown how velocity components u and v, represented by f' and f, respectively, behave in the boundary layer as the magnitude of pseudoplasticity (N) changes. In these figures, the curve 1 corresponds to a smaller value of N (= 0.3) while the curve 2 corresponds to relatively much higher value of N (= 0.9). It is easily observed that the effect of the non-Newtonian parameter N is to decrease the velocity in the boundary layer.

In the next set of Figs 3–5, we have illustrated (a) relative effects of retaining higher order terms (up to and including third order) and (b) non-Newtonian effects on the transverse velocity component. It may be noted that the curves labelled 1, 2 and 3 in each figure correspond to retaining one, two or three terms, respectively, in the perturbation expansion. Thus, these curves represent the following percentage magnitudes as compared to the Newtonian flow:

Curve 1: $100\, N f_1 / f_0$ (26)

Curve 2: $100\, (N f_1 + N^2 f_2) / f_0$ (27)

Curve 3: $100\, (N f_1 + N^2 f_2 + N^3 f_3) / f_0$ (28)

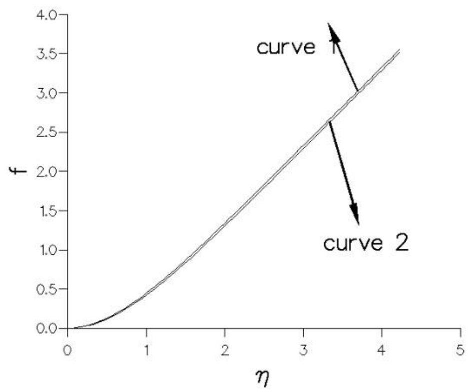

Figure 1: Variation of $f(\eta)$.

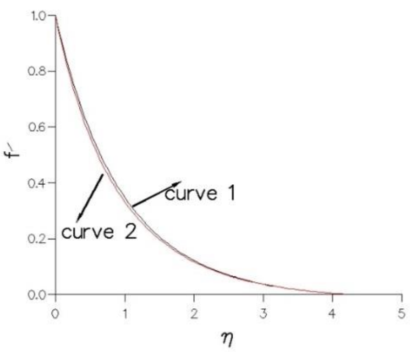

Figure 2: Variation of $f'(\eta)$.

The effect of increasing the value of the rheological parameter N is shown in Fig 3 (N = 0.3), Fig 4 (N = 0.6) and Fig 5 (N = 0.9). A closer examination of these curves leads to a very important conclusion: the inclusion of the higher order term beyond the second order (frequently used in the rheological flows literature) is necessary to assess the correct qualitative effects of the pseudo-plastic behavior vis-à-vis Newtonian counterpart. Incidentally, this noteworthy observation is quite similar to the one noted by the present authors in a similar study undertaken for an inelastic dilatant fluid [8]. Furthermore, this typical behavior is reflected apparently in the Figs. 4 and 5 too, with accompanying increase in the value of N. However, it is equally interesting to note that the magnitude of the rheological impact is no less significant as clearly borne out by various curves in the Figs 4 and 5, particularly near the boundary surface.

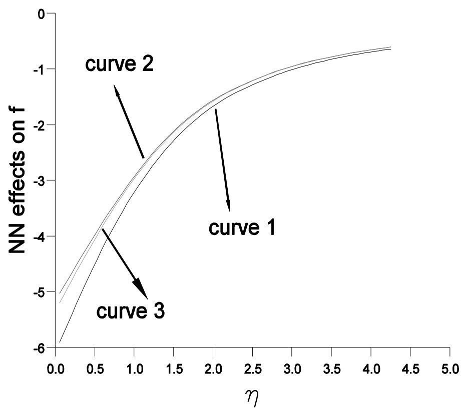

Figure 3: Higher order NN effects on relative percentage changes in f. $N = 0.3$

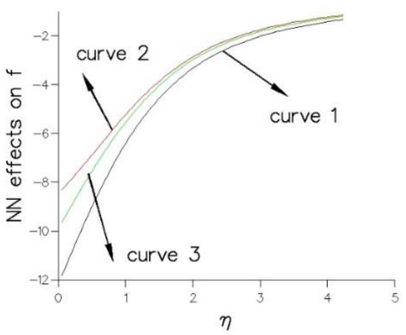

Figure 4: Higher order NN effects on relative percentage changes in f. $N = 0.6$

The next set of Figs 6 to 8, for longitudinal velocity, are the counterparts of the Figs 3–5 in terms of both higher order effects in the perturbation series as well as increasing non-Newtonian effects. The curves in these figures thus are the counterparts of quantities defined in Eqs (26)–(28) in which f is replaced by f'. The important conclusions drawn earlier for transverse velocity behavior are vindicated in these set of graphs too, albeit the extent of the non-Newtonian effects are slightly less pronounced here, particularly away from the boundary.

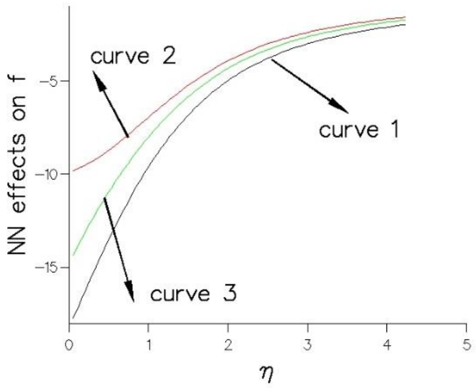

Figure 5: Higher order NN effects on relative percentage changes in f. $N = 0.9$

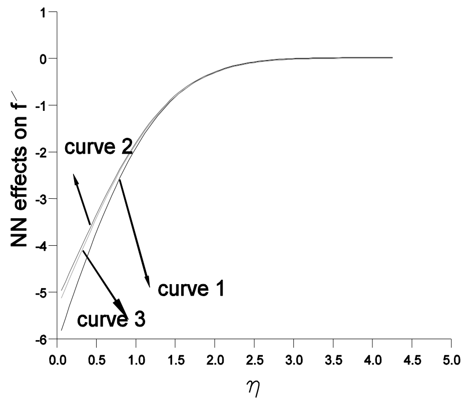

Figure 6: Higher order NN effects on relative percentage changes in f'. $N = 0.3$

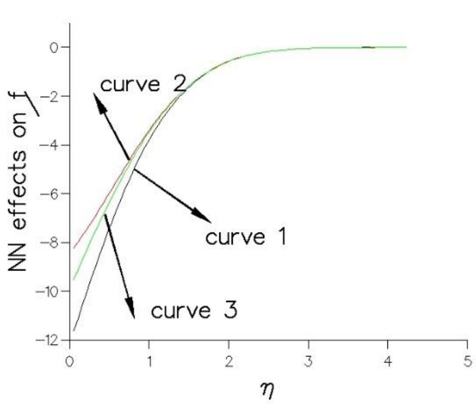

Figure 7: Higher order NN effects on relative percentage changes in f'. $N = 0.6$

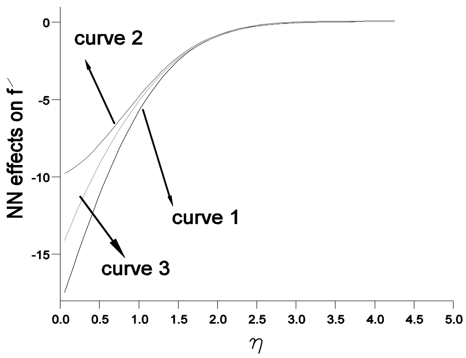

Figure 8: Higher order NN effects on relative percentage changes in f'. $N = 0.9$

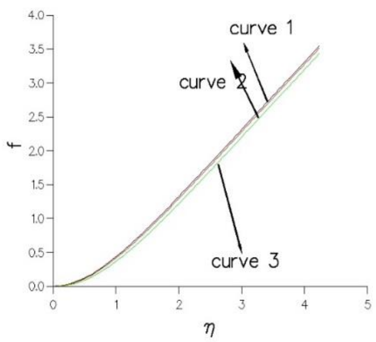

Figure 9: Variation of f. Effect of h.

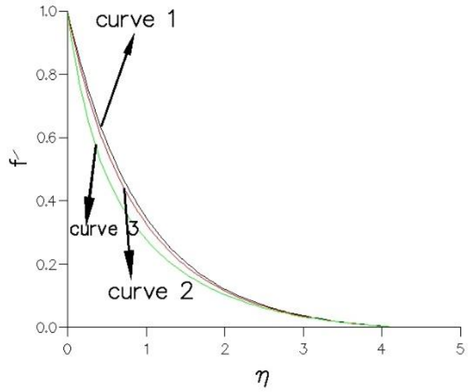

Figure 10: Variation of f'. Effect of h.

In order to assess the effect of the spatial parameter h on the boundary layer velocity profiles, keeping the parameter N fixed (= 0.5), we have included the corresponding similarity function profiles in Figs 9 and 10. The curves labelled 1, 2 and 3 in each figure here correspond to three values of h: 0.5, 1.0 and 2.0, respectively. It is obvious that the effect of increasing h, meaning as one gets farther from the stagnation point, is monotonic on both longitudinal and transverse velocity components, as also one would expect on the physical grounds. However, the extent of this effect is not high.

In order to implicitly evaluate the effect of the pseudoplastic parameter N on wall skin friction at $\eta = 0$, an important quantity of engineering interest, we have computed values of $f''(0)$ for N varying from 0 (Newtonian case) to 0.9. The subscripts 1, 2 and 3 in the table column headings refer to first second and third order effects, respectively. Thus

$$f''(0)_1 = f_0''(0) + N f_1''(0) \quad (29)$$

$$f''(0)_2 = f_0''(0) + N f_1''(0) + N^2 f_2''(0) \quad (30)$$

$$f''(0)_3 = f_0''(0) + N f_1''(0) + N^2 f_2''(0) + N^3 f_3''(0) \quad (31)$$

N	$f''(0)_1$	$f''(0)_2$	$f''(0)_3$
0	1.2326	1.2326	1.2326
0.3	1.6024	1.7133	1.7466
0.6	1.9721	2.4159	2.6821
0.9	2.3419	3.3403	4.2389

It is apparent that the skin friction at the bounding wall is quite sensitive to both N and the relative order employed in the perturbation expansion.

CONCLUSION

In this paper, we have analyzed the boundary layer flow of a class of inelastic fluids showing shear thinning effects. Such rheological behavior plays very important role in a host of engineering and industrial areas. Assuming the Williamson model constitutive equation for the pseudoplastic fluid considered in this study, we have analyzed the boundary layer flow near a stagnation point. The governing fluid dynamical partial differential equations have been transformed to a boundary value problem involving a third order nonlinear ordinary differential equation. There arises an important non-Newtonian parameter related to pseudoplastic behavior. A perturbation analysis followed by numerical integration has been carried out in terms of this rheological parameter. The focus of this study has been to bring out the relative order effects in the perturbation expansion. It is shown that as opposed to the second order effects in perturbation analysis, frequently employed in the literature, there is a need to include higher order effects to bring out more accurate qualitative and quantitative features of the flow.

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