

Effect of Diversity and Filtering on the Performance of Decorrelating Multiuser Detector Combining with Wavelet Packets Multicarrier CDMA system

Dr. Maryam M. Akho-Zahieh

*Associate Professor, Department of Electrical Engineering,
Applied Science Private University, Amman-11931, Jordan.*

Orcid Id: 0000-0003-0321-4648

Abstract

Wavelet Packets (WPs), multiuser detection, diversity and suppression filters are very effective in reducing the interferences in Coded Division multiple access communication (CDMA) systems. In this paper, we analyzed a WP Multi-Carrier (MC)-CDMA system that employs decorrelating multiuser detector. This type of detector, supports a higher number of users, suppresses the multiple access interference (MAI), thus relaxes the power control requirements. Since the sinusoidal sub-carriers are replaced by WPs sub-carriers, the proposed system will have a higher immunity against multicarriers and multipath interferences compared with the system which employs a sinusoidal subcarrier. The CDMA system can suppress a given amount of interference, but if the interference signal is powerful enough, the receiver is ineffective in mitigating this problem. In this paper, the receiver employs suppression filter (SF) to mitigate the effect of narrow-band jammer interference. The performance of the system is investigated in slow fading Nakagami channel by means of three types of diversity, which are Selection Diversity, Equal Gain Combining and Maximum Ratio Combining. For these combining techniques, Bit Error Rate (BER) and outage probability performances are investigated using several fading models and diversity orders. Also, we investigate how the performance is influenced by various parameters, such as the number of taps of the SF, the ratio of narrow-band interference bandwidth to the spread-spectrum bandwidth. Finally, the system performance is compared with WP-MC/SU-CDMA system and Sinusoidal (Sin)-MC/MU-CDMA system. Results reveal considerable performance improvement of our proposed system over the other two systems.

Keywords: multiuser, multicarrier, wavelet packets, bit error rate, outage probability, diversity techniques, suppression filter.

INTRODUCTION

A conventional Multi-Carrier Single User Coded Division Multiple Access (MC/SU-CDMA) system suffers from the problem of near-far effect. This is because; it considers each

user separately with the other users being as either Multiple Access Interference (MAI) or noise. Complex power control technique can be used to solve this problem. Another approach is the use of multiuser detection. A fundamental view of multiuser detection is that, by using joint detection all users are used for their mutual benefit instead of being interfering with each other. Optimum detectors can improve the system performance significantly because they are near-far resistant. But, their complexity increases exponentially with users' number. Alternatively, the complexity of suboptimum detectors increases linearly with users' number and can achieve near-optimum performance [1, 2]. The decorrelating multiuser detector is suboptimum and has attracted much attention among several multiuser detector because its attractive properties. For most among these properties are [1, 2, 3]:

- The demodulation of each user can be implemented completely independently
- Its performance is independent of the powers of the interfering users
- It does not require the knowledge of the users' power.

Wavelet packets (WP) have many attractive properties such as negligible sidelobe energy compared with sinusoid carriers; this property is effective in suppressing intercarrier interference (ICI) and MAI. Also, WPs are naturally orthogonal and well localized in both frequency and time domains. Because of that, there is no need for frequency/time guard between different user signals.

It is well known that the inherent processing gain of CDMA system, in many cases, provides the system with a sufficient degree of narrow-band interference (NBI) rejection capability. However, if the interference signal is powerful enough, the conventional receiver is ineffective in mitigating this problem. Interference suppression filter (SF) can be employed to reject the NBI. A wiener-type filter, described in [4], employs a tapped delay line structure to first predict and then subtract out the NBI.

The technique of diversity combining can provide an attractive means for improving system performance by

utilizing the signal components of different uncorrelated paths.

In this paper, a communication system employing multiple user detection, WP, MC and CDMA overlaid with a NBI is analyzed with and without the presence of SF. We refer to this system as Wavelet Packets based Multicarrier Multiuser (WP-MC/MU-CDMA) system. The Bit Error Rate (BER) and the outage probability (P_{out}) performances for the system in slow fading Nakagami channel are investigated using three types of diversities namely, Selection Diversity (SD), Equal Gain Combining (EGC) and Maximum Ratio Combining (MRC). The system performance is compared with the performances of wavelet packet based multicarrier single user CDMA systems denoted as WP-MC/SU-CDMA and Sinusoidal (Sin) based multicarrier multiuser CDMA system denoted as Sin-MC/MU-CDMA.

SYSTEM MODEL AND DESCRIPTION

The transceiver for WP-MC/MU-CDMA system is shown in Figure 1. At the transmitter, the BPSK modulated signal $d_k(t)$ for the k^{th} user is spread by PN signature sequence (processing gain) $c_k(t)$ corresponding to the k^{th} user. The spread signals is used to modulate H wavelet packets ,the h^{th} wavelet packet is given by $wp_h(t) = \sum_i w_h(t - iT_n)$, its support time and energy are T_n and 1, respectively [5]. The wavelet packets with different h indices represent different

subbands, because of that and due to flexibility of wavelet packets the bandwidth of each subband can be chosen arbitrarily. The whole signal spectrum is separated into H disjoint subbands, where the partition of subbands are determined by the channel characteristics and not limited by a minimum frequency distance. Note that:

- $d_k(t) = \sum_{i=-\infty}^{\infty} d_k^i \Pi_T(t - iT)$ has bit duration = T and $d_k^i \in \{-1,1\}$ is the binary data stream.
- $c_k(t) = \sum_{i=0}^{N_n-1} c_k^i \Pi_T(t - iT_n)$ has a length = N_n , $c_k^i \in \{-1,1\}$ is the i^{th} bit value with probabilities $P(1) = P(-1) = 0.5$ and $T_n = T/N_n$ is the chip duration. Therefore, the spread-spectrum system bandwidth $B_s = 2/T_n$. The energy of $c_k(t)$ is normalized to one, i.e., $\|c_k^i\|^2 = \int_0^T c_k^2(t) dt = 1$.

Let K denotes the number of active users and assuming identical power for all users, the transmitted signal of the k^{th} user is given by

$$s_k(t) = \left(\sqrt{2P} d_k(t) c_k(t) \sum_{h=1}^H wp_h(t) \right) \cos \omega_c t \quad (1)$$

where ω_c is the carrier frequency and P is the transmitted signal power.

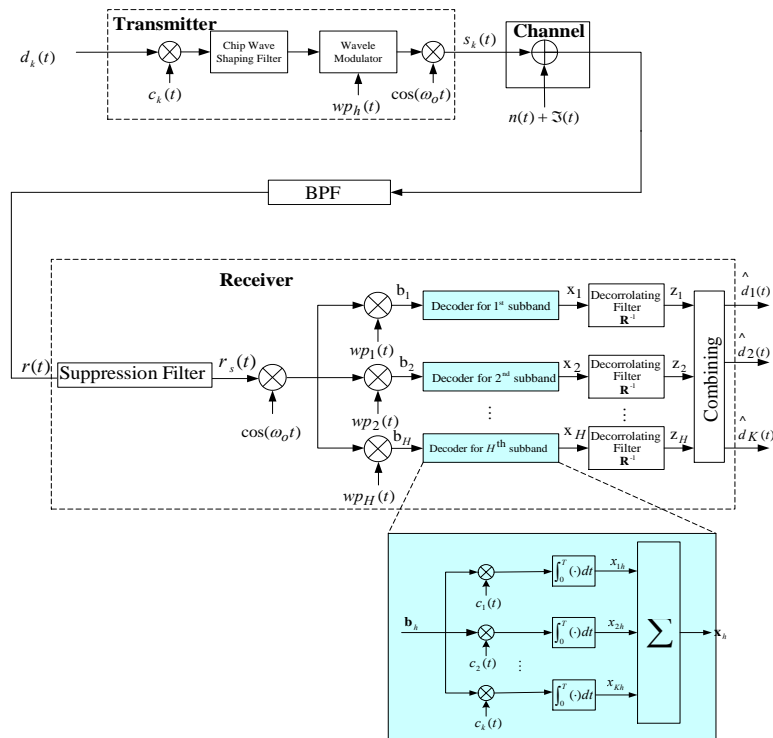


Figure 1: WP-MC/MU-CDMA transceiver system model

The channel used in this paper is assumed to be Nakagami fading channel with delay spread τ_h [6]. The impulse response for frequency selective fading channel of the conventional single-carrier decorrelating detector is given by [7, 8]

$$h_k(t) = \sum_{l=1}^L \zeta_{kl} e^{j\phi_{kl}} \delta(t - l\tilde{T}_n) \quad (2)$$

where $\tilde{T}_n = T_n/H$ is the chip duration for conventional single-carrier decorrelating detector, L is the number of resolvable paths = $\left\lfloor \frac{\tau_h}{\tilde{T}_n} + 1 \right\rfloor$ and $\lfloor x \rfloor$ is the integer part of x ,

ζ_{kl} is a zero mean complex valued stationary Gaussian Random Variable (RV) path gain, and ϕ_{kl} is the phase delay which is uniformly distributed over $[0, 2\pi]$. The channel model is characterized by the coherence bandwidth $\Delta f_c \approx 1/\tau_h$. For the decorrelating multiuser detector, each subband has no selectivity if the number of subband, H , satisfies the following condition

$$\frac{\tau_h}{T_n} = \frac{\tau_h}{HT_n} \leq 1 \quad (3)$$

Also, all subbands are subject to independent fading if bandwidth of each subband is greater than the coherence bandwidth [7], i.e.,

$$B_H = \frac{1}{T_n} (1 + \alpha) = \frac{1}{HT_n} (1 + \alpha) \geq \frac{1}{\tau_h} \quad (4)$$

where $\alpha (0 < \alpha \leq 1)$ is the roll-off factor of the chip waveform shaping filter. If we take $H = L = \left\lfloor \frac{\tau_h}{\tilde{T}_n} + 1 \right\rfloor$ and $\alpha \geq \frac{\tilde{T}_n}{\tau_h}$, we

can ensure that each subband of the system is subjected to independent fading and has no selectivity. Then, the impulse response of the h^{th} subband channel of the k^{th} user is given by [9, 10].

$$h_{kh}(t) = \beta_{kh} e^{j\theta_{kh}} \delta(t), \quad h=1, \dots, H \quad (5)$$

where β_{kh} is Independent Identically Distributed (*iid*) Nakagami distributed RV, and θ_{kh} is *iid* RV phase delay that is uniformly distributed over $[0, 2\pi]$. The output of the channel for the k^{th} user is given by

$$y_k(t) = s_k(t) * h(t) = \int_{-\infty}^{\infty} s(t - \tau) h(\tau) d\tau = \left(\sqrt{2P} d_k(t) c_k(t) \sum_{h=1}^H w p_h(t) \beta_{kh} e^{j\theta_{kh}} \right) \cos \omega_c t \quad (6)$$

The received signal can be written as

$$r(t) = \sum_{k=1}^K y_k(t) + n(t) + \mathfrak{I}(t) \quad (7)$$

where $n(t)$ is the Additive White Gaussian Noise (AWGN) and K is the number of active users. $\mathfrak{I}(t)$ is the narrow-band interference jammer which is given by

$$\mathfrak{I}(t) = \sqrt{2\mathfrak{I}} \xi(t) \cos[2\pi(f_o + \Delta)t + \psi] \quad (8)$$

where \mathfrak{I} and ψ are its power and phase, respectively. Δ is the offset of the interference carrier frequency with respect to signal carrier. The information sequence $\xi(t)$ has bit duration T_ξ and a bit rate $1/T_\xi$. The interference jammer has a bandwidth $B_\xi = 2/T_\xi$, we assume that $B_\xi < B_s$. The ratio of the jammer bandwidth to the system bandwidth is given by

$$p = \frac{B_\xi}{B_s} = \frac{T_n}{T_\xi}. \text{ The received signal is first passed through a}$$

Band-Pass Filter (BPF) having bandwidth B_s equal to the spread spectrum bandwidth; this filter removes the out-of band noise and let the desired signal and inferences pass without distortion. The filtered signal is then passed through a suppression filter (SF), its impulse response is given by

$$h_s(t) = \sum_{q=-Q_1}^{Q_2} \alpha_q \delta(t - qT_n) \quad (9)$$

where $\alpha_0 = 1$. The number of taps of the filter on the left and right of the center tap are represented by $Q_1 \geq 0$ and $Q_2 \geq 0$ [11]. The output of the filter for each tap is given by:

$$r_s(t) = r(t) * h_s(t) = \sum_{q=-Q_1}^{Q_2} \alpha_q \left\{ \sqrt{2P} \sum_{k=1}^K d_k(t - qT_n) c_k(t - qT_n) \times \sum_{h=1}^H w p_h(t - qT_n) \beta_{kh} e^{j\theta_{kh}} \cos[\omega_c(t - qT_n)] + \hat{n}(t - qT_n) + \mathfrak{I}(t - qT_n) \right\} \quad (10)$$

where $\hat{n}(t - qT_n)$ is the filtered AWGN and $\mathfrak{I}(t - qT_n) = \sqrt{2\mathfrak{I}} \xi(t - qT_n) \cos[2\pi(f_o + \Delta)(t - qT_n) + \psi]$.

The output signal of the filter, $r_s(t)$, is demodulated by the sinusoidal carrier, multiplied by the WPs, despread by the user specific code sequence, and correlated over a period T . Finally, the outputs of the correlators are decorrelated by means of decorrelating filters. The decorrelated signals for each user are then combined to recover the data signal. In the performance analysis, the perfect bit synchronization wavelet and code are assumed to be maintained. The decoder output

for the first data bit corresponding to the h^{th} diversity branch for the k^{th} user can be expressed as

$$\begin{aligned}
 x_{kh}(t) &= \int_0^T r_s(t) c_k(t) w_{p_h}(t) \cos(\omega_c t) dt \\
 &= \int_0^T \sum_{q=-Q_1}^{Q_2} \alpha_q \left\{ \sqrt{2P} \sum_{i=1}^K d_i^1 c_i(t - qT_n) \right. \\
 &\quad \times \sum_{u=1}^H w_{p_u}(t - qT_n) \beta_{iu} e^{j\theta_{iu}} \cos[\omega_c(t - qT_n)] \\
 &\quad \left. + \hat{n}(t - qT_n) + \tilde{\mathfrak{I}}(t - qT_n) \right\} \times c_k(t) w_{p_h}(t) \cos(\omega_c t) dt
 \end{aligned} \tag{11}$$

where d_i^1 denotes the first bit for the i^{th} user. The above expression can be decomposed into four components as

$$x_{kh}(t) = x_{DS} + x_{DSI} + \tilde{n} + \tilde{\mathfrak{I}} \tag{12}$$

Ignoring the double frequency and considering the orthogonality between different wavelet packets, the terms on right-hand side of (12) can be written as follow

- x_{DS} is the desired signal term of the reference user at the zeroth tap of the suppression filter and is given by

$$\begin{aligned}
 x_{DS} &= \int_0^T \left\{ \sqrt{2P} \sum_{i=1}^K d_i^1 c_i(t) \right. \\
 &\quad \times \sum_{u=1}^H w_{p_u}(t) c_k(t) w_{p_h}(t) \beta_{iu} e^{j\theta_{iu}} \cos^2(\omega_c t) \left. \right\} dt \tag{13} \\
 &= \sqrt{\frac{P}{2}} \sum_{i=1}^K d_i^1 \beta_{iu} e^{j\theta_{iu}} \int_0^T c_i(t) c_k(t) w_{p_h}^2(t) dt
 \end{aligned}$$

- x_{DSI} is an internal interference term. It is due to the reference user and caused by the taps of suppression filter excluding the zeroth tap, x_{DSI} is given by

$$\begin{aligned}
 x_{DSI} &= \int_0^T \sum_{q=-Q_1; q \neq 0}^{Q_2} \alpha_q \left\{ \sqrt{2P} \sum_{i=1}^K d_i^1 c_i(t - qT_n) \right. \\
 &\quad \times \sum_{u=1}^H w_{p_u}(t - qT_n) \beta_{iu} e^{j\theta_{iu}} \cos[\omega_c(t - qT_n)] \\
 &\quad \left. \times c_k(t) w_{p_h}(t) \cos(\omega_c t) \right\} dt \tag{14} \\
 &= \sum_{q=-Q_1; q \neq 0}^{Q_2} \alpha_q \sqrt{\frac{P}{2}} \sum_{i=1}^K d_i^1 \beta_{iu} e^{j\theta_{iu}} \cos(qT_n) \\
 &\quad \times \int_0^T c_i(t - qT_n) c_k(t) w_{p_h}^2(t) dt
 \end{aligned}$$

- \tilde{n} is the suppressed correlated AWGN term, it is given by

$$\begin{aligned}
 \tilde{n} &= \sum_{q=-Q_1}^{Q_2} \alpha_q \\
 &\quad \times \int_0^T \hat{n}(t - qT_n) c_k(t) w_{p_h}(t) \cos(\omega_c t) dt
 \end{aligned} \tag{15}$$

- $\tilde{\mathfrak{I}}$ is due to the narrow-band interference and is given by

$$\begin{aligned}
 \tilde{\mathfrak{I}} &= \sum_{q=-Q_1}^{Q_2} \alpha_q \\
 &\quad \times \int_0^T \mathfrak{I}(t - qT_n) c_k(t) w_{p_h}(t) \cos(\omega_c t) dt
 \end{aligned} \tag{16}$$

To represent the h^{th} decoder output in matrix form, we define the data bit sequence (\mathbf{d}) vector, PN signature sequence ($\mathbf{c}(t)$) vector and fading matrix ($\mathbf{\beta}_h$), respectively as follows:

$$\begin{aligned}
 \mathbf{d} &= [d_1^1 \quad d_2^1 \quad \dots \quad d_K^1]^T \\
 \mathbf{c}(t) &= [c_1(t) \quad c_2(t) \quad \dots \quad c_K(t)]^T \\
 \mathbf{\beta}_h &= \text{diag}[\beta_{1h} e^{j\theta_{1h}} \quad \beta_{2h} e^{j\theta_{2h}} \quad \dots \quad \beta_{Kh} e^{j\theta_{Kh}}]^T
 \end{aligned}$$

Then the h^{th} decoder output is given by

$$\begin{aligned}
 \mathbf{x}_h &= \sqrt{\frac{P}{2}} \mathbf{R}_h \mathbf{\beta}_h \mathbf{d} + \sum_{q=-Q_1; q \neq 0}^{Q_2} \alpha_q \sqrt{\frac{P}{2}} \mathbf{R}_h^q \mathbf{\beta}_h \mathbf{d} \\
 &\quad + \sum_{q=-Q_1}^{Q_2} \alpha_q \mathbf{n}_h + \sum_{q=-Q_1}^{Q_2} \alpha_q \mathbf{Y}_h
 \end{aligned} \tag{17}$$

where $\mathbf{n}_h = [\tilde{n}_1 \quad \tilde{n}_2 \quad \dots \quad \tilde{n}_K]^T$ is a Gaussian zero-mean noise vector and $\mathbf{Y}_h = [\tilde{\mathfrak{I}}_1 \quad \tilde{\mathfrak{I}}_2 \quad \dots \quad \tilde{\mathfrak{I}}_K]^T$ is a Gaussian zero-mean jamming vector. Note \tilde{n}_i and $\tilde{\mathfrak{I}}_i$ are defined in (15) and (16), respectively. The cross-correlation matrix, \mathbf{R}_h , is $(K \times K)$ square matrix given by

$$\mathbf{R}_h = \int_0^T [\mathbf{c}(t) w_{p_h}(t)] [\mathbf{c}(t) w_{p_h}(t)]^T dt$$

The ij (i.e., row and column) component of \mathbf{R}_h [12] is

$$\rho_{ij}^h = \int_0^T c_i(t) c_j(t) w_{p_h}^2(t) dt = \sum_{m=0}^{N_h-1} c_i^m c_j^m$$

The $\mathbf{R}_h^q = \int_0^T [\mathbf{c}(t - qT_n) \mathbf{w}_p(t)] [\mathbf{c}(t) \mathbf{w}_p(t)]^T dt$ is also a cross-correlation ($K \times K$) square matrix with ij component given by

$$\begin{aligned} \rho_{ij}^{qh} &= \int_0^T c_i(t - qT_n) c_j(t) \mathbf{w}_p^2(t) dt \\ &= \int_0^{T_n} \left[\sum_{u=0}^{N_n-1} c_i^{u+q} \mathbf{w}_h[t - (q+u)T_n] \sum_{l=0}^{N_n-1} c_j^l \mathbf{w}_h(t - lT_n) \right] dt \\ &= \int_0^{T_n} \left[\sum_{u=0}^{N_n-1} c_i^m \mathbf{w}_h[t - mT_n] \sum_{l=0}^{N_n-1} c_j^l \mathbf{w}_h(t - lT_n) \right] dt \\ &= \sum_{m=0}^{N_n-1} c_i^m c_j^m = \rho_{ij}^h \end{aligned}$$

note that ρ_{ij}^h is independent of the h subband, such that $\mathbf{R}_1 = \mathbf{R}_2 = \dots = \mathbf{R}_H = \mathbf{R}$.

The decoder outputs pass through the decorrelating filter to reduce MAI through matrix inversion. The output for the decorrelating filter for the h^{th} subband is given by

$$\begin{aligned} \mathbf{z}_h &= \mathbf{R}_h^{-1} \mathbf{x}_h = \sqrt{\frac{P}{2}} \mathbf{B}_h \mathbf{d} + \sum_{q=-Q_1; q \neq 0}^{Q_2} \alpha_q \sqrt{\frac{P}{2}} \mathbf{B}_h \mathbf{d} \\ &+ \sum_{q=-Q_1}^{Q_2} \alpha_q \mathbf{R}_h^{-1} \mathbf{n}_h + \sum_{q=-Q_1}^{Q_2} \alpha_q \mathbf{R}_h^{-1} \mathbf{Y}_h \quad (18) \\ &= \mathbf{z}_{DSH} + \mathbf{z}_{DSIh} + \sum_{q=-Q_1}^{Q_2} \alpha_q \mathbf{R}_h^{-1} \mathbf{n}_h + \sum_{q=-Q_1}^{Q_2} \alpha_q \mathbf{R}_h^{-1} \mathbf{Y}_h \end{aligned}$$

Notice that the k^{th} component of (18) is free from interference caused by other users, that is, it is independent of all $\{d_j\}$, $j \neq k$. The source of interferences are the user itself, noise and jamming. For Sin-MC/MU-CDMA the elements of the cross-correlation matrix, ρ_{ij}^h , is given by [3]

$$\rho_{ij}^s = \int_0^T c_i(t) c_j(t) dt = \frac{1}{N_n} \sum_{m=0}^{N_n-1} c_i^m c_j^m$$

For our proposed system, due to the wavelet packets orthogonality, $\rho_{ij} = \sum_{m=0}^{N_n-1} c_i^m c_j^m$. Since $\rho_{ij}^s \leq 1$, the elements

of the \mathbf{R}^{-1} for Sin-MC/MU-CDMA are generally > 1 , hence the noise and jamming powers are enhanced at the output of the decorrelating detector in this system. While in WP-MC/MU-CDMA $\rho_{ij} \geq 1$ which implies that the elements of the \mathbf{R}^{-1} are generally < 1 and thus the noise and jamming powers are reduced at the output of the decorrelating detector.

Signal-to-Interference plus Noise and Jamming Ratio

To find the signal-to-interference plus noise and jamming ratio (SINJR), we need to find the desired signal power, self-interference, noise and jamming variances. Using the fact that $(d_k^1)^2 = 1$, the signal power for the k^{th} user at the h^{th} diversity branch is given by

$$S_k = \left(\sqrt{\frac{P}{2}} \beta_{kh} e^{j\theta_{kh}} d_k^1 \right)^2 = \frac{P}{2} |\beta_{kh}|^2 \quad (19)$$

The variance for the self-interference at the h^{th} diversity branch for the k^{th} user is given by

$$\begin{aligned} \sigma_{DSIhk}^2 &= \text{var}[\mathbf{z}_{DSIh}]_k \\ &= \text{var} \left[\sqrt{\frac{P}{2}} \sum_{q=-Q_1; q \neq 0}^{Q_2} \alpha_q d_k^1 \beta_{kh} e^{j\theta_{kh}} \right] \\ &= \frac{P}{2} E \left[\sum_{q=-Q_1; q \neq 0}^{Q_2} \alpha_q d_k^1 \beta_{kh} e^{j\theta_{kh}} \right]^2 \quad (20) \\ &= \frac{P}{2} \sum_{q=-Q_1; q \neq 0}^{Q_2} \alpha_q^2 |\beta_{kh}|^2 \end{aligned}$$

where $E[\bullet]$ denotes the expected value.

Using same procedure as [12, chapter 8] we can show that the noise covariance matrix for the h^{th} diversity branch, denoted as $\text{cov}[\mathbf{n}_h]$ is given by

$$\text{cov}[\mathbf{n}_h] = \frac{N_o}{4} \mathbf{R}^{-1} \sum_{q=-Q_1}^{Q_2} \alpha_q^2 \quad (21)$$

The jamming covariance matrix for the h^{th} diversity branch, denoted as $\text{cov}[\mathbf{Y}_h]$, is given by

$$\begin{aligned} \text{cov}[\mathbf{Y}_h] &= E \left[\left(\sum_{q_1=-Q_1}^{Q_2} \alpha_{q_1} \mathbf{R}^{-1} \mathbf{Y}_h \right) \left(\sum_{q_2=-Q_1}^{Q_2} \alpha_{q_2} \mathbf{R}^{-1} \mathbf{Y}_h \right)^T \right] \\ &= \mathbf{R}^{-1} E \left[\sum_{q_1=-Q_1}^{Q_2} \sum_{q_2=-Q_1}^{Q_2} \alpha_{q_1} \alpha_{q_2} \mathbf{Y}_h \mathbf{Y}_h^T \right] \mathbf{R}^{-1} \quad (22) \\ &= \mathbf{R}^{-1} (E[\tilde{\mathbf{Y}}_h]) \mathbf{R}^{-1} \end{aligned}$$

Since $f_c \gg \frac{1}{T}$, the double frequency can be ignored. Thus the ij component of $E[\tilde{\mathbf{Y}}_h]$ can be written as

$$E[\tilde{\mathbf{Y}}_h]_{ij} = \frac{\tilde{\mathfrak{S}}}{2} \sum_{q_1=-Q_1}^{Q_2} \sum_{q_2=-Q_1}^{Q_2} \alpha_{q_1} \alpha_{q_2} \left(\int_0^T \xi(t - q_1 T_n) c_i(t) w_{p_h}(t) \cos(2\pi\Delta t + \psi) dt \right) \times \left(\int_0^T \xi(\lambda - q_2 T_n) c_j(\lambda) w_{p_h}(t) \cos(2\pi\Delta\lambda + \psi) d\lambda \right) \quad (23)$$

Using [11, eq. (18)], it can be shown that

$$E[\tilde{\mathbf{Y}}_h]_{ij} = \left(\frac{T^2 \tilde{\mathfrak{S}}}{4N_n} \sum_{q_1=-Q_1}^{Q_2} \sum_{q_2=-Q_1}^{Q_2} \alpha_{q_1} \alpha_{q_2} \sigma_{\xi}^2(q_1, q_2) \right) \mathbf{R}_{ij} \quad (24)$$

where $\sigma_{\xi}^2(q_1, q_2)$ is as given in [11,eq. (19)] as follows

$$\sigma_{\xi}^2(q_1, q_2) \approx \int_{-1}^1 \{ \text{sign}[1 - p |xN_n - q_1 + q_2|] \times \cos[2\pi\Delta T_n(xN_n - q_1 + q_2)](1 - |x|) \} dx \quad (25)$$

given that $N_n \gg 1$, $p = \frac{T_n}{T_{\xi}}$, ΔT_n represents the ratio of offset of the interference carrier frequency to half spread-spectrum bandwidth and

$$\text{sign}[x] = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Accordingly, $E[\tilde{\mathbf{Y}}_h]$ and $\text{cov}[\mathbf{Y}_h]$, respectively given by

$$E[\tilde{\mathbf{Y}}_h] = \left(\frac{T^2 \tilde{\mathfrak{S}}}{4N_n} \sum_{q_1=-Q_1}^{Q_2} \sum_{q_2=-Q_1}^{Q_2} \alpha_{q_1} \alpha_{q_2} \sigma_{\xi}^2(q_1, q_2) \right) \mathbf{R} \quad (26)$$

$$\text{cov}[\mathbf{Y}_h] = \mathbf{R}^{-1} \left(\frac{T^2 \tilde{\mathfrak{S}}}{4N_n} \sum_{q_1=-Q_1}^{Q_2} \sum_{q_2=-Q_1}^{Q_2} \alpha_{q_1} \alpha_{q_2} \sigma_{\xi}^2(q_1, q_2) \right) \mathbf{R} \mathbf{R}^{-1} = \left(\frac{T^2 \tilde{\mathfrak{S}}}{4N_n} \sum_{q_1=-Q_1}^{Q_2} \sum_{q_2=-Q_1}^{Q_2} \alpha_{q_1} \alpha_{q_2} \sigma_{\xi}^2(q_1, q_2) \right) \mathbf{R}^{-1}$$

Thus, the noise plus jamming variances for the k^{th} user at any h^{th} diversity branch, σ_k^2 is given by

$$\sigma_k^2 = \left(\frac{N_o}{4} \sum_{q_2=-Q_1}^{Q_2} \alpha_q^2 + \frac{T^2 \tilde{\mathfrak{S}}}{4N_n} \sum_{q_1=-Q_1}^{Q_2} \sum_{q_2=-Q_1}^{Q_2} \alpha_{q_1} \alpha_{q_2} \sigma_{\xi}^2(q_1, q_2) \right) (\mathbf{R}^{-1})_{kk} \quad (27)$$

where $(\mathbf{R}^{-1})_{kk}$ is equal to kk component of the cross-correlation matrix \mathbf{R} . The Signal-to-Interference plus Noise and Jamming Ratio (SINJR), γ_k , for the k^{th} user is given by

$$\gamma_k = \frac{S_k}{\sigma_{DSIhk}^2 + \sigma_k^2} = |\beta_{kh}|^2 \mathfrak{R} \quad (28)$$

where

$$\mathfrak{R}^{-1} = \sum_{q=-Q_1; q \neq 0}^{Q_2} \alpha_q^2 |\beta_{kh}|^2 + \left(\frac{\Omega T}{E_s/N_o} \sum_{q=-Q_1}^{Q_2} \alpha_q^2 \right) (\mathbf{R}^{-1})_{kk} + T_n^2 N_n \Omega \left(\frac{\tilde{\mathfrak{S}}}{\tilde{S}} \right) \sum_{q_1=-Q_1}^{Q_2} \sum_{q_2=-Q_1}^{Q_2} \alpha_{q_1} \alpha_{q_2} \sigma_{\xi}^2(q_1, q_2) (\mathbf{R}^{-1})_{kk} \quad (29)$$

with $E_s = 2P\Omega T$, $\tilde{S} = 2P\Omega$ and Ω is the mean path gain of the h^{th} path corresponding to k^{th} user.

Determination of Suppression Filter Coefficients

It is shown in [11] that the coefficients of the SF can be determined using

$$\sum_{q=-Q_1; q \neq 0}^{Q_2} \alpha_q \rho(v - qT_n) + \rho(vT_n) \quad (30)$$

$$v = -Q_1, \dots, -1, 1, \dots, Q_2$$

$\rho(vT_n)$ is a lowpass autocorrelation function consists of three components

$$\rho(vT_n) = \rho_s(vT_n) + \rho_n(vT_n) + \rho_{\xi}(vT_n) \quad (31)$$

where $\rho_s(vT_n)$, $\rho_n(vT_n)$ and $\rho_{\xi}(vT_n)$ are, the lowpass version of the desired signal, noise and interference functions of the input signal to the SF. From (10), the $\rho_s(vT_n)$ is given by

$$\rho_s(vT_n) = E \left\{ \left[\sqrt{2P} \sum_{k_1=1}^K \sum_{h_1=1}^H \beta_{k_1 h_1} d_{k_1}(t) c_{k_1}(t) w_{p_{h_1}}(t) \right] \times \left[\sqrt{2P} \sum_{k_2=1}^K \sum_{h_2=1}^H \beta_{k_2 h_2} d_{k_2}(t - vT_n) \times c_{k_2}(t - vT_n) w_{p_{h_2}}(t - vT_n) \right] \right\} = 2P \left\{ \sum_{k_1=1}^K \sum_{h_1=1}^H \sum_{k_2=1}^K \sum_{h_2=1}^H E \left[\beta_{k_1 h_1} \beta_{k_2 h_2} \right] \times E \left[d_{k_1}(t) d_{k_2}(t - vT_n) \right] \times \sum_{i_1=0}^{N_n-1} \sum_{i_2=0}^{N_n-1} E \left[c_{k_1}^{i_1} w_{h_1}(t - i_1 T_n) \times c_{k_2}^{i_2} w_{h_2}(t - \{v - i_2\} T_n) \right] \right\}$$

Since different user's codes are orthogonal when $k_1 \neq k_2$, or $h_1 \neq h_2$, this implies

$$E \left[c_{k_1}^{i_1} w_{h_1}(t - i_1 T_n) c_{k_2}^{i_2} w_{h_2}(t - \{v - i_2\} T_n) \right] = 0$$

Hence, for iid case $\rho_s(vT_n)$ is given by

$$\begin{aligned} \rho_s(vT_n) = 2P \left\{ \sum_{k=1}^K \sum_{h=1}^H E \left[\beta_{kh}^2 \right] E \left[d_k(t) d_k(t - vT_n) \right] \right. \\ \times \sum_{i_1=0}^{N_n-1} \sum_{i_2=0}^{N_n-1} E \left[c_{k_1}^{i_1} w_{h_1}(t - i_1 T_n) \right. \\ \left. \left. \times c_{k_2}^{i_2} w_{h_2}(t - \{v - i_2\} T_n) \right] \right\} \end{aligned} \quad (32)$$

Due to orthogonality of wavelet the term

$$\sum_{i_1=0}^{N_n-1} \sum_{i_2=0}^{N_n-1} \left[c_{k_1}^{i_1} w_{h_1}(t - i_1 T_n) c_{k_2}^{i_2} w_{h_2}(t - \{v - i_2\} T_n) \right]$$

in (32) reduces to $\sum_{i=1}^{N_n-1} (c_k^i)^2 w_h^2(t - iT_n)$. Also, since

$$E \left[\sum_{i=1}^{N_n-1} (c_k^i)^2 w_h^2(t - iT_n) \right] = 1, \text{ then}$$

$$\rho_s(vT_n) = \begin{cases} 2KHP\Omega & v = 0 \\ 0 & v \neq 0 \end{cases} \quad (33)$$

The $\rho_n(vT_n)$ and $\rho_\xi(vT_n)$ are given in [11] by

$$\rho_n(vT_n) = \begin{cases} 2N_o/T_n & v = 0 \\ 0 & v \neq 0 \end{cases} \quad (34)$$

$$\rho_\xi(vT_n) = \begin{cases} \Im(1 - |v|p)(2\pi v \Delta T_n) & |v| \leq \text{int}[1/p] \\ 0 & |v| > \text{int}[1/p] \end{cases} \quad (35)$$

where $\text{int}[x]$ is defined as the integer part of x . Therefore, from (33), (34) and (35), one obtains

$$\rho(vT_n) = 2P\Omega \begin{cases} KH + 2N_n[E_s/N_o]^{-1} + \Im/\tilde{S} & v = 0 \\ \Im(1 - |v|p)(2\pi v \Delta T_n) & |v| \leq \text{int}[1/p] \\ 0 & |v| > \text{int}[1/p] \end{cases} \quad (36)$$

From (30) and (36), we can obtain the coefficients α_q .

BER and Outage Probability Performance

The average bit error rate, \bar{P}_e , and the outage probability, P_{out} , are used to measure the performance for our systems. The \bar{P}_e is obtained by averaging the instantaneous BER of

SINJR γ over the channel fading functions. The outage probability represents the probability of unsatisfactory reception of the signal over the intended coverage area. It is defined as the probability that γ falls below certain specified

threshold, γ_{th} [13]. That is, $P_{out} = \Pr(\gamma < \gamma_{th}) = \int_0^{\gamma_{th}} f_\gamma(\gamma) d\gamma$

The \bar{P}_e and P_{out} depend on the method of diversity combining employed, as explained below.

Selection Diversity

For selection diversity, the overall output SINJR γ of the receiver is given by

$$\gamma^{SD} = \gamma_{\max} = \max(\gamma_1, \dots, \gamma_i, \dots, \gamma_H) \quad (37)$$

where γ_i is the SINJR at the i^{th} branch given by (28). It is assumed that the channel does not change significantly over one symbol period and the selection is continuous. If the channel path gain is assumed to be Nakagami distributed with parameter (m, Ω) , then each input SINJR, γ_i , will be gamma distributed. In [12], it was shown that, \bar{P}_e^{SD} and P_{out}^{SD} are respectively given by

$$\begin{aligned} \bar{P}_e^{SD} = \frac{H [\tilde{G}(x, m)]^{H-1}}{\Gamma(m)} \\ \times \left[\int_0^\infty \left(\frac{m}{\Omega \Re_h} \right)^m \gamma^{m-1} \exp(-x) \right. \\ \left. \times \left(\frac{1}{2} \text{erfc} \left(\sqrt{\frac{\gamma^{SD}}{2}} \right) \right) d\gamma \right] \end{aligned} \quad (38)$$

$$P_{out}^{SD} = \frac{H}{\Gamma(m)} \int_0^{\frac{m\gamma_{th}}{\Omega \Re_h}} [\tilde{G}(x, m)]^{H-1} x^{m-1} \exp(-x) dx \quad (39)$$

where $x = \frac{m\gamma}{\Omega \Re_h}$, $\Gamma(m)$ is the gamma function and $[\tilde{G}(x, m)]$

is the incomplete gamma function and \Re is as given by (29).

Equal Gain Combining

For equal gain combining (EGC), each branch contributes equally to the overall output SINJR [14]. The decision variable can be written as

$$z^{EGC} = \sum_{h=1}^H z_h = \sum_{h=1}^H (z_{DSH} + z_{DSH} + \sigma_k^2)$$

where z_{DSH} and z_{DSIH} are as given in (18), σ_k^2 as given in (27). In [12] it was found that γ^{EGC} , \bar{P}_e^{EGC} and P_{out}^{EGC} are given respectively by

$$\gamma^{EGC} = \frac{\Re}{H} \left(\sum_{h=1}^H \beta_{kh} \right)^2$$

$$\bar{P}_e^{EGC} = \frac{1}{\Gamma(Hm)} \left[\int_0^\infty x^{Hm} \gamma^{Hm-1} \exp(-x\gamma) \times \left(\frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{\gamma^{EGC}}{2}} \right) \right) d\gamma \right] \quad (40)$$

$$P_{out}^{EGC} = \tilde{G}(x\gamma_{th}, Hm) \quad (41)$$

where $x = \frac{m}{\Omega \Re \left(1 - \frac{1}{5m} \right)}$.

Maximal Ratio Combining

Maximal Ratio Combining is the optimum diversity combining technique [15]. It depends on the idea that components of the received signal with high amplitudes contain relatively low noise power level. So, their effect on the decision process can be increased by squaring their amplitude [14]. In MRC technique, the diversity branches are weighted and combined. For the k^{th} user, the output of the combiner is given by

$$z^{MRC} = \sum_{h=1}^H \omega_h (\mathbf{z}_h)_k = \sum_{h=1}^H \omega_h (z_{DSH} + z_{DSIH} + \sigma_k^2)$$

where ω_h is the weight function. In [12] it was shown that

$$\gamma^{MRC} = \Re \sum_{h=1}^H \beta_{kh}^2$$

$$\bar{P}_e^{MRC} = \frac{1}{\Gamma(Hm)} \left[\int_0^\infty x^{Hm} \gamma^{Hm-1} \exp(-x\gamma) \times \left(\frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{\gamma^{MRC}}{2}} \right) \right) d\gamma \right] \quad (42)$$

$$P_{out}^{MRC} = \tilde{G}(x\gamma_{th}, Hm) \quad (43)$$

where $x = \left(\frac{m}{\Omega \Re} \right)$.

Analytical simulation in the following section is used to evaluate the \bar{P}_e and P_{out} . Note that in the flowing section we will refer to \bar{P}_e as BER.

RESULTS AND DISCUSSIONS

In this section, using the above analytical results and by means of the MATLAB program, we evaluate the BER and P_{out} for the system. The performance of the system is tested in presence of narrow-band interference. Unless otherwise mentioned, the numerical results were generated using:

- MRC diversity with mean path gain, $\Omega = 10$, diversity order, $H = 4$ and Nakagami parameter, $m = 1$.
- Processing gain length, $N_n = 63$ with chip duration, $T_n = 10^{-6}$
- Number of users, $K = 10$.
- Ratio of the interference bandwidth to the system bandwidth $p = 0.1$.
- Ratio of the offset of interference carrier frequency to half spread-spectrum bandwidth $\Delta T_n = 0$. This means that the narrowband interference exists at the middle of the CDMA spectrum.
- Interference jamming power to signal power, $\tilde{\mathfrak{I}}/\tilde{\mathfrak{S}} = 20$ dB.
- Threshold of SINJR, $\gamma_{th} = 10$ dB.
- Signal to noise ratio, $E_s/N_o = 25$ dB

Two types of SF were used:

- Double Sided (DS) filter: DS 1 has $Q_1 = 1$ and $Q_2 = 1$, thus it has three taps; DS 2 has $Q_1 = 2$ and $Q_2 = 2$, thus it has five taps.
- Single Sided (SS) filter: SS 3 has $Q_1 = 0$ and $Q_2 = 3$; SS 5 has $Q_1 = 0$ and $Q_2 = 5$.

Effect of Number of Filter Taps

Figure 2-a shows the BER performance as a function of E_s/N_o , while Figure 2-b shows the P_{out} performance as a function of γ_{th} . The two Figures show the performances with and without SF. As expected, the BER and also P_{out} performances are improved by using SF. It can be noted that as the number of taps increases for SS or DS filters, the performances are improved. For 3 taps, SS filter outperforms DS filter. For 5 taps, DS filters has better BER performance.

The P_{out} performances for SS 5 and DS 2 filters are approximately the same.

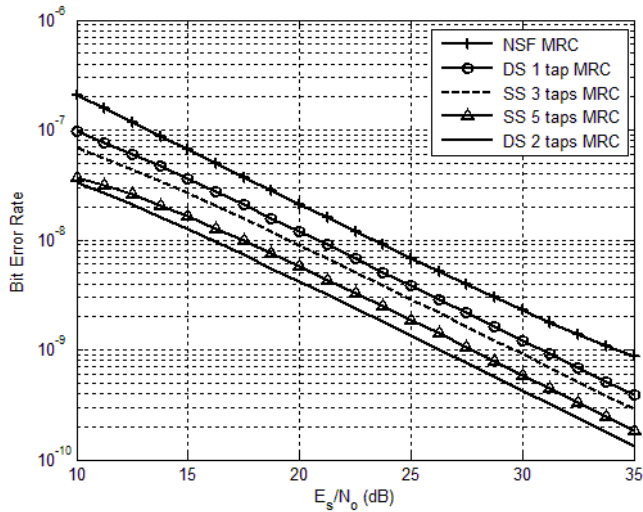


Figure 2-a: Effect of filter on BER

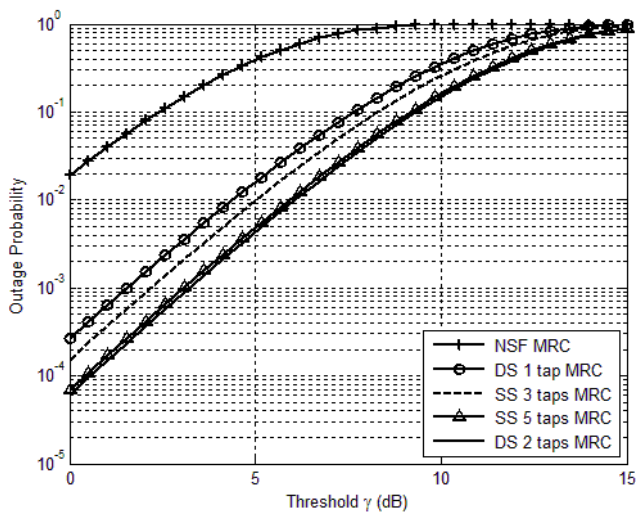


Figure 2-b: Effect of filter on P_{out}

Effect of Diversity

Figure 3-a and Figure 3-b shows the effect of diversity on BER performance and P_{out} performance, respectively. From Figure 3-a and Figure 3-b, it can be noted that: tremendous improvement can be achieved by using diversity. As discussed previously, MRC is the optimum diversity because of that MRC technique shows the best performance. Selection diversity shows the worst performance, however, one benefit of SD is its simple implementation..

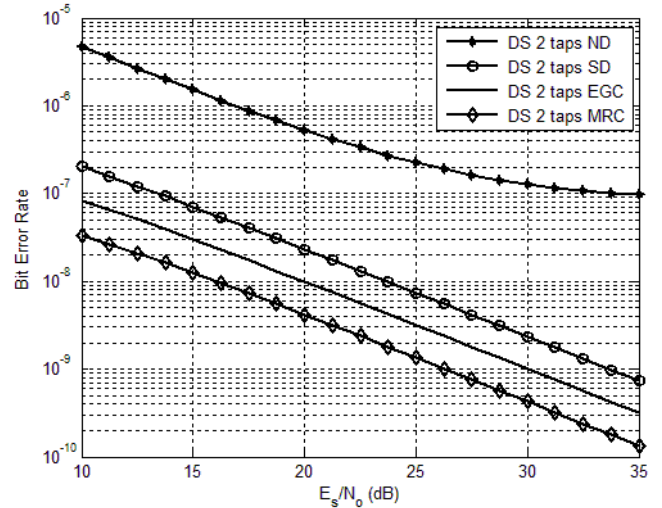


Figure 3-a: Effect of diversity on BER

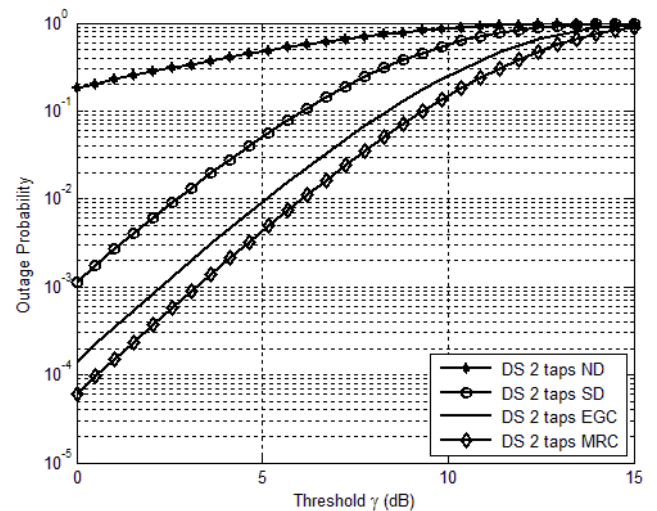


Figure 3-b: Effect of diversity on P_{out}

Effect of Interference Jamming Power to Signal Power

The effect of \mathfrak{I}/\tilde{S} on BER for DS and SS filters is represented in Figure 4. As expected, the BER performance is degraded by increasing \mathfrak{I}/\tilde{S} . Also it can be noted that: DS 2 has the best performance regardless the value \mathfrak{I}/\tilde{S} . The SS filters outperform the DS 1 filter when $\mathfrak{I}/\tilde{S} < 25$ dB, while for $\mathfrak{I}/\tilde{S} > 25$ dB DS 1 has better performance than SS filters. For $27 < \mathfrak{I}/\tilde{S} < 33$ dB, system with no SF has a very little better performance than systems with SS filter. The number of taps for certain type of filter has no significant effect if $\mathfrak{I}/\tilde{S} > 40$ dB.

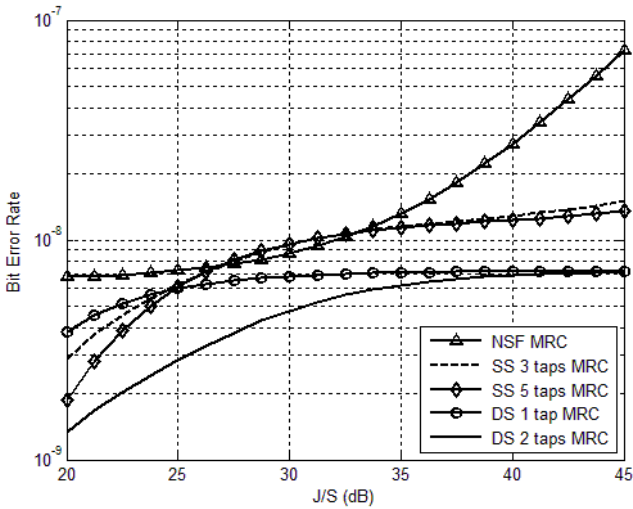


Figure 4: Effect of J/S on BER

Effect of Channel Fading

Figure 5 shows the effect of channel fading parameter, m , on BER. Three values of m are chosen, namely, $m = 0.5$, which approximate one sided Gaussian model, $m = 1$, which approximate the Rayleigh fading and $m = 3$, which approximated Ricean model. The performance is investigated for SS 5. As expected, increasing m improves the performance of the system.

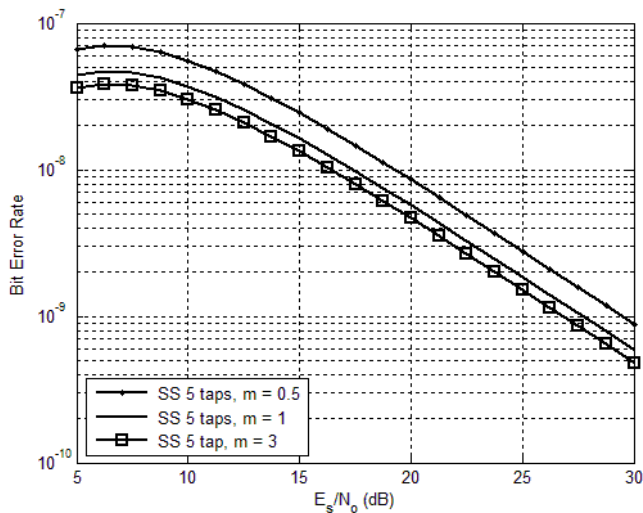


Figure 5: Effect of channel fading, m , on BER

Effect of Diversity Order

The effect of diversity order, H , on P_{out} is investigated in Figure 6. The performance is investigated for DS filters. Provided that $\gamma_{th} < 15$, as expected, increasing H improves

the performance of the system and for a given value of H , the DS 2 filter outperforms the DS 1 filter.

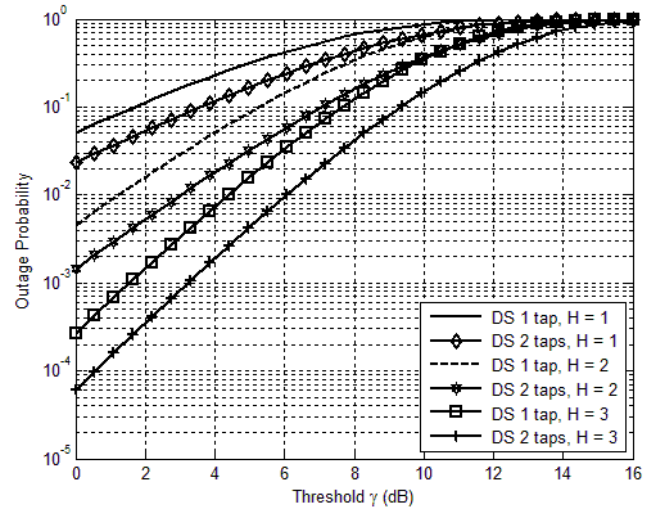


Figure 6: Effect of diversity order, H , on P_{out}

Effect of Ratio of the Offset of Interference Carrier Frequency to Half Spread-Spectrum Bandwidth, ΔT_n

Figure 7 shows the effect ΔT_n on BER using DS 2 with different types of diversity and without diversity. As expected, the MRC has the best BER performance. Since the variances of self-interference, jamming and noise depends on filter coefficients, which depends on $\cos(2\pi\nu\Delta T_n)$ (ν integer) we can note that: the performance of the system as a function of ΔT_n has the shape of cosine function.

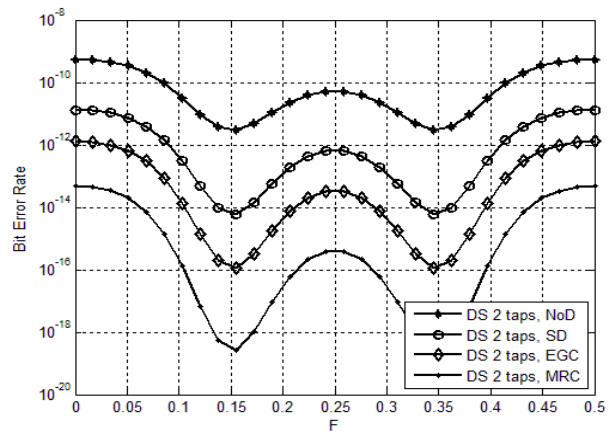


Figure 7: Effect of ΔT_n on BER

Effect of Ratio of the Interference Bandwidth to the System Bandwidth, p

Figure 8, illustrates the BER performance as a function of p using DS 2 filter for different type of diversities and with no

diversity. As useful, MRC diversity outperforms the other diversities. For $p < 0.5$ we can note that BER performance is almost insensitive to the value of p , but for $p > 0.5$, significant improvement at BER can be noted.

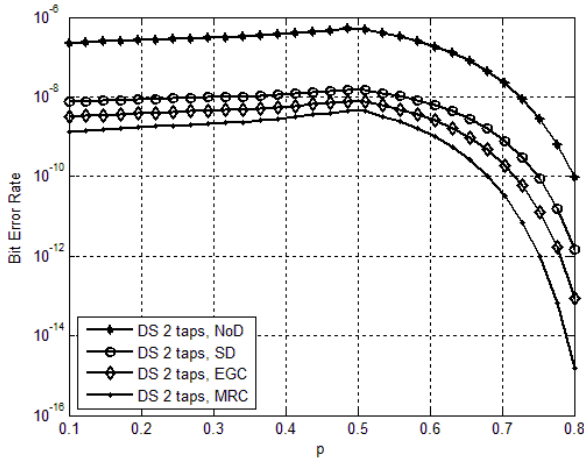


Figure 8: Effect of p on BER.

Performance Comparison

In Figure 9, with $K = 3$, the BER performance is compared for three CDMA based systems, namely WP-MC/MU-CDMA, Sin-MC/MU-CDMA and WP-MC/SU-CDMA with and without SF filter. Observe that:

- Our proposed system WP-MC/MU-CDMA outperforms Sin-MC/MU-CDMA. The reasons for this performance is as follows:

For Sin-MC/MU-CDMA, the noise and jamming powers are enhanced at the output of the decorrelating detector because the elements of $\mathbf{R}^{-1} > 1$. For our proposed system, due to the wavelet packets orthogonality the elements of $\mathbf{R}^{-1} < 1$, thus the noise and jamming powers are reduced at the output of the decorrelating detector. This means that Sin-MC/MU-CDMA system at the output of the decorrelating detector completely eliminates the MAI, but it enhance the noise and jamming powers. However, our proposed system, WP-MC/MU-CDMA, due the orthogonality of wavelet packets, not only eliminates the MAI, but also reduces the noise and jamming powers.

- Since tremendous improvement can be achieved by using SF and multiuser detection, a significant difference can be observed between BER performance of WP-MC/SU-CDMA with and without SF and also between WP-MC/SU-CDMA and the other two systems. This is because:

1. SF is very effective in suppression the jamming interference.

2. Single user detector is unable to exploit the structure of MAI (interference from other paths and other users), while in multiuser detector instead of users interfering with each other; they are all being used for their mutual benefit by joint detection. Note that in this part we take $K = 3$ to reduce MAI.

- The drawback of multiuser systems is its complexity, which grows as K increased. This is due to increasing of the dimension of \mathbf{R} matrix.

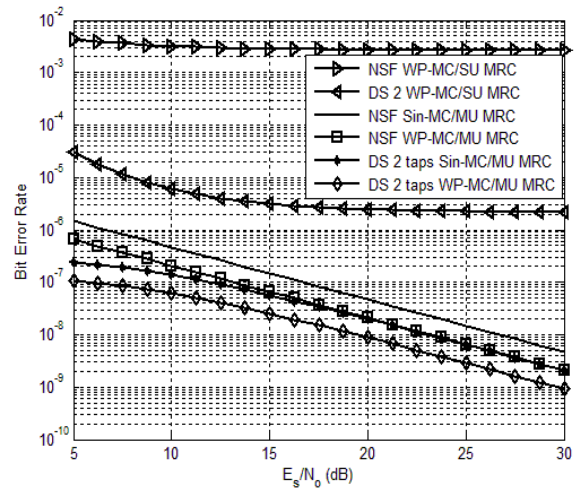


Figure 9: BER performance for WP-MC/MU-CDMA and Sin-MC/MU-CDMA with and without SF

CONCLUSIONS

The performance of WP-MC/MU-CDMA system employing SF is analyzed in this paper. Analytical framework is provided and system performance is analyzed in terms of BER and P_{out} in a frequency selective Nakagami fading channel. Different analytical results were used to illustrate the influence of diversity and fading on the system performance. Performance our system, WP-MC/MU-CDMA, is compared to the performance of the Sin-MC/MU-CDMA and WP-MC/SU-CDMA systems with and without SF. It is found that, in all cases our new system outperformed the other systems and the performance is improved by using SF and diversity. The double-sided SF is superior to single-sided SF for the same number of total taps and increasing the number of taps improved the system performance. The MRC has better performance than SD and EGC. The performance of the system is improved by increasing the diversity order and the fading parameter. The performance of the system is improved when p increased and degraded when $\mathfrak{S}/\tilde{\mathfrak{S}}$ increased. The complexity of the multiuser system is increased as the number of users increased.

ACKNOWLEDGMENT

The author is grateful to the Applied Science Private University, Amman, Jordan, for the full financial.

REFERENCES

- [1] A. D. Hallen, J. Holtzman and Z. Zvonar, "Multiuser detection for CDMA systems", *IEEE Personal Communications.*, pp. 46-58, April 1995.
- [2] S. Verdu, "Adaptive multiuser detection", IEEE Third International Symp. Spread Spectrum Techniques and Applications, vol. 1, pp. 43-50, July 1994.
- [3] S. Verdu, "Multiuser Detection", Cambridge University Press, New York, 1998.
- [4] Wang, J.Z. (1999) "On the Use of Suppression Filter for CDMA Overlay", *IEEE Transactions on Vehicular Technology*, 48, 405-414.
- [5] S. Mallat, "A Wavelet Tour of Signal Processing", Academic Press, USA, 1998.
- [6] N. Nakagami, "The m-distribution, a general formula for intensity distribution of rapid fading", In Statistical Methods in Radio Wave Propagation, W. G Hoffman, ed. Oxford, U.K: Pergamon, pp.3-35, 1960.
- [7] S. Kondo and L. B. Milstein, "Performance of multicarrier DS-SS systems," *IEEE Trans. Commun.*, vol. 44, no. 2, pp. 238-246, February 1996.
- [8] E. Sourour and M. Nakagawa, "Performance of orthogonal multicarrier CDMA in a multipath fading channel", *IEEE Transactions on Communications.*, vol. 44, pp. 356-367, March 1996.
- [9] A. Kothiram, T. Sirimak and S. Sittichivapak, "Performance of multiuser detection combining with multicarrier on Nakagami fading channels for DS/SS transmission systems", SCORED Proc. Student Conf. on Research and Development, pp. 313-318, August 2003.
- [10] W. Choi and J. Y. Kim, "Performance of multiuser detection with multicarrier transmission for DS/SS system," *Wireless Personal Communications*, pp. 71-87, 2002.
- [11] Wang Jiangzhou, "On the use of suppression filter for CDMA overlay", *IEEE Transactions Vehicular Technolog.*, vol. 48, no. 2, pp. 405-414, March 1999.
- [12] M. M. Akho-Zahieh, "Design and Analysis of Multicarrier Multicode Wavelet Packets Based CDMA Communication Systems With Multiuser Detection", PhD Dissertation, The University of Akron, August 2006.
- [13] M. K. Simon and M. S. Alouini, "Digital Communication Over Fading Channel", 2nd edition. John Wiley & Sons, Hoboken, New Jersey, 2005.
- [14] J. G. Proakis, "Digital Communications", 3rd edition, McGraw-Hill, New York, 1995.
- [15] E. K. Al-Hussaini and A. M. Al-Bassiouni, "Performance of MRC diversity systems for the detection of signals with Nakagami fading," *IEEE Transaction on Communications*, vol. 33, pp.1315-1319, December 1985.