

Development of Mathematical Model of Heat and Mass Transfer in Soil, with Provision for the Gradients of soil-water and soil-salt potentials. Part 3

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Abstract

The building of underground pipelines, shallow foundation and roads on a heaving soils is associated with risks of malfunction. Frost heaving of soils can develop a huge pressure, which is practically impossible to balance with the weight of the construction. Irregularity of the soil deformation leads to a deterioration of the stress-strain state of the construction or to total failure.

This article was written in three parts. In the first part, the authors investigated the thermodynamic parameter soil-water and soil-salt potentials. The study showed that the potentials can include chemical potential. However, soil-water and soil-salt potentials must contain a part related to the potential of external mass forces in order to explain the displacement of soil-water solution against the forces of gravity and this part is not zero. In addition, an equation of the pore soil-water solution dynamics was derived. This equation at zero velocity is equivalent to the equation of thermodynamic equilibrium.

In the second part of the article, the authors used the laws of nonequilibrium thermodynamics and derived thermodynamic driving forces and flows, which take into account the cross-effects of heat and mass transfer and the gradients of soil-water and soil-salt potentials. Moreover, in the total energy equation (the sum of the potential, kinetic and internal energies), the energy of intramolecular interaction of water, salt and solid particles of the ground (in fact, this is soil-water and soil-salt potentials) was considered separately.

In the third part, the authors estimated the magnitude of the cross effects, wrote expressions for the kinetic coefficients, which take into account the empirical laws of heat and mass transfer. The authors derived a system of differential equations of heat and mass transfer, which takes into account: convective motion and diffusion motion of water and salts, dynamic change of groundwater level, kinetics of crystallization, melting and dissolution according to the first-order chemical reaction equation. The value of the equation system lies in the possibility of direct use in engineering practice.

Keywords: frost heaving, thermodynamic driving forces and fluxes, heat and mass transfer in soil, groundwater level, cross effects, kinetic coefficients

INTRODUCTION

In the first part of the article the authors used the basic postulates of equilibrium thermodynamics to prove that the soil-water and soil-salt potentials can consist of Gibbs chemical potential and the potential of external mass forces (1). Moreover, the potential of external mass forces is not equal to zero [1]:

$$\psi_i = \psi_i^{ch} + \psi_i^{evf}, \text{ where } \psi_i^{evf} \neq 0 \quad (1)$$

$$\sum_{i=1}^m (\omega_i d\psi_i^{ch}) = 0, \quad (2)$$

$$\sum_{i=1}^m (\omega_i d\psi_i) = \sum_{i=1}^m (\omega_i d\psi_i^{evf}) \quad (3)$$

$$\varphi_{ws} \frac{d\vec{W}_{ws}}{dt} = -n_{ws} \nabla P - \varphi_{ws} \sum_{i=1}^m \omega_i \nabla \psi_i + \varphi_{ws} (\vec{F}_i^{c,ws} + \vec{F}_{fr}^{ws}), \quad (4)$$

where ψ_i - decreasing of the potential of the i -th component in soil solution in comparison with free solution, J/kg; ψ_i^{evf} - part of ψ_i belong to potential of external volume forces, J/kg; ψ_i^{ch} - part of ψ_i , belong to chemical potential (for different concentration), J/kg; ω_i - mass concentration of the i -th component in solution; φ_{ws} - content of water-salt solution in soil, kg/m³; \vec{W}_{ws} - vector of true solution velocity as a single body, m/s; n_{ws} - volume fraction of water-salt solution in the soil, u.f.; P - hydrostatic pressure, Pa; $\vec{F}_i^{c,ws}$ - specific conservative force acting on the i -th component of water-salt solution, N/kg; \vec{F}_{fr}^{ws} - specific friction force acting on the water-salt solution, N/kg; m - number of components of water-salt solution.

In the second part of the article, the authors used the postulates of nonequilibrium thermodynamics and equations

(1) - (3) to derive the system of Onsager linear equations for thermodynamic driving forces and fluxes:

$$\begin{aligned} \vec{J}_{ws} &= L_{ws,ws}\vec{X}_{ws} + L_{ws,q}\vec{X}_q + \sum_{i=1}^{m-1} (L_{ws,i}(\vec{X}_i - \vec{X}_m)), \\ \vec{J}_q^{ws} &= L_{q,ws}\vec{X}_{ws} + L_{q,q}\vec{X}_q + \sum_{i=1}^{m-1} (L_{q,i}(\vec{X}_i - \vec{X}_m)), \\ \vec{J}_1^{ws} &= L_{1,ws}\vec{X}_{ws} + L_{1,q}\vec{X}_q + \sum_{i=1}^{m-1} (L_{1,i}(\vec{X}_i - \vec{X}_m)), \end{aligned} \quad (4)$$

...

$$\begin{aligned} \vec{J}_j^{ws} &= L_{j,ws}\vec{X}_{ws} + L_{j,q}\vec{X}_q + \sum_{i=1}^{m-1} (L_{j,i}(\vec{X}_i - \vec{X}_m)), \\ \dots, \\ \vec{J}_{m-1}^{ws} &= L_{m-1,ws}\vec{X}_{ws} + L_{m-1,q}\vec{X}_q + \sum_{i=1}^{m-1} (L_{m-1,i}(\vec{X}_i - \vec{X}_m)), \\ \vec{X}_{ws} &= \vec{F}_{fr}^{ws} = v_{ws}\vec{\nabla}P + \sum_{i=1}^m (\omega_i\vec{\nabla}\psi_i^{evf}) - \sum_{i=1}^m \omega_i\vec{F}_i^{c,ws}, \end{aligned} \quad (5)$$

$$\vec{X}_q = \frac{\vec{\nabla}T}{T}, \quad (6)$$

$$\vec{X}_i = \vec{\nabla}\psi_i^{evf} + v_i^p\vec{\nabla}P + \vec{\nabla}(\mu_i^c + \psi_i^{ch})_T - \vec{F}_i^{c,ws}, \quad (7)$$

where $L_{j,i}$ – kinetic transport coefficients, \vec{J}_q^{ws} – heat flux, W/m^2 ; \vec{J}_{ws} – convective flow of solution, $kg/(s \cdot m^2)$; \vec{J}_i^{ws} – diffusion flux of the i -th component of water-salt solution, $kg/(s \cdot m^2)$; v_{ws} – specific volume of water-salt solution, m^3/kg ; v_i^p – specific volume of the i -th component in solution, as the ratio of volume to mass of the i -th component in solution, m^3/kg ; $\mu_i^c = \left. \frac{\partial G_i}{\partial \omega_i} \right|_{T,P,\omega_j,\varphi_i}$ – decrease of Gibbs free energy of the i -th component in free solution, due to the change of the concentration, J/kgT - temperature, K ;

The system of equations (4) describes the fluxes of heat and matter, taking into account cross effects and the influence of external mass forces, pressure gradients, temperature, soil-water and soil-salt potentials of all components.

The third part completes a series of articles devoted to the development of the mathematical model of heat and mass transfer in soils for forecasting frost heaving and taking into account the gradients of soil-water and soil-salt potentials of all components of the solution.

OBJECT AND PURPOSE

In the third part, the authors of the article solved the following problems:

- cross-effects of heat and mass transfer were estimated;
- empirical laws of heat and mass transfer and of the rate of ice and undissolved salt crystallization were written
- differential equations of heat and mass transfer was derived.

The main advantage of the equations is the possibility of their direct use in engineering practice. The parameters used in the equations can be obtained by a standard laboratory equipment for geotechnical surveys.

ESTIMATION OF THE CROSS EFFECTS

First of all authors estimated the cross-effects of heat and mass transfer. In engineering practice, it is usually considered aggregate transfer of salts, so we will consider only binary water-salt solution (subscript w refers to water, subscript s refers to the dissolved salt). We will refer only gravity \vec{g} to conservative forces $\vec{F}_i^{c,ws}$.

Soil-water and soil-salt potentials depend on content of relevant component, i.e. $\psi_i = f(\varphi_i)$. In this case $\psi_i^{ch} = 0$ because the chemical potential is an isobaric-isothermal derivative with respect to concentration in accordance with the Gibbs equation:

$$\left. \frac{\partial G}{\partial \omega_i} \right|_{P,T,\omega_j,\varphi_i} d\omega_i = (\mu_w^c + \psi_i^{ch})d\omega_i = \mu_w^c d\omega_i \quad (8)$$

Then, in accordance with (8), we got, that $\psi_i^{evf} = \psi_i$. Using (4) – (8) we derived:

$$\vec{J}_{ws} = L_{ws,ws}\vec{X}_{ws} + L_{ws,q}\vec{X}_q + L_{ws,w}(\vec{X}_w - \vec{X}_s), \quad (9)$$

$$\vec{J}_q^{ws} = L_{q,ws}\vec{X}_{ws} + L_{q,q}\vec{X}_q + L_{q,w}(\vec{X}_w - \vec{X}_s), \quad (10)$$

$$\vec{J}_w^{ws} = L_{w,ws}\vec{X}_{ws} + L_{w,q}\vec{X}_q + L_{w,w}(\vec{X}_w - \vec{X}_s), \quad (11)$$

$$\vec{J}_s^{ws} = -\vec{J}_w^{ws} \quad (12)$$

$$\vec{X}_{ws} = \vec{F}_{fr}^{ws} = v_{ws}\vec{\nabla}P + \omega_w\vec{\nabla}\psi_w + \omega_s\vec{\nabla}\psi_s - \vec{g}, \quad (13)$$

$$\vec{X}_q = \frac{\vec{\nabla}T}{T}, \quad (14)$$

$$\begin{aligned} (\vec{X}_w - \vec{X}_s) &= \vec{\nabla}(\psi_w - \psi_s) + \\ &+ (v_w^p - v_s^p)\vec{\nabla}P + \vec{\nabla}(\mu_w^c - \mu_s^c)_T, \end{aligned} \quad (15)$$

In the tabular form, equations (9) - (15) are presented in Table 1.

Table 1: Names of thermodynamic fluxes

Force \ Flow	X_{ws}		X_T		$X_w - X_s$
\vec{J}_{ws}	$L_{ws,ws}$ Filtration $\vec{j} = -\frac{\lambda_p}{g}\vec{\nabla}P$	+	$L_{ws,q} \approx 0$ Thermal filtration (direct effect)	+	$L_{ws,w} = 0$
\vec{J}_q^{ws}	$L_{q,ws} \approx 0$ Thermal filtration (reverseeffect)	+	$L_{q,q}$ Fourier Law $\vec{j} = -\lambda_T^{ws}\vec{\nabla}T$	+	$L_{q,w} \approx 0$ Thermo-diffusion (Dufour effect)
\vec{J}_w^{ws}	$L_{w,ws} = 0$	+	$L_{w,q} \approx 0$ Thermo-diffusion (Soreteffect)	+	$L_{w,w}$ Fick's Law $\vec{j} = -D_{ws}\varphi_{ws}\vec{\nabla}\omega_w$

The question about the value of the soil-salt potential of the dissolved salt (ψ_s) remains insufficiently studied. There are no formulas to predict this value for soils in a wide range of chemical and granulometric texture. However, the available data on the thicknesses of the adsorption part of the double electric layer (10 nm), where ψ_s is maximal, and the pore size involved in mass transfer during the frost heaving (about 10 μm), allow us to conclude that the influence of ψ_s to the convective motion of the water-salt solution is small.

The expressions (11), (15) show us that the gradients of soil-water $\vec{\nabla}\psi_w$ and soil-salt $\vec{\nabla}\psi_s$ potentials are the opposing factors of diffusion. Usage of one factor and absence of another can lead to a spatial separation over the components. Therefore, the expression for the diffusion flow of water, we should put $\vec{\nabla}(\psi_w - \psi_s) = 0$. The investigation of soil-salt potential ψ_s remains relevant today and will be the subject of further research of authors.

The coefficients $L_{ws,w}$ and $L_{w,ws}$ in the equation (9) are zero in accordance with the equation of the dynamics (3). The values of thermodiffusion and thermal filtration are small, in accordance with the experimental research [8]. It allow to equate to zero the coefficients $L_{ws,q}$, $L_{q,ws}$, $L_{w,q}$, $L_{q,w}$.

The practice of physical modeling of heat and mass transfer in the base of pipelines allows to conclude that pressure and stress-strain state of the soil have small influence to the value of the convective flux of the solution and the diffusion fluxes of solution components [7]. Thus, in the first approximation, we can consider the stress-strain state of the soil separately from heat and mass transfer, i.e. $\vec{\nabla}P = 0$.

However, the expression for convective flux of the solution (9) must take into account the impossibility of motion in the saturated soil when the $\vec{\nabla}\psi_w = 0$ and $v_{ws}\vec{\nabla}P - \vec{g} = 0$. Thus, the gradient of the hydrostatic pressure balances the gravity and the system remains stationary below the water level if there is no exit from system. To take into account the water level, authors assumed that water level coincides with the mark on which the entire volume of pores is filled with ice, water, dissolved and undissolved salts. Below water level, the motion is possible only under the action of the soil-water potential gradient. The equation (3) allows us to write the final equations for the fluxes:

$$\vec{J}_{ws} = L_{ws,ws}(\omega_w \vec{\nabla}\psi_w - (1 - hav(\varepsilon_V))\vec{g}), \quad (16)$$

$$\vec{J}_q^{ws} = L_{q,q} \frac{\vec{\nabla}T}{T}, \quad (17)$$

$$\vec{J}_w^{ws} = -\vec{J}_s^{ws} = \frac{L_{w,w}}{\omega_s} (\vec{\nabla}\mu_w^c)_T, \quad (18)$$

$$\varepsilon_V = \frac{\wp_{ws}}{\rho_{ws}} + \frac{\wp_{ice}}{\rho_{ice}} + \frac{\wp_{ns}}{\rho_{ns}} - \frac{e}{e+1} \quad (19)$$

where ρ_{ws} – real density of solution, kg/m^3 ; \wp_{ice} – content of ice in soil, kg/m^3 ; ρ_{ice} – real density of ice, kg/m^3 ; \wp_{ns} – content of undissolved salt in soil, kg/m^3 ; ρ_{ns} – real density of

undissolved salt, kg/m^3 ; e – porosity ratio (volume of pore divided by volume of solid particles), u.f.; ε_V – bulk frost heaving deformation of soil, u.f.; $hav(x)$ – Heaviside function of argument x .

In this case, the use of ε_V is most convenient, because frost heaving implies the excess of the pore volume by interstitial substances. This approach allow to take into account the dynamic change in the water level.

EMPIRICAL LAWS OF MATTER TRANSPORTATION

At the next stage, the authors wrote the empirical laws which determine a connection between kinetic transport coefficients and empirical laws. The kinetic coefficient $L_{ws,ws}$ corresponds to the convective motion under the action of the pressure gradient (table 1):

$$L_{ws,ws} = -\frac{\lambda_P \rho_{ws}}{g} \quad (20)$$

where λ_P – transmissibility coefficient, m/s.

To describe soil-water potential, Kalyuzhnyy I.L., Lavrov S.A [5] recommended the exponential law which allows approximating the dependence on water content by two experimental points and has acceptable accuracy:

$$\psi_w = \psi_{ww} \left(\frac{\wp_{ww}}{\wp_w} \right)^\gamma, \quad (21)$$

$$\gamma = \log \left(\frac{\wp_{ww}}{\wp_{wl}} \right) \left(\frac{\psi_{wl}}{\psi_{ww}} \right), \quad (22)$$

$$\frac{\partial \psi_w}{\partial \wp_w} = \frac{-\gamma \psi_{ww} (\wp_{ww})^\gamma}{(\wp_w)^{\gamma+1}}, \quad (23)$$

where γ – exponent of soil-water potential; \wp_{ww} – water content at wilt, correspond $\psi_{ww} \approx -1500$ J/kg; \wp_{wl} – water content at least water capacity, correspond $\psi_{wl} \approx -33$ J/kg.

Transmissibility coefficient describes by Maulem's expression [9]:

$$\lambda_P = \frac{\lambda_{P0}}{S_f^2} \left(\frac{\wp_w}{\wp_w^{max}} \right)^{0.5} \left(\frac{\int_0^{\wp_w} \wp_w d\wp_w}{\int_0^{\wp_w^{max}} \wp_w d\wp_w} \right)^2 \quad (24)$$

where λ_{P0} – permeability coefficient in saturated soil, m/s; S_f – specific surface of filtration channel; $\wp_w^{max} = \rho_w e / (1 + e)$, – maximal water content in saturated soil, kg/m^3 .

Specific surface of filtration channel in frozen soil is approximately twice as large of specific surface of thawed soil [5]. Then the specific surface of filtration channel can be found by the following expression:

$$S_f = 1 + \frac{(e+1)\wp_{ice}}{e\rho_{ice}} + \frac{(e+1)\wp_{ns}}{e\rho_{ns}} \quad (25)$$

The use of (21) - (25) allowed us to derive the following expression for transmissibility coefficient:

$$\lambda_p = \lambda_{p0} \frac{\left(\frac{\rho_w}{\rho_w^{max}}\right)^{2\gamma+2.5}}{\left(1 + \frac{(e+1)\rho_{ice}}{e\rho_{ice}} + \frac{(e+1)\rho_{ns}}{e\rho_{ns}}\right)^2} \quad (26)$$

The kinetic coefficient $L_{w,w}$ describes diffusion according to Fick's law:

$$L_{w,w} = -D_{ws} \frac{\rho_{ws} M_w \omega_s (1-f\omega_s)}{RTf} \quad (27)$$

$$\mu_w^c = \frac{RT}{M_w} \ln(1 - f\omega_s), \quad (28)$$

where coefficient $f = 0,86$ for NaCl-H₂O solution with an error no more than 3,2% at a temperature from -21°C to 20°C; D_{ws} – diffusion coefficient, m²/s; R – universal gas constant, J/(mole·K); M_w – molar mass of water, kg/mole.

Literary review of experimental data, made by Komarov I.A., showed that diffusion coefficient D_{ws} in thawed soils is between 10⁻¹⁰ ... 10⁻¹² m²/s. At the same time in frozen soils the diffusion coefficient is higher: 10⁻⁹ ... 10⁻¹¹ m²/s [6].

HEAT AND MASS TRANSFER EQUATIONS

At the last stage, the authors used equations (16) - (28) and derived the classical laws of conservation of the mass of water and dissolved salt:

$$\frac{\partial \rho_w}{\partial t} = \nabla \cdot (-D_{ws} \rho_{ws} \vec{\nabla} \omega_s) + \nabla \cdot \left(\frac{\omega_s \lambda_P \rho_{ws}}{g} (\omega_w \vec{\nabla} \psi_w - (1 - hav(\varepsilon_V)) \vec{g}) \right) - \frac{\partial \rho_{ice}}{\partial t} \quad (29)$$

$$\frac{\partial \rho_s}{\partial t} = \nabla \cdot (D_{ws} \rho_{ws} \vec{\nabla} \omega_s) + \nabla \cdot \left(\frac{\omega_s \lambda_P \rho_{ws}}{g} (\omega_w \vec{\nabla} \psi_w - (1 - hav(\varepsilon_V)) \vec{g}) \right) - \frac{\partial \rho_{ns}}{\partial t} \quad (30)$$

The usage of equations (29) и (30) more convenient with an another form. We expressed ρ_w interms of ψ_w and ρ_s interms of ω_s and derived laws of conservation in potential-concentration form:

$$\frac{\partial \rho_w}{\partial \psi_w} \frac{\partial \psi_w}{\partial t} = +\nabla \cdot (-D_{ws} \rho_{ws} \vec{\nabla} \omega_s) + \nabla \cdot \left(\frac{\omega_s \lambda_P \rho_{ws}}{g} (\omega_w \vec{\nabla} \psi_w - (1 - hav(\varepsilon_V)) \vec{g}) \right) - \frac{\partial \rho_{ice}}{\partial t} \quad (31)$$

$$\left(\frac{\rho_s + \rho_w}{\omega_w} \right) \frac{\partial \omega_s}{\partial t} = - \frac{\omega_s}{\omega_w} \left(\frac{\partial \rho_w}{\partial \psi_w} \right) \frac{\partial \psi_w}{\partial t} + \nabla \cdot (D_{ws} \rho_{ws} \vec{\nabla} \omega_s) + \nabla \cdot \left(\frac{\omega_s \lambda_P \rho_{ws}}{g} (\omega_w \vec{\nabla} \psi_w - (1 - hav(\varepsilon_V)) \vec{g}) \right) - \frac{\partial \rho_{ns}}{\partial t} \quad (32)$$

The law of thermal energy conservation was written taking into account earlier assumptions about neglecting pressure gradient $\vec{\nabla} P$, neglecting the motion of matter in the gas phase

and $\vec{F}_i^{c,ws} = \vec{g}$. Also we used expression (15) from the second part of this article [2]:

$$\rho_{ws} \frac{dh_{ws}}{dt} + \rho_{sk} \frac{dh_{sk}}{dt} = -\nabla \cdot (\vec{J}_q) - \nabla \cdot (\vec{j}_w^{ws} (h_w^{ws} - h_s^{ws})) - \rho_{ws} \vec{W}_{ws} \cdot \vec{F}_{fr}^{ws} - \vec{j}_w^{ws} \cdot \vec{\nabla} \psi_w^{evf} \quad (33)$$

where h_{ws} , h_{sk} – enthalpy of water-salt solution and soil skeleton, J/kg; ρ_{sk} – density of soil skeleton; h_w^{ws} , h_s^{ws} – enthalpy of water and dissolved salt, J/kg.

The value of dissipative processes in soils is small, because the velocity of the solution is small too, i.e. $\rho_{ws} \vec{W}_{ws} \cdot \vec{F}_{fr}^{ws} \approx 0$, $\vec{j}_w^{ws} \cdot \vec{\nabla} \psi_w^{evf} = \vec{j}_w^{ws} \cdot \vec{\nabla} \psi_w \approx 0$. Using the correlation between isobaric heat capacity and enthalpy, the following equation was derived:

$$\left(\rho_{sk} c_p^{sk} + \rho_w c_p^w + \rho_{ice} c_p^{ice} + \rho_s c_p^s + \rho_{ns} c_p^{ns} \right) \frac{\partial T}{\partial t} = \nabla \cdot (\lambda_T \vec{\nabla} T) - \{ \vec{j}_w^{ws} (c_p^w - c_p^s) + \vec{j}_{ws} (\omega_w c_p^w + \omega_s c_p^s) \} \vec{\nabla} T + \left(L_w + (c_p^w - c_p^{ice})(T - T_{wi}) \right) \frac{\partial \rho_{ice}}{\partial t} + L_s \frac{\partial \rho_{ns}}{\partial t}, \quad (34)$$

where c_p^{sk} , c_p^w , c_p^{ice} , c_p^s , c_p^{ns} – heat capacity at constant pressure of the soil skeleton, water, ice, dissolved salt, J/(kg·K); L_w – the heat of the phase transition water → ice at atmospheric pressure, J/kg; T_{wi} – temperature of the phase transition water → ice at atmospheric pressure, K.

The coefficient of thermal conductivity of the soil is calculated by the following expression:

$$\lambda_T = \lambda_T^{th} \frac{\rho_w}{\rho_w + \rho_{ice}} + \lambda_T^{fr} \frac{\rho_{ice}}{\rho_w + \rho_{ice}} \quad (35)$$

where λ_T^{th} , λ_T^{fr} – coefficient of thermal conductivity of thawed and frozen soil, W/(m·K).

The generation rate of ice is described by the chemical reaction equation of the first order:

$$\frac{\partial \rho_{ice}}{\partial t} = \frac{\Omega_w - \Omega_{ice}}{\tau_{\Omega 1}} \cdot hav(\Omega_w - \Omega_{ice}) \cdot hav(\rho_w) + \frac{\Omega_w - \Omega_{ice}}{\tau_{\Omega 2}} \cdot (1 - hav(\Omega_w - \Omega_{ice})) \cdot hav(\rho_{ice}), \quad (36)$$

where $\tau_{\Omega 1}$ – the parameter of water crystallization, J·s·m³/kg²; $\tau_{\Omega 2}$ – the parameter of ice melting, J·s·m³/kg²; $\Omega_w - \Omega_{ice}$ – difference of the total thermodynamic potentials of water and ice, J/kg.

Equation (36) take into account the possibility of water → ice phase transitions only below the freezing point. The parameter water crystallization is estimated by the values $\tau_{\Omega 1} = 1000 - 100000$ J·s·m³/kg². Phase transitions ice → water possible only at a temperature above the freezing point, and the process is characterized by the parameter of ice melting $\tau_{w2} \rightarrow 0$. The Heaviside function allow to take into account the

impossibility of negative values of water or ice content in soil. The difference of the total thermodynamic potentials of water and ice was written as follows:

$$\Omega_w - \Omega_{ice} = \psi_w + \mu_w^c - \frac{L_w(T-T_{wi})}{T_{wi}} \quad (37)$$

The generation rate of undissolved salt, like ice, is described by the chemical reaction equation of the first order:

$$\frac{\partial \rho_{ns}}{\partial t} = \frac{\rho_{ws}(\omega_s - \omega_s^{max})}{\tau_{s1}} \cdot hav(\omega_s - \omega_s^{max}) \cdot hav(\rho_s) + \frac{\rho_{ws}(\omega_s - \omega_s^{max})}{\tau_{s2}} \cdot (1 - hav(\omega_s - \omega_s^{max})) \cdot hav(\rho_{ns}) \quad (38)$$

where ω_s^{max} – maximal concentration, u.f; τ_{s1} – relaxation time for undissolved salt crystallization, s; τ_{s2} – relaxation time for undissolved salt dissolution, s.

Maximal concentration of NaCl+H₂O solution as the function of temperature is described by following expression:

$$\omega_s^{max} = \frac{(T-273)+5523}{21000} hav(T - 273) + \frac{(T-252)+163.1}{700} hav(T - 252)(1 - hav(T - 273)) \quad (39)$$

The system of differential equation of heat and mass transfer (31), (32), (34), (36), (38) could be calculated by the finite difference method with implicit scheme. The correlation between time step and space steps, providing stability and convergence of solution, was shown in the article [3].

CONCLUSION

The authors of the paper derived the system of five differential equations (31), (32), (34), (36), (38) which describes the transfer of heat, water and salts, crystallization and melting of ice, crystallization and dissolution of salts from supersaturated solutions. Equation system allow describing the motion of water in a zone of intensive ice generation and forecast frost heaving of the soil.

The equation for the transfer of water and salt takes into account the convective motion of water-salt solution under the action of soil-water potential gradient and gravity, and diffusion motion under the action of concentration gradient. In addition, the equation takes into account the dynamic change of water level. The ice generation equation and undissolved salt generation equation take into account the kinetics of crystallization, melting and dissolution with the help of first order chemical reaction equations.

Practical value of the work lies in the possibility of direct application in the practice of mathematical modeling of differential equations (31), (32), (34), (36), (38). For this, it is necessary to know the classical parameters of heat capacity and thermal conductivity of the soil and its components. In addition, it is necessary to carry out a study of the dependence of the capillary-sorption potential of water on the water

content in the soil. The methodology of this study is well developed [4].

Practical significance of article consist in possibility of direct application of differential equations (31), (32), (34), (36), (38) in the practice of mathematical modeling. For this, it is necessary to know the classical parameters of heat capacity and thermal conductivity of the soil. In addition, it is necessary to measure water content for two soil-water potential values ($\psi_w = -1500$ J/kg and $\psi_w = -33$ J/kg). The methodology of this measure is well developed [4].

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