

Using Entropy and 2-D Correlation Coefficient as Measuring Indices for Impulsive Noise Reduction Techniques

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Abstract

Several measuring indices have been used in the literature of image restoration to examine the performance of the proposed schemes. In this paper, entropy and two-dimensional correlation coefficient have been studied and used as performance measuring indices for image quality. An adaptive two-stage algorithm for restoration of images corrupted by impulse noise proposed in one of our previous works is used in the simulation experiments. The impulse noise used is also called salt and pepper noise which is typically introduced due to sharp and sudden disturbance in the image. Several images of different characteristics that have been subject to a wide range of noise densities are tested. Experimental results show the superiority of such indices in measuring the performance of the adaptive filtering scheme to reduce the noise corrupting the images.

Keywords: Entropy, Correlation coefficient, Correlation factor, Image restoration, Noise reduction, Impulse noise.

INTRODUCTION

Images are often contaminated by several types of noise such as impulse noise, Gaussian noise, speckle noise and Poisson noise. Impulse noise, in particular, could seriously degrade the image quality in digital cameras or video sequences of DTV in several processes such as image acquisition, recording, and transmission. Moreover, atmospheric disturbances and household electrical appliances normally cause impulsive noise contaminating standard television broadcast signals. It is of a crucial importance to reduce noise in the images before any subsequent processing such as image segmentation, edge detection, and object recognition.

Various methods have been proposed over the years in the literature of image restoration to remove this type of noise [1-7]. Several performance measuring indices have been used in these proposed methods such as Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR), Mean Absolute Error (MAE), and Structural Similarity Index (SSIM). In this paper, two different measuring indices are studied and examined using our previous work [5], namely the entropy and the two-dimensional correlation coefficient.

Entropy is one of statistical measures of randomness that can be used to characterize the texture of images. It has been used by several researchers in their work [8-16]. In [8], a robust channel-calibration technique based on weighted minimum entropy is introduced for the multi-channel in azimuth high-resolution and wide-swath synthetic aperture radar imaging. In [9], the proposed new similarity index for images takes into consideration an entropy-based function and group theory. In [10], a small-target detection scheme based on weighted image entropy is discussed. This scheme aims to improve the signal to noise ratio through weighting the local entropy measure by the multiscale grayscale difference followed by an adaptive threshold operation. The framework introduced in [11] is an entropy-based trilateral filter that integrates a geometric, radiometric, median-metric information, and an entropy function to balance the contribution between the weights. Local intensity variations on each pixel are detected using an entropy-based function.

In the proposed adaptive recursive median filter (ARMF) [12], the size of the sliding window varies adaptively based on the entropy value of that moving (sliding) window. The proposed scheme in [13] uses an entropy-based performance measuring index to evaluate context models employed to code wavelet-transformed images. An algorithm called local mutation weighted information entropy (LMWIE) is proposed in [14] to detect small infrared targets under the complex background of the image. An entropy-based method with a new range kernel which contains a new range distance is proposed in [15]. The new range distance is robust to noise by exploiting the information from the difference between the noisy image and its restored estimate. Local statistics of images have been taken into consideration by applying local entropy to adaptively select the range distances.

A fast method for the computation of the Structural Similarity (SSIM) index for images contaminated by a global distortion using an entropy-based formula is introduced in [16].

The 2-D correlation coefficient is typically used to detect similarities between 2-D signals, which are often saved in matrices. This correlation coefficient between two matrices, that represent two images of the same size, has been examined by some researchers in the area of image processing.

A novel image restoration scheme is proposed in [17] based on the discrete 2-D wavelet transform and the correlation coefficient. In this scheme, the reference image and the noisy image are decomposed into several levels by wavelet transform. The correlation coefficients are then calculated between the reference sequences and the comparative sequences (the approximation and detail coefficients of the reference image and the noisy image, respectively).

An adaptive interpolation-based function is used in the proposed algorithm in [18] to eliminate impulse noise. The proposed method is examined using several performance measuring indices, one of which is the correlation factor.

This paper is outlined as follows. The next section describes and analyzes the entropy and the two-dimensional correlation coefficient as performance measuring indices for image quality. Then, experimental results of using such indices in a specific adaptive filtering scheme are presented. The conclusion is drawn in the last section.

ENTROPY AND 2-D CORRELATION COEFFICIENT

Many existing image quality indices can be represented as mathematical quantities. These include Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR), Mean Absolute Error (MAE), and Root Mean Squared (RMS) error. In this paper, two different measuring indices are studied and examined, namely the entropy and the two-dimensional (2-D) correlation coefficient.

Entropy

The entropy of an image is the average information generated from its pixels. It can also be defined as a statistical measure of the randomness that can be used to characterize the texture of that image.

The entropy E of an image is defined as

$$E = - \sum_0^{N-1} p_k \log_2(p_k)$$

where N is the number of gray levels and p_k is the probability associated with gray level k .

The maximum value of entropy is obtained when the probability distribution is uniform. In other words, if $N = 2^n$, then p_k is constant and given by

$$p_k = \frac{1}{N} = 2^{-n}$$

The maximum value of entropy is then computed as

$$E = - \sum_0^{N-1} 2^{-n} \log_2(2^{-n}) = -\log(2^{-n}) = n$$

The minimum value of entropy is obtained when the image itself is constant with little contrast, i.e., if all the pixels have the same gray levels k . For that gray level, $p_k = 1$, and $E = -\log(1) = 0$.

2-D Correlation Coefficient

The 2-D correlation coefficient between images A and B is defined as

$$R = \frac{\sum_m \sum_n (A_{mn} - \bar{A})(B_{mn} - \bar{B})}{\sqrt{(\sum_m \sum_n (A_{mn} - \bar{A})^2) (\sum_m \sum_n (B_{mn} - \bar{B})^2)}}$$

where A_{mn} is the intensity of the (m,n) th pixel in image A , B_{mn} is the intensity of the (m,n) th pixel in image B , \bar{A} is the mean (average) intensity of image A , and \bar{B} is the mean (average) intensity of image B .

The above mentioned measuring indices are applied to our previous work [5]. In this image noise reduction method, a sliding window of adaptive size $W \times W$ is used where W is computed based on an estimated value of the impulsive noise density corrupting the image being examined. Many researchers proposed different techniques to restore images corrupted with impulsive noise [15-18].

The size of the sliding window used in [5] is determined based on the estimated value of the noise density in the image, denoted by η , which can be defined as

$$\eta = \frac{K}{M \cdot N}$$

where K is the number of pixels which are determined to be noisy in the first stage of the method, and the product $(M \cdot N)$ is the total number of image pixels. The size of the sliding window is then computed as follows [5].

$$W = 2 \left\lceil \sqrt{\frac{2.2}{1 - \eta}} \right\rceil + 1$$

where $W = \lfloor X \rfloor$ is the floor value of X .

EXPERIMENTAL RESULTS

A variety of computer simulations are carried out to test the performance of the indicated image restoration method using two indices: entropy and two-dimensional correlation coefficient. Many images corrupted by a wide range of impulsive noise densities have been tested. However, results obtained as a result of applying the adaptive scheme on two images of different characteristics and levels of details, namely Lena and Bridge, are shown below.

Table 1 shows the entropy of the original clean Lina image (T), entropy of the same image after being corrupted by noise

(A), and entropy of the filtered image (C), i.e., entropy of the output image obtained as a result of applying the image restoration scheme.

Table 2 shows the correlation coefficient of the original clean Lina image (T) and the noisy image (A), of the filtered image (C) and the noisy image (A), and of the filtered image (C) and the original clean image (T). This table clearly shows that even for high noise densities, the correlation coefficient between the restored signal and the original signal is very close to 1 which demonstrates the effectiveness of the filtering method in reducing the noise.

Figure 1 shows a plot of the difference between the entropy of the filtered image and that of the noisy image versus the noise density. As the noise density increases this difference between the entropies increases.

Figure 2 shows the entropy of both the noisy image and the filtered image. The figure depicts that as the noise density increases, the entropy of the noisy image severely decreases while the entropy of the restored image slightly decreases which indicates the superiority of the method in reducing the noise.

Figure 3 shows the original clean Lina image, noisy image with noise density =30%, and the restored image respectively, and their corresponding values of entropy and 2-D correlation coefficient.

Table 1. Entropy of the original clean image (T), noisy image (A) corrupted with a wide range of noise densities, filtered image (C). (Lina image is used)

η	W	Entropy of T	Entropy of A	Entropy of C
10%	3	7.4451	7.2660	7.4423
20%	3	7.4451	6.8805	7.4396
30%	3	7.4451	6.3848	7.4370
40%	3	7.4451	5.8477	7.4366
50%	5	7.4451	5.2209	7.4209
60%	5	7.4451	4.5492	7.4167
70%	5	7.4451	3.8216	7.4150
80%	7	7.4451	3.0120	7.3964
90%	9	7.4451	2.1081	7.3737

Table 2. Correlation coefficient of the original clean image (T), noisy image corrupted with a wide range of noise densities (A), and filtered image (C). (Lina image is used)

η	W	Correlation Coefficient (T, A)	Correlation Coefficient (C, A)	Correlation Coefficient (T, C)
10%	3	0.7073	0.7090	0.9992
20%	3	0.5350	0.5376	0.9983
30%	3	0.4147	0.4169	0.9972
40%	3	0.3243	0.3262	0.9947
50%	5	0.2451	0.2501	0.9906
60%	5	0.1869	0.1907	0.9884
70%	5	0.1322	0.1353	0.9845
80%	7	0.0840	0.0875	0.9741
90%	9	0.0393	0.0414	0.9580

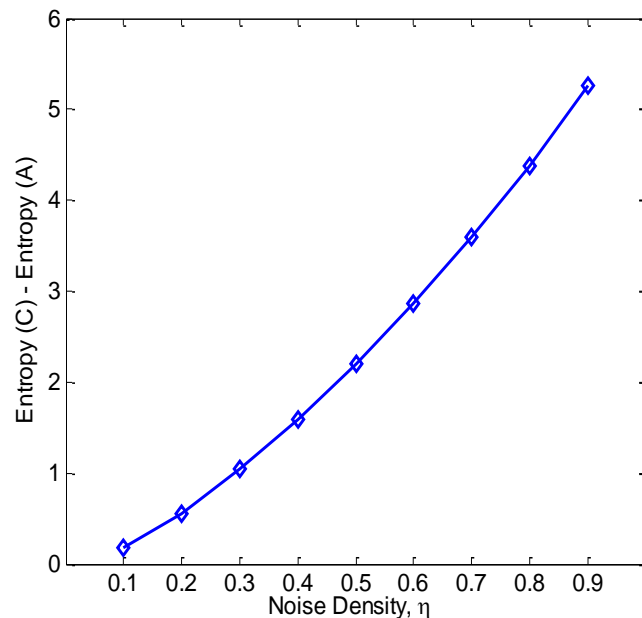


Figure 1. Entropy difference between the filtered Lina image and the noisy image versus noise density.

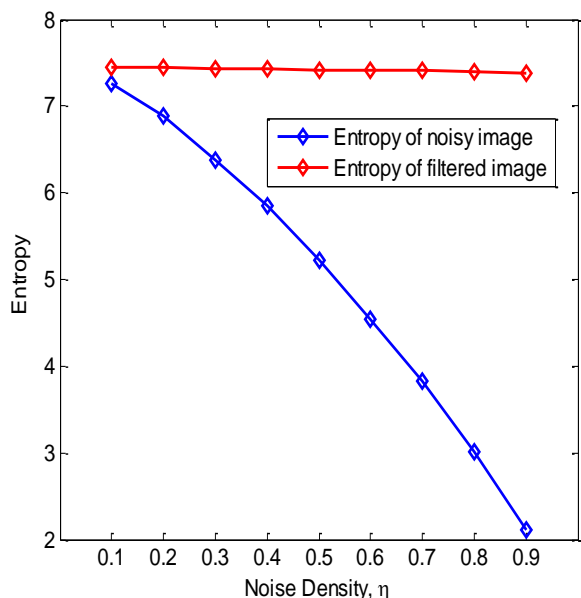


Figure 2. Entropies of the filtered Lina image and the noisy image versus the noise density.

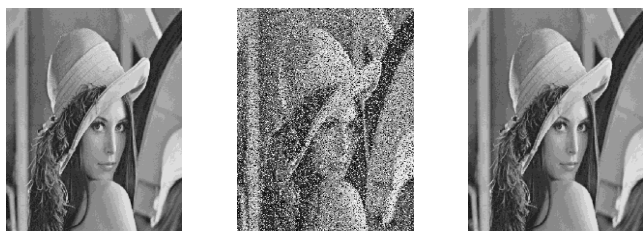


Figure 3. From Left to right: $(E, R) = (7.4451, 0.4147)$, $(6.3848, 0.4169)$, $(7.4370, 0.9972)$ for the original clean Lina image, noisy image with $\eta=30\%$, and restored image, respectively.

Similar experiments were carried out on another image (Bridge image) and similar results to those in Table 1 and Table 2 are shown in Table 3 and Table 4, respectively. Figure 4 shows the 2-D correlation coefficient R of the filtered image (C) and the noise image (A), i.e., $R(C, A)$, versus noise density, η . While η increases, $R(C, A)$ decreases as shown in this figure, note that the correlation coefficient of the filtered image (C) and the original clean image (T), i.e., the value of $R(C, T)$, is still close to one even for high values of noise density as shown in Table 4.

Table 3. Entropy of the original clean image (T), noisy image corrupted with a wide range of noise densities (A), filtered image (C). (Bridge image is used)

η	W	Entropy of T	Entropy of A	Entropy of C
10%	3	7.6830	7.4756	7.6803
20%	3	7.6830	7.0590	7.6779
30%	3	7.6830	6.5543	7.6751
40%	3	7.6830	5.9673	7.6738
50%	5	7.6830	5.3307	7.6487
60%	5	7.6830	4.6501	7.6428
70%	5	7.6830	3.8848	7.6354
80%	7	7.6830	3.0577	7.6052
90%	9	7.6830	2.1276	7.5770

Table 4. 2-D Correlation coefficient, R , of the original clean image (T), noisy image (A) corrupted with a wide range of noise densities, and filtered image (C). (Bridge image is used)

η	W	$R(T, A)$	$R(C, A)$	$R(T, C)$
10%	3	0.7454	0.7489	0.9965
20%	3	0.5792	0.5836	0.9932
30%	3	0.4541	0.4606	0.9894
40%	3	0.3556	0.3618	0.9841
50%	5	0.2755	0.2852	0.9729
60%	5	0.2061	0.2147	0.9661
70%	5	0.1449	0.1525	0.9575
80%	7	0.0934	0.1003	0.9383
90%	9	0.0476	0.0504	0.9124

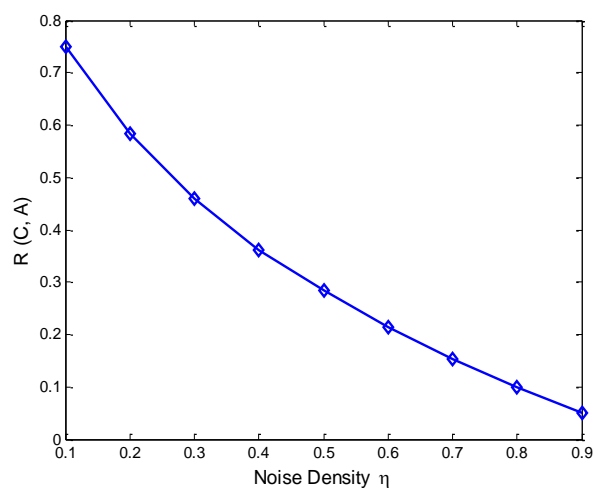


Figure 4. 2-D correlation coefficient of the filtered Bridge image and the noisy image, i.e., $R(C, A)$, versus the noise density.

Figure 5 shows the original clean Bridge image, noisy image with noise density =30%, and the restored image respectively, and their corresponding values of entropy and 2-D correlation coefficient.

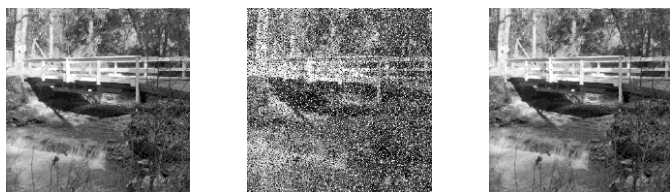


Figure 5. From Left to right: $(E, R) = (7.6830, 0.4565), (6.5491, 0.4615), (7.6754, 0.9894)$ for the original clean Bridge image, noisy image with $\eta=30\%$, and restored image, respectively.

CONCLUSION

In this paper, two indices are used to measure the performance of image denoising method. These indices are the entropy and two-dimensional correlation coefficient. Impulsive noise with a wide range of densities from 10% up to 90% have been used. A sliding window of adaptive size depending on an estimated value of noise density is used in the method. Simulation results are carried out using many images, two of which are shown in this paper. The entropy of the original clean image, after it is being corrupted, and of the filtered image are all obtained for each of these two images. Moreover, for each image, the two-dimensional correlation coefficient is computed between the clean image, noisy image, and the filtered image. Simulation results show the superiority of the chosen indices in measuring the quality of the filtered images.

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