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Unsteady Free Convection Flow of a Viscous Incompressible Polar Fluid past a Semi Infinite Vertical Porous Moving Plate

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Abstract

This paper considers the dynamics of unsteady free convection flow of a viscous incompressible polar fluid past a semi-infinite vertical porous flat moving plate which is subjected to move with a uniform velocity in the upward direction in its own plane and the free stream velocity follows an exponentially increasing or decreasing small perturbation law. The porous surface absorbs the polar fluid with a suction velocity varying with time. The governing equations for velocity, angular velocity and temperature of the flow field have been formulated and solved employing perturbation technique. The effects of the pertinent flow parameters on the velocity, angular velocity and temperature of the flow field across the boundary layer have been analyzed with the help of figures.

Keywords: unsteady, free convection, viscous, polar fluid, moving plate.

INTRODUCTION

Flows and heat transfer in fluids past a porous plate have several applications in various fields of science, engineering and technology such as in plasma studies, nuclear reactors, oil exploration, oceanic circulation, atmospheric studies, geothermal energy extractions and in the studies of boundary layer control in the field of aerodynamics. Polar fluids are fluids with microstructure and belong to a class of fluids with non-symmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium (Lukaszewicz [1]).

A limited number of researchers have carried out their investigations on polar fluids under different physical conditions. Das and his associates [2], Das and Tripathy [3], Das and his coworkers [4] have analyzed MHD/ non-MHD flow past a vertical porous plate under different physical situations. Panda *et al.* [5] discussed the MHD free convection transient flow past an infinite vertical porous flat plate in presence of mass transfer. Char and Chang [6] estimated the effect of wall conduction on natural convection flow of micropolar fluids along a flat plate.

Helmy [7] discussed the state space approach to unsteady free convection flow of a micropolar fluid. Chang [8] approached numerically the simulation of the natural convection plume about a line heat source in micropolar fluid. Chaudhary and Jain [9] studied the effect of combined heat and mass transfer on the flow of a magneto-micropolar fluid from radiate surface with variable permeability in the slip-flow regime. Patil and Kulkarni [10] estimated the effect of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation.

The objective of this study is to investigate the dynamics of an unsteady free convection flow of a viscous incompressible polar fluid past a semi-infinite vertical porous flat moving plate which is subjected to move with a uniform velocity in upward direction in its own plane and the free stream velocity follows an exponentially increasing or decreasing small perturbation law. The plate absorbs the polar fluid with a suction velocity varying with time. The governing equations of the flow field have been solved employing perturbation technique. The effects of the pertinent flow parameters on the velocity, angular velocity and temperature of the flow field across the boundary layer have been analyzed with the help of figures.

FORMULATION OF THE PROBLEM

Consider an unsteady flow of a viscous incompressible polar fluid past a semi-infinite vertical porous moving plate in the presence of a pressure gradient. Due to the semi-infinite plane surface assumption, the flow variables are functions of y^* and t^* only. It is assumed that the porous plate moves with constant velocity ($u_p{}^*$) in the longitudinal direction and the free stream velocity ($U_\infty{}^*$) follows an exponentially increasing or decreasing small perturbation law. Under these conditions, the governing equations, i.e., the mass, momentum and energy conservation equations can be written in a Cartesian frame of reference, as:

Continuity:

$$\frac{\partial v^*}{\partial y^*}=0, \tag{1}$$

Linear momentum:

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}+(\nu+\nu_r)\frac{\partial^2 u^*}{\partial y^{*2}}+g\beta_f(T-T_\infty)+2\nu_r\frac{\partial \omega^*}{\partial y^*}, \tag{2}$$

Angular momentum:

$$\rho j^*\left(\frac{\partial \omega^*}{\partial t^*}+v^*\frac{\partial \omega^*}{\partial y^*}\right)=\gamma\frac{\partial^2 \omega^*}{\partial y^{*2}}, \tag{3}$$

Energy:

$$\frac{\partial T}{\partial t^*}+v^*\frac{\partial T}{\partial y^*}=\alpha\frac{\partial^2 T}{\partial y^{*2}}, \tag{4}$$

where x^* and y^* are the dimensional distances longitudinal and perpendicular to the plate respectively, u^*, v^* the components of dimensional velocities along x^* and y^* directions respectively, t^* the time, ρ the density, ν the kinematic viscosity, ν_r the kinematic rotational viscosity, g the acceleration due to gravity, β_f the coefficient of volumetric thermal expansion of the fluid, j^* the micro-inertia density, ω^* the component of the angular velocity vector normal to xy-plane, γ the spin-gradient viscosity, T the temperature, and α is the effective fluid thermal diffusivity and the subscripts p, w, ∞ denote the condition on plate, wall and the free stream respectively.

The heat due to viscous dissipation is neglected for small velocities in equation (4). We assume that the plate temperature (T) and suction velocity (v^*) vary exponentially with time. Under these assumptions, the appropriate boundary conditions for velocity and temperature fields are

$$u^*=u_p^*, T=T_w+\varepsilon(T_w-T_\infty)e^{n^*t^*}, \frac{\partial \omega^*}{\partial y^*}=-\frac{\partial^2 u^*}{\partial y^{*2}} \text{ at } y^*=0$$

$$u^*\to U_\infty^*=U_0\left(1+\varepsilon e^{n^*t^*}\right), \ T\to T_\infty, \omega^*\to 0 \text{ as } y^*\to\infty \tag{5}$$

where n^* is a scalar constant, and U_0 is a scale of free stream velocity.

From the continuity equation (1), it is clear that the suction velocity normal to the plate is a function of time only and we shall take it in the form:

$$v^*=-V_0\left(1+\varepsilon A e^{n^*t^*}\right), \tag{6}$$

where A is the suction velocity parameter (a real positive constant), ε (<<1) and εA are small and less than unity and V_0 is a non-zero positive constant representing the scale of suction velocity. Outside the boundary layer, equation (2) gives

$$-\frac{1}{\rho}\frac{dp^*}{dx^*}=\frac{dU_\infty^*}{dt^*} \tag{7}$$

We now introduce the dimensionless variables, as follows:

$$u=\frac{u^*}{U_0}, v=\frac{v^*}{V_0}, y=\frac{V_0 y^*}{\nu}, U_\infty=\frac{U_\infty^*}{U_0}, U_p=\frac{u_p^*}{U_0},$$

$$\omega=\frac{\nu}{U_0V_0}\omega^*, t=\frac{t^*V_0^2}{\nu}, \theta=\frac{T-T_\infty}{T_w-T_\infty}$$

$$n=\frac{n^*\nu}{V_0^2}, j=\frac{V_0^2}{\nu^2}j^*,$$

$P_r=\frac{\nu}{\alpha}$ is the Prandtl number, $G_r=\frac{\nu\beta_f g(T_w-T_\infty)}{U_0V_0^2}$ is the Grashof number for heat transfer. (8)

Further, the spin-gradient viscosity γ which gives some relationship between the coefficients of viscosity and micro-inertia, is defined as

$$\gamma=(\mu+\frac{A}{2})j^*=\mu j^*(1+\frac{1}{2}\beta), \tag{9}$$

where $\beta=\frac{\Lambda}{\mu}$, the dimensionless viscosity ratio (10)

and Λ is the coefficient of gyro-viscosity (or vortex viscosity) and μ is fluid dynamic viscosity.

Using equations (6)-(10), the governing equations (2)-(4) reduce to the following non-dimensional form:

$$\frac{\partial u}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial u}{\partial y}=\frac{dU_\infty}{dt}+\left(1+\beta\right)\frac{\partial^2 u}{\partial y^2}+G_r\theta+2\beta\frac{\partial\omega}{\partial y} \quad (11)$$

$$\frac{\partial w}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial\omega}{\partial y}=\frac{1}{\eta}\frac{\partial^2\omega}{\partial y^2} \quad (12)$$

$$\frac{\partial\theta}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial\theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} \quad (13)$$

where $\eta=\frac{\mu j^*}{\gamma}=\frac{2}{2+\beta}$

The corresponding boundary conditions are reduced to

$$u=U_p,\theta=1+\varepsilon e^{nt},\frac{\partial\omega}{\partial y}=-\frac{\partial^2 u}{\partial y^2} \quad \text{at } y=0,$$

$$u\to U_\infty,\theta\to 0,\omega\to 0 \text{ as } y\to\infty \quad (14)$$

METHOD OF SOLUTION

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in non-dimensional form, we assume the linear and angular velocities and temperature as

$$u=u_0(y)+\varepsilon e^{nt}u_1(y)+O\left(\varepsilon^2\right)+\cdots \quad (15)$$

$$\omega=\omega_0(y)+\varepsilon e^{nt}\omega_1(y)+O\left(\varepsilon^2\right)+\cdots \quad (16)$$

$$\theta=\theta_0(y)+\varepsilon e^{nt}\theta_1(y)+O\left(\varepsilon^2\right)+\cdots \quad (17)$$

On substitution of equation (15)-(17) in equation (11)-(14) and equating the harmonic and non-harmonic term, neglecting the coefficient of O (ε^2), we get the following pairs of equations for (u_0,ω_0,θ_0) and (u_1,ω_1,θ_1).

$$\left(1+\beta\right)u_0''+u_0'=-G_r\theta_0-2\beta\omega_0' \quad (18)$$

$$\left(1+\beta\right)u_1''+u_1'-u_1 n=-n-Au_0'-G_r\theta_1-2\beta\omega_1' \quad (19)$$

$$\omega_0''+\eta\omega_0'=0 \quad (20)$$

$$\omega_1''+\eta\omega_1'-n\eta\omega_1=-A\eta\omega_0' \quad (21)$$

$$\theta_0''+P_r\theta_0'=0 \quad (22)$$

$$\theta_1''+P_r\theta_1'-nP_r\theta_1=AP_r\theta_0', \quad (23)$$

where the primes denote differentiation with respect to *y*.

The corresponding boundary conditions can be written as:

$$u_0=U_p,u_1=0,\omega_0'=-u_0'',\omega_1'=-u_1'',\theta_0=1,\theta_1=1 \text{ at } y=0$$

$$u_0=1,u_1=1,\omega_0\to 0,\omega_1\to 0,\theta_0\to 0,\theta_1\to 0 \text{ as } y\to\infty \quad (24)$$

The solutions of equations (18)-(23) with satisfying boundary conditions (24) are given by

$$u_0(y)=A_2e^{-P_r y}-A_3A_5e^{-\eta y}-A_4e^{-y}, \quad (25)$$

$$u_1(y)=A_7+A_8e^{-P_r y}+A_9e^{-\eta y}-A_{10}e^{-m_1 y}+A_{11}A_{13}e^{-m_3 y}+A_{12}e^{-y}+A_{14}e^{-\lambda_1 y}, \quad (26)$$

$$\omega_0(y)=A_5e^{-\eta y}, \quad (27)$$

$$\omega_1(y)=-A_6e^{-\eta y}+A_{14}e^{-m3y}, \quad (28)$$

$$\theta_0(y)=e^{-P_r y}, \quad (29)$$

$$\theta_1(y)=e^{-m1y}, \quad (30)$$

where,

$$m_1=\frac{1}{2}\left(P_r+\sqrt{P_r^2+4\eta P_r}\right),m_2=\frac{1}{2}\left(P_r+\sqrt{P_r^2+4\eta P_r}\right),$$

$$m_3=\frac{1}{2}\left(\eta+\sqrt{\eta^2+4n\eta}\right),m_4=\frac{1}{2}\left(-\eta+\sqrt{\eta^2+4n\eta}\right),$$

$$\lambda_1=\frac{1}{2}\left(1+\sqrt{1+4n(1+\beta)}\right),\lambda_2=\frac{1}{2}\left(-1+\sqrt{1+4n(1+\beta)}\right),$$

$$A_1=\frac{AP_r^2}{(m_1-P_r)(m_2+P_r)},$$

$$A_2=\frac{G_r}{P_r(1-P_r)},A_3=\frac{2\beta\eta}{\eta(1-\eta)},A_4=A_2-A_3A_5-U_p,$$

$$A_5=\frac{A_2P_r^2-\left(A_2-U_p\right)}{A_3\eta^2-A_3-\eta},A_6=\frac{A\eta^2A_5}{(m_3-\eta)(m_4+\eta)},$$

$$A_7=\frac{n}{\lambda_1\lambda_2},A_8=\frac{AA_2P_r}{(P_r-\lambda_1)(P_r+\lambda_2)},$$

$$A_9=\frac{\eta(AA_3A_5+2\beta A_6)}{(\lambda_1-\eta)(\lambda_2+\eta)},A_{10}=\frac{G_r(1-A_1)}{(m_1-\lambda_1)(m_1+\lambda_2)},$$

$$A_{11}=\frac{2\beta m_3}{(m_1-m_3)(m_3+\lambda_2)},A_{12}=\frac{AA_4^2}{(1-\lambda_1)(1+\lambda_2)},$$

$$A_{13}=\frac{\eta A_6+A_8P_r^2+A_9\eta^2-A_{10}m_1^2-A_{12}}{m_3-A_{11}\left(m_3^2+\lambda_1^2\right)}+\frac{\lambda_1^2\left(A_7+A_8+A_9-A_{10}-A_{12}\right)}{m_3-A_{11}\left(m_3^2+\lambda_1^2\right)},$$

$$A_{14}=A_7+A_8+A_9-A_{10}+A_{11}A_{13}-A_{12}. \quad (31)$$

Skin friction

The skin friction at the wall of the plate is given by

$$\tau_w = \frac{\tau_w^*}{\rho U_0 V_0} = \left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{32}$$

Using equations (15), (25) and (26) in equation (32), we obtain

$$\tau_w = -P_r A_3 + \eta A_4 A_6 + A_5 + \varepsilon e^{nt}\left(-P_r A_9 - \eta A_{10} + m_1 A_{11} - m_3 A_{12} - A_{13} - \lambda_1 A_{14}\right) \tag{33}$$

Rate of heat transfer

The rate of heat transfer or the heat flux at the wall in terms of Nusselt number is given by

$$N_u = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} \tag{34}$$

Using equations (17), (29) and (30) in equation (34), we get

$$N_u = -P_r - \varepsilon e^{nt} m_1 \tag{35}$$

DISCUSSIONS AND RESULTS

The problem of a two dimensional unsteady free convection flow of a viscous incompressible polar fluid past a semi-infinite vertical porous flat moving has been formulated. The governing equations of the flow field are solved employing perturbation technique. The effects of the pertinent flow parameters characterizing the velocity, angular velocity and temperature profiles of the flow field have been presented and analyzed with the aid of Figures 1-6.

Velocity field

The effect of the various flow parameters on the velocity profiles of the flow field is discussed with the aid of Figures 1-3. Figure 1 analyzes the effect of Grashof number for heat transfer G_r on the velocity profiles of the flow field. A growing Grashof number for heat transfer is found to have an accelerating effect on the velocity of the flow field near the plate thereafter the effect is very much insignificant. Figure 2 depicts the effect of exponential index n on the velocity profiles of the flow field. Comparing the curves of the figure, it is observed that an increase in exponential index has a retarding effect on the velocity of the flow field at all points. Figure 3 indicates the variation in the velocity of the flow field due to viscosity ratio β. Like exponential index, the viscosity ratio is also observed to retard the velocity of the flow field at all points.

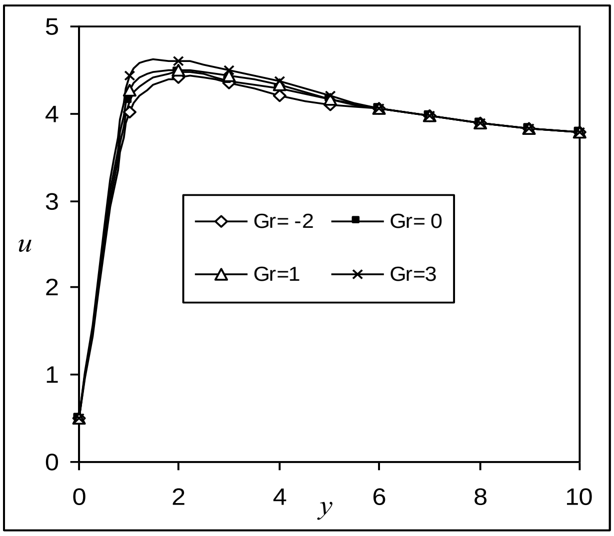

Figure 1: Effect of G_r on Velocity profiles against y

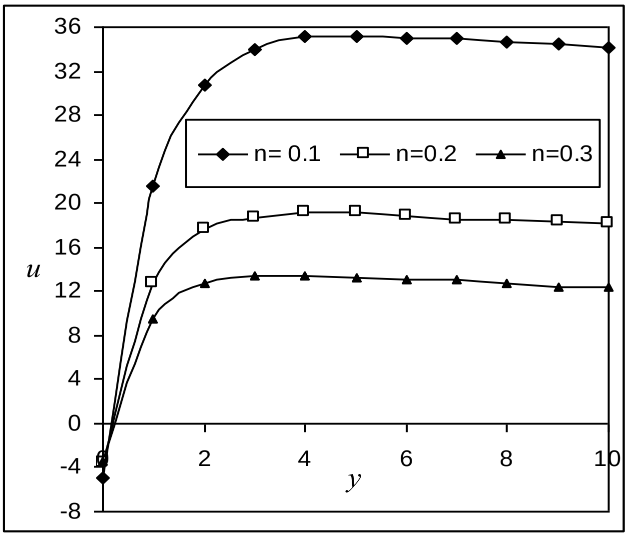

Figure 2: Effect of n on Velocity profiles against y

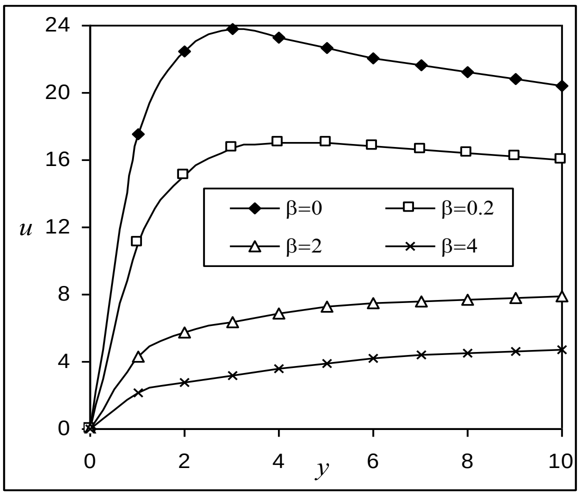

Figure 3: Effect of β on Velocity profiles against y

Angular velocity

Figures 4 and 5 elucidate the effect of flow parameters on the angular velocity of the flow field. Figure 4 shows the variation of angular velocity due to change in Grashof number for heat transfer G_r on the flow field. A growing Grashof number for heat transfer G_r is found to enhance the angular velocity of the flow field at all points. In Figure 5, we analyze the effect of viscosity ratio β on the angular velocity of the flow field. The viscosity ratio β has a retarding effect on the angular velocity profiles of the flow field.

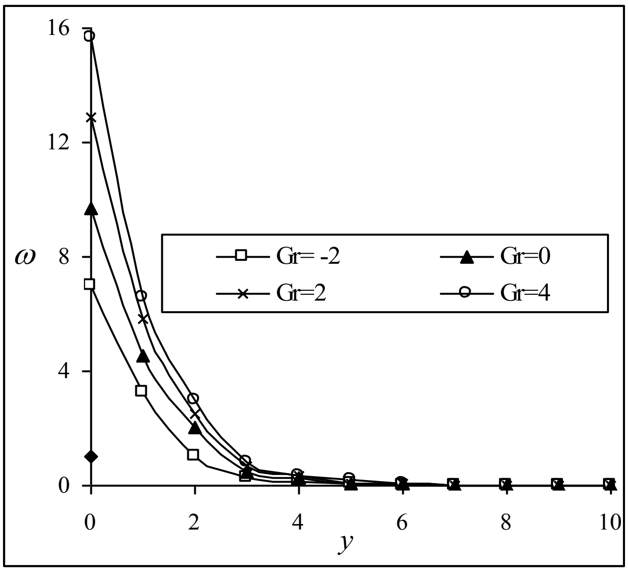

Figure 4: Effect of *Gr* on Angular Velocity profiles against y

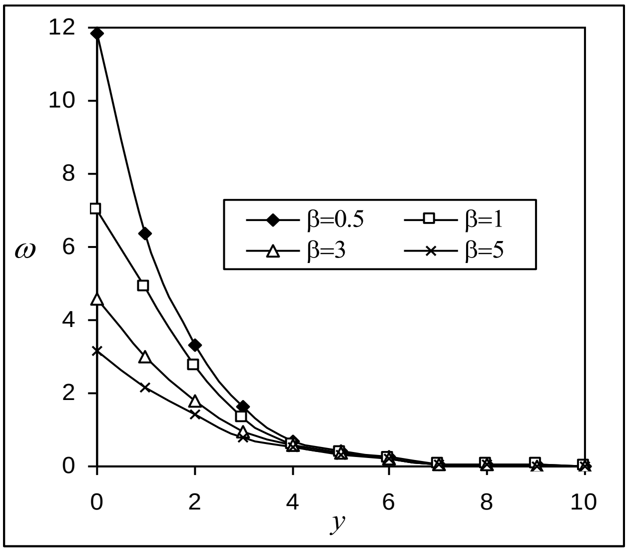

Figure 5: Effect of β on Angular Velocity profiles against *y*

Temperature field

Figure 6 depicts the variation of temperature of the flow field against *y* for different values of the Prandtl number P_r. On close observation of the curves of the figure, it is clearly seen that an increase in Prandtl number decreases the temperature of the flow field at all points.

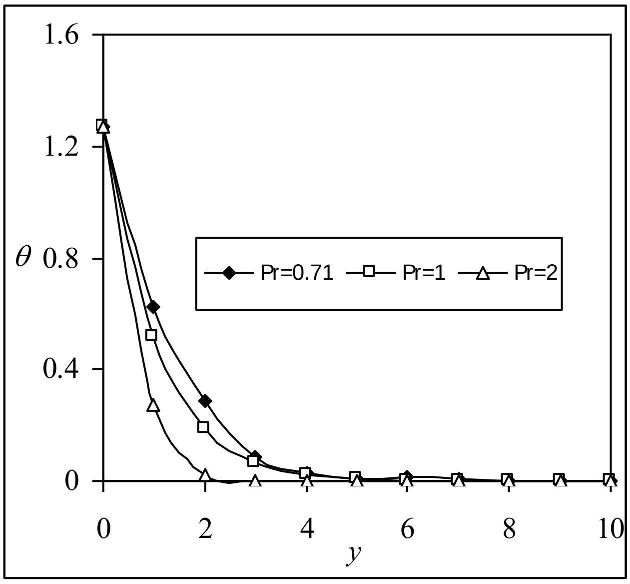

Figure 6: Effect of *Pr* on Temperature profiles against y

CONCLUSIONS

We summarize the following results of physical interest on the flow field from the above analysis:

1. A growing Grashof number for heat transfer G_r is found to have an accelerating effect on the velocity of the flow field near the plate thereafter the effect is very much insignificant.
2. An increase in exponential index *n* / viscosity ratio β has a retarding effect on the velocity of the flow field at all points.
3. A growing Grashof number for heat transfer G_r is found to enhance the angular velocity of the flow field at all points while the viscosity ratio β has a retarding effect on the angular velocity profiles of the flow field.
4. The effect of Prandtl number P_r is to decrease the temperature of the flow field at all points.

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