Optimization of Myoelectric Elbow Prosthesis Transmission

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Abstract

Limb prosthesis can be cosmetic or functional and latter active or passive. The active prosthetic arms can be body powered or have external source of power. Myoelectric elbows can profitably substitute the passive ones, with patient willing, only if they can guarantee: durability, low noise, adequate torque, low power consumption, low weight, easy motion control and natural movements. Most of these goals can be achieved by good electro-mechanical design. Currently two manufacturers share out most of the myoelectric elbows market but a few other producers are present and contributes to the improvement of the active prosthetic arm design. The evolution of the project of an Italian artificial elbow during the last ten years is shown and four different prostheses are compared in order to draw attention to the most critical aspects of the prosthetic design. The kinematic analysis of each device is described, highlighting the mechanical efficiency. A great amount of founds have been recently injected into this research area with the aim to design very sophisticated prosthetic upper limbs. Up to now no commercial products have been produced yet and only very few prototypes have been shown, but they are very promising for the future.

Keywords: Prosthesis, elbow, mechatronics, mechanism, mechanical efficiency

INTRODUCTION

The prosthetic arms, and consequently the artificial elbow joint, have a very long history [1]. Prostheses can be cosmetic or functional and since cosmetic prosthesis recover only a very limited portion of functionality of amputated limb, most of the prosthetic limbs are functional prostheses. They can be fixed or articulated and, in the latter case, the joints movement can be active or passive [2]. The passive motion of an elbow can be obtained, for instance, using the other arm and the position can be maintained by means of a friction between the joint surfaces. On the contrary, the active prostheses can be driven exploiting the patient energy (body powered), or by an external source of power (eg. battery) [3].

A good prosthesis design has to take into account all the problems related to the interaction between human and machines: since the very first need of an amputee is the social and physical rehabilitation, patients should have a good feeling with their prosthesis and they should be able to perform daily activities without stress and excessive mental load [4].

Despite the evolution of myoelectric prosthesis, body powered prostheses are still preferred by many patients. The main reasons are essentially the higher cost and less durability of myoelectric prostheses, also myoelectrically controlled prostheses require more maintenance and are not suitable for heavy works with respect to cable driven prosthesis [5]. Therefore, body powered prostheses, that usually employ cables for transmitting to the elbow the force produced by the motion of other natural joints, are still current and under development [6].

Patient refusal is the main cause limiting the use of active prostheses [7]. It can be related to many factors related to prosthesis features as: excessive weight, limited speed, noise, poor reliability and high power consumption. The latter means that patient has to carry big batteries or can not use the prosthesis for a long time. However, myoelectric prostheses are very appreciated for their easiness in controlling the movements, absence of control cables and being more cosmetic acceptance [8]. They usually exploit electromyographic signals of two residual antagonist muscles of the stump to command a single degree of freedom of the prosthesis.

As the myoelectric hand market is dominated by the German company Otto Bock [9][10], most of the prosthetic elbows market is shared by two American companies that produces the Utah Arm and the Boston Elbow. Their first commercial devices were designed about 30 years ago and their upgrades mainly include the electronic and the control [11][12]. The Boston Elbow is a myoelectrically controlled prosthesis and uses EMG signals from residual biceps and triceps muscles through electrodes which are located in prosthesis socket. The Utah Arm like the Boston Elbow is myoelectric and proportional but former has more attractive appearance and it is less noisy in comparison to the latter. The Utha Arm is completely free swing while The Boston Elbow has only 30 degrees of free swing. Boston Elbow weighs more but also it
is able to lift more weight, moreover both of them can use Otto Bock hands or hooks as terminal devices [13].

Non of above prostheses represent a complete arm while in 1990 the Edinburgh team began to design a prosthesis arm with electrically powered shoulder, elbow and hand. The Prosthesis could only be built for male adults amputees due to use of a lengthy linear recirculating ball-screw actuators driven by d.c motors. In 1998 the team developed another prototype for maximizing the number of amputees who can use the prosthesis arm. The mechanical transmission of this version was based on gearboxes coupled to worm gear and wheel linkages in order to be compact and have a good speed reduction ratio [15].

Another recent work in this field is “Revolutionizing Prosthetics program” which is doing by DARPA (The U.S Defense Advanced Research Projects Agency). Its aim is the development of a functional upper limb that responds to direct neural control. DARPA’s prosthetic arm aims to work much like a regular arm [16].

Up to now high tech new prototypes have been developed and some sophisticated upper limb components, including elbows, have been produced. Other new design approaches to the elbow system are oriented toward a more human-skeleton like structures for the forearm: for instance, the prosthesis designed at the Bioelectronic section of CINVESTAV-IPN Mexico [17] the forearm prono-supination, together with the elbow flexion, are obtained by four linear actuators, two of which replace ulna and radio physiological actions.

Beyond those already mentioned, other little groups, in the last decades, designed, tested and produced other artificial elbows and they mostly covered certain local markets.

This paper shows the evolution of the myoelectric elbows available on the Italian market highlighting the designers efforts in the development of the mechanical transmission to improve efficiency, increase performances and the patient comfort. In the following sections, 4 different prostheses are de- scribed: the oldest one was developed in 1973 for INAIL (Bologna) while the others were designed and realized by the Research Group of Man-Machine Systems of Politecnico di Milano in the following years. The work puts its basis on the idea that a good mechanical design is the key factor for a prosthetic device directed towards a functional and psychological rehabilitation of the amputee.

ELBOW PHYSIOLOGY AND PROSTHESIS REQUIREMENTS

A good mechanical design of a prosthesis must take into account the natural elbow physiology and the patient requirements in terms of comfort and easiness of use. The elbow allows two main different movements [19]:

The hinge-like bending and straightening of the elbow (flexion and extension) happens at the articulation ("joint") between the humerus and the ulna. Amplitude of this movement is about $135^\circ - 150^\circ$.

The complex action of turning the forearm over (pronation or supination) happens at the articulation between the radius and the ulna (this movement also occurs at the wrist joint). In the anatomical position (with the forearm supine), the radius and ulna lie parallel to each other. During pronation, the ulna remains fixed, and the radius rolls around it at both the wrist and the elbow joints. In the prone position, the radius and ulna appear crossed.

While the elbow prosthesis should reproduce the flexion-extension movement, pronation and supination is usually reproduced by a prothetic wrist.

Moreover the prosthesis should have some general requirements as:

- Comfort: the prosthesis must be noiseless, wearable, and the movements should reproduce the natural ones as much as possible.
- Weight and dimensions: the device should be light in order to be easily coupled to the stump without modify the patient posture (natural arm weight is approximately 3.2 kg). Moreover, the higher is the weight the motor has to move, the heavier is the battery the patient has to carry.
- Performance: the device should reproduce human movement with, at least, the same speed of a natural arm ($\approx 0.45 \text{ rad/s}$) in a physiological way. The patient should be able to carry a mass of at least 0.6 kg and the range of motion should be as wide as possible to allow the patient to reach objects in space.
- Reliability: the prosthesis should work at least 5000 hours/year without maintenance. Both mechanical and electrical components must work correctly, avoiding breaking and breakdown.
- High mechanical efficiency: to reduce power consumption and batteries dimensions, the transmission should guarantee high efficiency, in order to minimize loss of energy. Moreover a well designed and effective transmission is often silent because of reduced backlash.
- Backdriving: it must be avoided in order to reduce the motor workload when the patient is extending the artificial arm.

Appearance: since it has a very important social function, the device dimension should be the same of a natural arm and color of all the visible parts should reproduce human skin.

A good design of the prosthesis should start from these considerations.
THE STARTING POINT OF THE MYOELECTRIC ELBOW PROSTHESIS

The elbow prosthesis most used in Italy in ’90 is represented in Fig.9,10. The device is constituted by two D.C. motors (Tab.1) which are connected to a transmission realized with 5 couples of gears (Tab.2).

### Table 1: DC Motors features

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Power</td>
<td>3.1 [W]</td>
</tr>
<tr>
<td>Mechanical efficiency ($\eta$)</td>
<td>0.85</td>
</tr>
<tr>
<td>Maximum speed ($\omega$)</td>
<td>8000 [rpm]</td>
</tr>
<tr>
<td>Nominal torque ($T_{M,N}$)</td>
<td>$10^{-2}$ [Nm]</td>
</tr>
<tr>
<td>Momentum of Inertia ($J_M$)</td>
<td>1.92 [gcm²]</td>
</tr>
<tr>
<td>Weight</td>
<td>61 [g]</td>
</tr>
</tbody>
</table>

Two gears ($Z_1$) are linked with motors shafts and connected to a cog wheel ($Z_2$). It moves a worm gear ($Z_3$) and then a helical gear ($Z_4$). Three couples of helical gears ($Z_5 - Z_6$, $Z_7 - Z_8$, $Z_9 - Z_{10}$) reduce the angular speed and the rotation of $Z_{10}$ coincides with the elbow movement. To increase efficiency and reduce noise, gears are made of different materials, considering the different load they have to carry.

### Table 2: Gears features

<table>
<thead>
<tr>
<th>Wheel no.</th>
<th>Tooth</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$</td>
<td>14</td>
<td>steel</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>60</td>
<td>nylon</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>3</td>
<td>steel</td>
</tr>
<tr>
<td>$Z_4$</td>
<td>41</td>
<td>nylon</td>
</tr>
<tr>
<td>$Z_5$</td>
<td>12</td>
<td>steel</td>
</tr>
<tr>
<td>$Z_6$</td>
<td>60</td>
<td>aluminum</td>
</tr>
<tr>
<td>$Z_7$</td>
<td>19</td>
<td>copper</td>
</tr>
<tr>
<td>$Z_8$</td>
<td>27</td>
<td>steel</td>
</tr>
<tr>
<td>$Z_9$</td>
<td>12</td>
<td>steel</td>
</tr>
<tr>
<td>$Z_{10}$</td>
<td>35</td>
<td>steel</td>
</tr>
</tbody>
</table>

Global transmission ratio is:

$$\tau = \frac{Z_1 \cdot Z_2 \cdot Z_3 \cdot Z_4 \cdot Z_5}{Z_2 \cdot Z_3 \cdot Z_4 \cdot Z_5 \cdot Z_6} = \frac{1}{1214} \quad (1)$$

It is noted the choice of using spur gears, instead of helical ones, in the faster part of the gear reduction unit, is inconsistent with the need to have a low noise level. Also coupling between worm gear and helical gear ($Z_3 - Z_4$) makes a high transmission ratio and avoids backdrivability.

If the mechanical efficiency of each couple of gears were known, the global transmission mechanical efficiency could be calculated as:

$$\eta_{Inail} = \eta_{1-2} \cdot \eta_{3-4} \cdot \eta_{5-6} \cdot \eta_{7-8} \cdot \eta_{9-10} \quad (2)$$

Differently, to measure the mechanical efficiency of the transmissions, experimental tests has been carried out. Let’s consider:

$$W_1 = T_1 \omega_1; \quad W_2 = T_2 \omega_2; \quad (3)$$

$$\tau = \frac{\omega_2}{\omega_1}; \quad \eta = \frac{W_2}{W_1} \quad (4)$$

where $\tau$ is the transmission ratio, $W_1$, $W_2$ are the powers incoming and outcoming the transmission, $T_1$ and $T_2$ are the torques on the incoming and
Being known the transmission ratio \( \tau \), the transmission mechanical efficiency can be obtained applying a torque \( T_2 \) and measuring \( T_1 \):

\[
\eta = \frac{\tau}{T_2} \frac{T_1}{T_1} \tag{5}
\]

To measure the transmission mechanical efficiency, a test rig has been developed, its layout is shown in Fig. 3.

A known torque \( T_2 \) is applied on the prosthesis (-1-) outcoming shaft through a mass placed at a known distance (-2-), while the incoming shaft (-3-) is moved with constant speed by a linear actuator (-5,6-) through a flexible wire. Its tension, and therefore the torque \( T_1 \), is measured by a dynamometer (-4-) connected to a data acquisition system (-7,8-). Trials have been conducted with different velocities and loads to verify the mechanical efficiency is not affected by these two parameters. Its average value (considering 10 tests) is:

\[
\eta_0 = 0.078; \tag{6}
\]

As predictable, the long chain of gears conducts to a very low transmission efficiency. This is the starting point of a new mechanical design whose aim is to improve the performance of the device increasing the efficiency of the mechanism.

**FIRST PROSTHESIS PROTOTYPE**

The first artificial elbow developed has the aim of improving the performance of the commercial prosthesis previously used in Italy. Main disadvantages of the INAIL elbow prosthesis consist of its poor efficiency and its complex design. Power losses in transmission lead to very high power consumption and to the battery oversizing. Moreover, the long chain of gears results to be noisy. The first new design of the artificial elbow considered three important functional features:

1. a simple design allows to reduce costs and to increase the system reliability,
2. high transmission ratios should be reached without affecting the transmission efficiency,
3. backdriving has to be avoided without decreasing mechanical efficiency or adding new devices

Since brushless motor dimension roughly depends by its nominal torque, using small actuators the available torque is very low, while rotational speed is high. To obtain high torques and low speeds, as required by the application, a large transmission ratio is needed. Such a transmission can be obtained using suitable mechanism. Since the dimension of the device should be as small as possible, a linkage is designed along prosthesis longitudinal axis (Fig. 4, 5). It means the motion has to be transmitted from the motor’s shaft (parallel to longitudinal axis) to the elbow axis (normal to longitudinal one), having a large transmission ratio and good mechanical efficiency. A good result can be achieved designing a linkage in series to a ball-screw as shown in Fig. 4, 5.

This configuration reduces the loss of energy related to the friction in revolute pairs and ball-screw. The system allows a \( 130^\circ \) flexion-extension movement.

**Kinematical Analysis**

The analysis of the transmission kinematic can be carried out considering the system as made of two mechanism in series: a crank and a four bar linkage.

Considering the scheme shown in Fig. 6, the equation of closure for the crank is:

\[
ce^{i\gamma} = t_1 e^{i\gamma/2} + xe^{i\pi} + be^{i\beta} \tag{7}
\]
that can be projected on real and imaginary axes:

\[
\begin{align*}
& c \cos \gamma = t_1 - x + b \cos \beta \\
& c \sin \gamma = t_2 + b \sin \beta
\end{align*}
\] (8)

Squaring both equation and summing to eliminate \(\beta\):

\[
\begin{align*}
& c^2 + t_1^2 + t_2^2 - b^2 + x^2 - 2t_1x - 2cx + 2ct_1 \cos \gamma - 2ct_2 \sin \gamma + 2cx \cos \gamma = 0
\end{align*}
\] (9)

One can reach the \(\gamma\) angle as a function of \(x\):

\[
\gamma = 2 \text{atan} \frac{-b \pm \sqrt{b^2 - 4Ac}}{2A}
\] (10)

where:

\[
\begin{align*}
A &= c^2 + t_1^2 + t_2^2 - b^2 + x^2 - 2t_1x - 2cx + 2ct_1 \\
B &= -4ct_2 \\
C &= A - 4ct_1 + 4cx
\end{align*}
\] (11)

Deriving Eq. (8) with respect to time one gets:

\[
\begin{align*}
& c\dot{\gamma} \sin \gamma = \dot{x} + b\dot{\beta} \sin \beta \\
& c\dot{\gamma} \cos \gamma = b\dot{\beta} \cos \beta
\end{align*}
\] (12)

and then:

\[
\begin{align*}
& \dot{\beta} = \frac{c \cos \gamma}{b \cos \beta} \dot{\gamma} \\
& \dot{\gamma}(c \sin \gamma - c \cos \gamma \tan \beta) = \dot{x}
\end{align*}
\] (13)
the transmission ratio of the crank can be achieved as:

\[ \tau_1 = \frac{1}{x} = \frac{1}{c(\sin \gamma - \cos \gamma \tan \beta)} \]  

(14)

Let’s consider the four bar linkage. The equation of closure is:

\[ ce^{i\gamma} + de^{ib} = t_3e^{i\pi/2} + fe^{i\phi} \]  

(15)

that can be projected on real and imaginary axes:

\[
\begin{align*}
  c \cos \gamma - f \cos \varphi &= -d \cos \delta \\
  -c \sin \gamma + f \sin \varphi + t_3 &= d \sin \delta
\end{align*}
\]  

(16)

Squaring both equation and summing to eliminate \( \delta \):

\[ c^2 + t_3^2 + f^2 - d^2 - 2cf \cos \gamma \cos \varphi - 2ct_3 \sin \gamma + 2t_3f \sin \varphi - 2cf \sin \gamma \sin \varphi = 0 \]  

(17)

One can reach the \( \varphi \) angle as a function of \( \gamma \):

\[ \varphi = 2 \tan^{-1} \frac{c \pm \sqrt{c^2 + 2ct_3 \sin \gamma}}{A - B} \]  

(18)

where:

\[ A = c^2 + t_3^2 + f^2 - d^2 - 2ct_3 \sin \gamma \quad B = -2cf \cos \gamma \]  

(19)

and then:

\[
\begin{align*}
  c \dot{\gamma} \sin \gamma + d \dot{\delta} \sin \delta &= f \dot{\phi} \sin \varphi \\
  c \dot{\gamma} \cos \gamma + d \dot{\delta} \cos \delta &= f \dot{\phi} \cos \varphi
\end{align*}
\]  

(20)

The transmission ratio of the four bar linkage is then:

\[ \tau_2 = \frac{\dot{\varphi}}{\dot{\gamma}} = \frac{c}{f} \left( \frac{\sin \gamma \cos \gamma \tan \delta}{\sin \varphi \cos \varphi \tan \delta} \right) \]  

(22)

Finally, the transmission ratio of the total mechanism, and therefore the transmission ratio between the motor and the elbow speed is:

\[ \tau = \tau_1 \tau_2 = \frac{\dot{\gamma}}{\dot{\varphi}} = \frac{1}{f} \left( \frac{\sin \gamma \cos \gamma \tan \delta}{\sin \varphi \cos \varphi \tan \delta} \right) \]  

(23)

Mechanism optimization

It is noted the designed mechanism has a transmission ratio that changes with the position of the elbow. In order to avoid sudden variation of arm speed, transmission ratio should be constant as much as possible and fluctuation must be limited. Therefore lengths of each element of the linkage and the angles they form have been optimized taking into account the dimension of the system. The two objective functions are:

\[ F_1 = \frac{\max \tau - \min \tau}{\max \tau} \]  

(24)

\[ F_2 = \max \left| \frac{\partial \tau}{\partial \gamma} \right| \]  

(25)

where \( \gamma \) is the flexion-extension angle of the elbow.

The two functions represent respectively the percentage variation of the transmission ratio and the maximum inclination of the curve as a function of the angle \( \gamma \). Assuming the motor speed is constant, the variations in the speed of the elbow are small enough to be accepted (Fig. 7).

![Figure 7: Optimized transmission ratio](image_url)
A good parameter to evaluate the linkage capability to transmit motion is the angle of transmission. It is defined as the angle between the direction of the transmitted force and that related to the speed of the moving link. The lower is the transmission angle, the lower is the mechanical efficiency. In four-bar linkages it’s the smaller angle between connecting rod and rocker arm directions. The adopted linkage is made of two four-bar mechanisms in series, whose transmission angles are $\alpha$ and $\beta$ (fig.8). When the transmission angle goes under $40^\circ$, the effects of backlashes and flexibility are amplified, causing vibrations and malfunctions. Figure 9 shows the trends of $\alpha$ and $\beta$ as a function of $\gamma$. Their values show the linkage is always able to transmit motion properly.

To obtain the desired transmission ratio, a small epicyclic reduction gear has been adopted to guarantee a discrete mechanical efficiency ($\tau = 1/14, \eta = 0.80$). Besides, backdriving is avoided by a couple of gears which have special corrections. They allow to have different efficiency for direct and reverse cases.

Mechanical efficiency is measured with the described apparatus. Its mean value is:

$$\eta_1 = 0.64;$$

Using the linkage in substitution of gears, mechanical efficiency has a remarkable increase. It permits to reduce batteries dimensions needed to move the prosthesis and to make the system more silent.

**SECOND PROSTHESIS PROTOTYPE**

A multi-stages transmission compromises mechanical efficiency, but is often required to have an high transmission ratio. Common brushless motors, in fact, supplies low torque at high speed. Moreover, their traditional shape does not allow to mount the motor itself on the same axis of the ball-screw. It implies to use a couple of gears to transmit the rotation. This problem is solved using a *pancake* brushless motor: its reduced dimensions and its different shape allows to simplify the design of the elbow. In order to exploit better this feature, some elements of the transmission system described in the previous section have been modified. The transmission is the most important element of the prosthesis and the performances of the system highly depends on how it works: since a multi-stages transmission is often required to have an high transmission ratio and to adequate the elbow speed to the motor one, it’s fundamental to realize a system characterized by high mechanical efficiency. To avoid a long chain of gears, usually used in this application, transmission is made by an innovative mechanism (Fig.10,11).

Figure 10: Elbow prosthesis 3D model

Figure 11: Prosthesis layout
As shown in Fig.11, the mechanism is made of two linkages in series: a crank and slotted link (1-6-3) and a four bar linkage (2-4-5-1). A “pancake” brushless motor, used because of its reduced dimension along longitudinal axis, is directly coupled with a ball-screw. The system (motor, ball-screw) swings around the joint (3) and the nut screw (6) is connected to the top of the follower bar (1-6). This element actuates the four bar linkage and elbow speed dovetails with angular velocity of the crank (2-4).

This kind of transmission allows to achieve some important results:

- high transmission ratio ($\tau \approx 1/140$)
- mechanical efficiency is very high ($\eta \approx 0.86$): it’s made possible because of reduced friction of the system (in joints and in the ball-screw) and the absence of gears.
- the system is constituted of few elements: it’s easy to be assembled and it has a good mechanical reliability;
- overall dimensions and weight are reduced thanks to motor dimension and mechanism configuration.

**Kinematical analysis**

A kinematical analysis of the system is needed to optimize the mechanism transmission ratio. To perform it, the system can be divided into two linkages: the four bar one and the crank and slotted link.

Referring to Fig. 13, it’s possible to substitute the four bar linkage with vectors:

$$z_1 = a \cdot e^{i\alpha} \quad z_2 = b \cdot e^{i\theta} \quad z_3 = c \cdot e^{i\gamma} \quad z_4 = d \quad (27)$$

and write the equation:

$$z_1 - z_2 - z_3 - z_4 = 0 \quad (28)$$

$$a \cdot e^{i\alpha} - b \cdot e^{i\theta} - c \cdot e^{i\gamma} - d = 0 \quad (29)$$

that can be projected on real and imaginary axes:

$$\begin{cases} b \cos \theta = a \cos \alpha - c \cos \gamma - d \\ b \sin \theta = a \sin \alpha - c \sin \gamma \end{cases} \quad (30)$$

Squaring both equation and summing to eliminate $\theta$:

$$a^2 - b_1^2 + c^2 + d^2 - 2ac \cos \gamma \cos \alpha - 2ac \sin \gamma \sin \alpha - 2ad \cos \alpha + 2cd \cos \gamma = 0 \quad (31)$$

and substituting:

$$A = -2ac \sin \gamma \quad B = -2ad - 2ac \cos \gamma \quad C = a^2 - b_1^2 + c^2 + d^2 + 2cd \cos \gamma$$

$$D = \sqrt{A^2 + B^2 - C^2}$$

it’s possible to highlight the function $\alpha = a(\gamma)$:

$$\sin \alpha = -\frac{AC-BD}{A^2+B^2} \quad \alpha = \arcsin \left( -\frac{AC-BD}{A^2+B^2} \right) \quad (33)$$

Angle $\theta$ can be obtained from equations (30)(4)(33):

$$\sin \theta = \frac{c \sin \gamma - a \sin \alpha}{b} \quad \cos \theta = \frac{d + c \cos \gamma - a \cos \alpha}{b} \quad (34)$$

from which:

$$\theta = \arctan \left( \frac{c \sin \gamma - a \sin \alpha}{d + c \cos \gamma - a \cos \alpha} \right) \quad (35)$$
Let's now consider the crank and slotted link. Referring to its geometry (fig.12), it's possible to determine the relationship between angles:

\[ \alpha_1 = \alpha + \psi \quad \beta_1 = \beta + \psi \]  

(36)

and the vectorial equation:

\[ b_1 \cdot e^{i \alpha_1} - l \cdot e^{i \beta_1} - d_1 = 0 \]  

(37)

It can be projected on real and imaginary axes:

\[ l = \sqrt{b_1^2 + d_1^2 - 2b_1d_1 \cos \alpha_1} \]  

(38)

\[ \sin \beta_1 = \frac{b_1 \sin \alpha_1}{l} \]  

(39)

where \( l \) is the translation of the nut screw along the ball-screw. Since \( \beta_1 \) can't be less than 90°:

\[ \sin \beta_1 = \pi - \arcsin \left( \frac{b_1 \sin \alpha_1}{l} \right) \]  

(40)

Since motor shaft is directly coupled with the ball-screw, nut screw position (\( l \)) and speed (\( \dot{l} \)) can be expressed as a function of motor rotation (\( \theta_m \)) and speed (\( \dot{\theta}_m \)):

\[ l = \frac{p_{\text{screw}} \cdot \theta_m}{2\pi} \quad \dot{l} = \frac{p_{\text{screw}} \cdot \dot{\theta}_m}{2\pi} \]  

(41)

where \( p_{\text{screw}} \) is the screwball pitch. Mechanism transmission ratio can be expressed as the ratio between the elbow speed (\( \dot{\gamma} \)) and motor speed (\( \omega_m = \dot{\theta} \)).

The first parameter can be found deriving equation (29) and projecting it on real and imaginary axes:

\[ \dot{\gamma} = \dot{\alpha} \frac{a \sin(\alpha - \theta)}{c \sin(\gamma - \theta)} \]  

(42)

which is a function of the four bar linkage driver speed (\( \dot{\alpha} \)):

\[ \dot{\alpha} = -\frac{l}{b_1 \sin(\alpha_1 - \beta_1)} \]  

(43)

Finally, substituting equations (41)(42)(43), the elbow transmission ratio is:

\[ T_{\text{elbow}} = \frac{\dot{\gamma}}{\omega_m} = \frac{p_{\text{screw}} a \sin(\alpha - \theta)}{2nb_1c \sin(\gamma - \theta) \sin(\alpha - \beta)} \]  

(44)

**Mechanism optimization**

The designed mechanism has a transmission ratio that changes with the position of the elbow. In order to avoid sudden variation of arm speed, transmission ratio should be constant as much as possible. Therefore, the lengths of each element of the linkage and the angles they form have been optimized, taking into account all the mechanical and the geometrical constraints. The two objective functions are:

\[ F_1 = \frac{\max(\tau) - \min(\tau)}{\max(\tau)} \]  

(45)

\[ F_2 = \max \left| \frac{\partial \tau}{\partial \gamma} \right| \]  

(46)

where \( \gamma \) is the flexion-extension angle of the elbow.

The two functions represent respectively the percentage variation of the transmission ratio and the maximum inclination of the curve as a function of the angle. Assuming the motor speed is constant, the variations in the speed of the elbow are small enough to be accepted (fig.14). To better understand how geometry and dimensions affect transmission ratio, its trend is represented as a function of mechanism rod length (fig.15).
A good parameter that shows the linkage capability to transmit motion is the angle of transmission [20]. In four-bar linkages it’s defined as the smaller angle between connecting rod and rocker arm directions, while in the crank and slotted link it’s the angle between the two rods (fig.16).

The lower is the transmission angle, the lower is the mechanical efficiency. Moreover, if transmission angle goes under 40°, the effects of backlashes and flexibility are amplified, causing vibrations and malfunctions. Figure 17 shows the transmission angles trends: their values confirm the linkage is always able to transmit motion properly.
THIRD PROSTHESIS PROTOTYPE

The aim of the last designed prosthesis is to reduce the dimension of the device, maintaining a good efficiency of the system. A new commercial transmission has been used: the harmonic drive. This system is a epicyclic reduction gear which is very small and has an high transmission ratio ($\tau = 1/80$) that is enough if combined with the brushless motor used in the previous version of the prosthesis.

Since the transmission is constituted only by the harmonic drive, dimensions are reduced and its mechanical efficiency is very high. It can be directly obtained from the data sheets. Its value is:

$$\eta_3 = 0.80$$  \hspace{1cm} (47)

This value is a little lower than the previous one, but the system has achieved some important features:

- harmonic drive is a commercial system: it’s available and easy to be assembled to the motor,
- the system is more reliable and maintenance is not needed,
- the prosthesis is compact and well-built.

Finally, the prosthesis uses the same motor-transmission system which has been used for shoulder actuation (Fig. 26). This configuration allows cost reduction and makes the device simpler and modular.
CONCLUSION

Four different active elbow prostheses have been presented. The kinematic analysis and experimental tests show that a good mechanical design can improve the system mechanical efficiency. It conducts to the reduction of batteries dimension and weight and to a more noiseless prosthesis. Functional design allows to optimize the mechanism in order to reduce device dimensions and to achieve high performances and reliability. The first prosthesis developed by the Research Group of Man-Machine Systems of Politecnico di Milano has been adopted by patients. The other two following prostheses are still under test together with a new driving system.

REFERENCES


