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Determining the Inventory Level of Spare Parts according to System Utilization in a Two-Echelon Distribution System

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Abstract

Repairing broken systems to improve productivity and utilization has been a major issue in recent years. Generally, a high system utilization requires large amount of spare parts. Therefore, it is crucial to determine the appropriate level of inventory with respect to system utilization. This study proposes a methodology to determine appropriate levels of inventory in a two-echelon spare parts distribution system. The objectives are to satisfy customer demands at a distribution center (DC) and to maintain the utilization of the whole system for customers. The proposed methodology minimizes the total inventory cost for both the DC and the customers.

Keywords: Inventory model, Spare parts, System Utilization, Two Echelon System

INTRODUCTION

With advances in technology, products are designed to offer high precision and various additional services. Typically, these products consist of many components. Examples of such products are airplanes, trains, naval vessels engines, and high-end electronic products; moreover, the nature of these products demands that many repairable items must be kept in stock in case a breakdown occurs. However, it is not possible have large quantities of each item in stock due to budget limitations. On the other hand, when there is a lack of stock, the whole system becomes impossible to operate. Therefore, effective ways are required to maintain an appropriate inventory level for the repairable items composing each product. Furthermore, if such products are used for military, vehicular, aircraft, life-saving or restoration purposes, an inventory level for the repairable items must be chosen that will both minimize cost and facilitate the utilization of the whole system.

Studies concerning system utilization have mainly focused on system reliability. However, most did not consider the relationship between system utilization and inventory. Rotab Khan [10] and Seal et al. [11] proposed a method that did not consider the whole system as a combination of reparable item, but instead reviewed a number of possible systems according to system failure itself.

On the other hand, studies on repairable items inventory and distribution supply chain have been done thoroughly. The most fundamental of these researches was the METRIC model developed by Sherbrooke [12] which applied the (S-1, S) policy to the stochastic demand of the repairable-items. Thereafter, many have extended the METRIC model. Hopp et al. [6, 7] proposed a heuristic method to ascertain a (r, Q) policy which minimizes the inventory holding cost under the constraints of both the order frequency and the level of customer service at a single DC. Also, Hopp et al. [6, 7] extended its range to a two-echelon distribution supply chain and presented a heuristic method that minimizes the inventory holding cost in a model that considers the constraints that delay the customer's annual order amount, the constraints of order frequency, and the level of customer services at the DC. Moinzadeh and Lee [8] fixed the inventory level and batch size that minimize the backorder penalty cost, and Svoronos and Zipkin [14] developed a METRIC model having stochastic transportation time. Topan et al. [16] found the policy parameters to minimize expected system-wide inventory holding and fixed order costs in which the central warehouse operates under a (Q, R) policy and local warehouse implement (S-1, S) policy. Through the case in practice, they showed that it is possible to keep the cost-benefit of using aggregate service levels while preventing long individual response time. Costantino et al. [3] studied the problem of allocation of spare parts of aeronautical system. They developed the model to minimize the system-wide expected backorder while fitting constraints on limited budget and on a specific operational availability target. Tsai and Zheng [17] presented hybridization approach using simulation and optimization for solving the problem that minimizes the total investment costs while satisfying the expected response time targets for each filed depot. They showed that the proposed algorithm is more adaptive and can be applied to any multi-item multi-echelon inventory system.

Moreover, concerning the studies on customer service, Nahmias [9] divided services into Types Ⅰ and Ⅱ and defined them according to backorder occurrence during lead-time. Hopp et al. [6] proposed Types Ⅰ and Ⅱ, and a hybrid heuristic model to determine the (r,Q) value based on Nahmias [9]'s model. Ganeshan [4] identified out the near-optimal (r,Q) value based on both the service rate of the DC and customer at the supply chain network consisting of multi - suppliers, a single DC, and multi - customers. Cohen et al. [2] compared three policies; a central policy, a local policy, and mixed policy in order to minimize spare parts purchases and repair costs for maintaining a fleet of mission-critical systems. Van den Berg et al. [18] studied a multi-item, two-echelon, and continuous-review inventory problem. They utilized the column generation method with building blocks for single-item models as columns and found the optimal allocation of service parts under an aggregate waiting time constraint.

MODEL DESCRIPTION

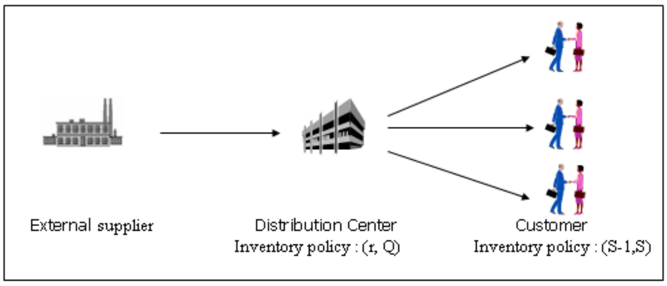

Figure 1: DC and customer model

The subject of this study is a model consisting of an external supplier, a DC which purchases items from an external supplier and then supplies them to several customers who consume the items and reorder them through the DC as shown in figure 1. The inventory policy at the DC is (r, Q), and the customers use the (S-1, S) policy. The lead-time of an item supplied to the distribution center from the external supplier was thought to be excessive, and although lost sales are not reflected, backorders are allowed. Also, customers are required to maintain a certain level of system utilization, and the DC must retain a level of customer service for in which items are efficiently supplied.

Definition of system utilization

Before defining system utilization, if one assumes that no more than one kind of repairable item is present in one system, and that distinct items in distinct systems may fail to operate, cannibalization becomes possible as seen in figure 2.

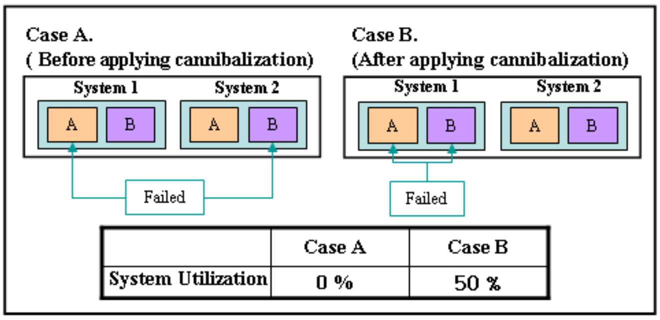

Figure 2: Cannibalization and System Utilization

The system consisting of single item or multiple items is seen as follows. First of all, when consisting single item, the fact that there are k number of systems means that k number of spare parts exist. When a system fails, spare parts will be substituted with the failed spare parts, and the number of operational systems will be maintained as k. If the number of spare parts is less than number of failed spare parts, the number of systems corresponding to the missing items will be non-operational, and backorder corresponding to the number of missing items will occur. Then, the customer orders the number of items under non-operational conditions though the DC. Therefore, when backorder occurs, the number of operational systems becomes (k – backorder). The system utilization is defined as {(k – number of backorders) / k}. Moreover, the utilization of the whole system may result in cannibalization; therefore, the utilization of whole system is calculated as follows:

$$System\ utilization = \frac{k - \max_{i=1,\dots,N}(backorders\ of\ item\ i)}{k}$$

Mathematical Formulation

We first summarize the notations for our formation:

General notation

N = number of items;

c_i = cost of item i ($);

K = number of systems;

F = average order frequency at the DC;

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Decision variables

Q_o^i = order quantity for item i at the DC;

r_o^i = reorder point for item i at the DC;

S_m^i = base stock level for item i for customer m.

Customer notation

M = number of customer;

O_m = minimum allowable system utilization for customer m;

d_m = shipping time of parts from DC to customer m (day);

λ_m^i = expected annual demand for item i for customer m (parts/yr);

l_m^i = expected replenishment lead time for item i for customer m (day);

θ_m^i = expected lead time for item i for customer m (parts);

$B_m^i(S_m^i)$ = expected number of backorders for item i for customer m at any point in time (parts);

$h_m^i(S_m^i) = S_m^i - \theta_m^i + B_m^i(S_m^i)$ = expected on hand inventory of item i for customer m at any point in time (parts).

DC notation

$\lambda_o^i = \sum_{m=1}^{M} \lambda_m^i$ = expected annual demand for item i at the DC (parts/yr);

l_o^i = replenishment lead time for item i at the DC (day);

$\theta_o^i = \lambda_o^i l_o^i$ = expected demand for item i at the DC during lead time l_o^i (parts);

$A_o^i(r_o^i, Q_o^i)$ = probability of stockout for item i at the DC;

$B_o^i(r_o^i, Q_o^i)$ = expected number of backorders for item i at the DC at any point in time (parts);

$h_o^i(r_o^i, Q_o^i) = 1 / Q_o^i \cdot \sum_{y=r_o^i+1}^{r_o^i+Q_o^i} y - \theta_o^i + B_o^i(r_o^i, Q_o^i)$ = expected on hand inventory of item i at the DC at any point in time (parts);

$S_o^i(r_o^i, Q_o^i) = 1 - A_o^i(r_o^i, Q_o^i)$ = service level of item i at the DC.

Note that $B_m^i(\cdot)$ denotes the expected number of backorders of item i for customer m at any point in time. We can now formulate the combined DC and customer problem to minimize total inventory investment in the system subject to a constraint on order frequency and at the DC and constraints on the utilization of the whole system for customer as:

$$Min \sum_{i=1}^{N} c_i \cdot \left\{ h_o^i(r_o^i, Q_o^i) + \sum_{m=1}^{M} h_m^i(S_m^i) \right\} \quad (1)$$

s.t $\lambda_o^i / Q_o^i \le F \ \forall i$ (2)

$$1 - A_o^i(r_o^i, Q_o^i) \ge S \ \forall i \quad (3)$$

$$1 - B_m^i(S_m^i) / K \ge O_m \ \forall i, \forall m \quad (4)$$

$$r_o^i, Q_o^i, S_m^i \ge 0 \ r_o^i, Q_o^i, S_m^i \text{ : integer} \quad (5)$$

The average backorders for customers, $B_m^i(S_m^i)$, can be computed using the method suggested by Axsäter [1]. However, the complexity of the recursive procedure limits its practical use. So we sought approximations. The simplest way to approximate $B_m^i(S_m^i)$ is to replace the replenishment lead time for customer by their expectations as suggested by many researchers [1,13]. While this is very simple, it is easy to show that it consistently causes backorders to be understated, and in some cases the discrepancy can be large. So, instead we estimate the variance of the lead times and approximate the lead time demand distribution for customers with a negative binomial distribution suggested by Graves[5] and Svoronos and Zipkin[15].

HEURISTIC APPROACH

Equation (1) - (5) represents a large-scale, nonlinear, discrete optimization problem. Hence, there is no practical exact solution method and our goal is to find a good approximation. We first decompose the problem by item and customer. We then develop closed-form expressions for the control parameters for customers and DC, and a heuristic algorithm to find the parameters in closed-forms.

Customer heuristic

$$Min \quad \sum_{i=1}^{N} c_i \cdot \left\{ \sum_{m=1}^{M} h_m^i (S_m^i) \right\} \tag{6}$$

Subject to: $1 - B_m^i (S_m^i)/K \geq O_m, i = 1,..., N, \quad m = 1,..., M$ (7)

S_m^i : integer , $\quad i = 1,..., N, \quad m = 1,..., M$ (8)

Objective equation (6) concerns customers in equation (1).Solving problem (6)-(8) is equivalent to solving the following $N \times M$ subproblems:

$$Min \quad c_i \cdot \left\{ h_m^i (S_m^i) \right\} \tag{9}$$

Subject to: $1 - B_m^i (S_m^i)/K \geq O_m$ (10)

S_m^i : integer, $\quad i = 1,..., N, \quad m = 1,..., M$ (11)

Equation (9) is an objective equation of single item, single customer that minimizes the inventory holding cost. Equation (10) means to be greater than the minimum system utilization, and it is applied only if the demand during the lead-time is greater than the backorder. If the demand is less than the backorder, then base stock level, S_m^i becomes 0.

Using these approximations and differentiating the Lagrangian yield::

$$S_m^i = \begin{cases} \theta_m^i + \Phi^{-1}\{1 - (c_i K)/\rho\} \cdot (\sigma_m^i)^{1/2}, & if\ c_i K \leq \rho \\ 0 & , \ otherwise \end{cases} \tag{12}$$

Where ρ , is the Lagrange multiplier. Note that the inverse cumulative distribution function (CDF) of the normal is included n most spreadsheets or can easily be calculated using very accurate polynomial approximations. To approximate ρ , we adjust ρ , and compute S_m^i using (12) to achieve the lowest utilization that satisfies the minimum system utilization constraint. In order to ensure a feasible solution to the original problem, we use the exact expression for system utilization for customers when searching for appropriate Lagrange multipliers.

We can derive simple formula for S_m^i by using the following approximation:

$$B_m^i (S_m^i) = \sum_{u=S_m^i}^{\infty} (u - S_m^i) \cdot \Phi\{(u - \theta_m^i)/(\sigma_m^i)^{1/2}\}$$

$$= \sum_{u=S_m^i+1}^{\infty} [1 - \Phi\{(u - \theta_m^i)/(\sigma_m^i)^{1/2}\}]$$

$$= \sum_{u=0}^{\infty} [1 - \Phi\{(u - \theta_m^i)/(\sigma_m^i)^{1/2}\}] - \sum_{u=S_m^i}^{\infty} [1 - \Phi\{(u - \theta_m^i)/(\sigma_m^i)^{1/2}\}]$$

(13)

In equation (13), number of backorders during lead-time, $B_m^i(\cdot)$, assumes the probability density function (pdf) as normal distribution with the demand for item i during lead time. Where σ_m^i is the standard deviation of the demand distribution.

$$l_m^i = d_m + B_o^i (r_o^i, Q_o^i)/\lambda_o^i \tag{14}$$

$$\theta_m^i = \lambda_m^i \cdot l_m^i \tag{15}$$

The expected replenishment lead time, l_m^i becomes equation (14) by adding the shipping time from the DC to customer with the waiting time as occurs in the DC backorder. Here, the waiting time at DC is calculated by Little's law. The expected demand during lead time is ascertained thorough equation (15).

By using solutions $S_m^i, B_m^i(S_m^i)$, solution $h_m^i(S_m^i)$ for the objective equation (6) can be obtained.

3.2 DC heuristics

$$Min \quad \sum_{i=1}^{N} c_i \cdot \; h_o^i(r_o^i, Q_o^i) \tag{16}$$

Subject to: $\lambda_o^i / Q_o^i \le F \;\; \forall i$ (17)

$$1 - A_o^i(r_o^i, Q_o^i) \ge S \;\; \forall i \tag{18}$$

$$r_o^i, Q_o^i, S_o^i \ge 0 \quad r_o^i, Q_o^i, S_m^i \;\text{: integer} \tag{19}$$

To find a policy for the DC, we formulate a single level problem hat minimizes the total inventory cost at the DC subject to constraints on average order frequency and service level:

$$Min \quad c_i \cdot \; h_o^i(r_o^i, Q_o^i) \tag{20}$$

Subject to: $\lambda_o^i / Q_o^i \le F$ (21)

$$1 - A_o^i(r_o^i, Q_o^i) \ge S \tag{22}$$

$$r_o^i, Q_o^i, S_o^i \ge 0 \quad r_o^i, Q_o^i, S_m^i \;\text{: integer} \tag{23}$$

In equation (22), S represents the customer service level, and $\{1 - A_o^i(r_o^i, Q_o^i)\}$ means the probability of receiving services. The exact formula for the probability of being out of stock is equation (24). However, the latter can be represented as equation (25) by using the Type II Service applied in Nahmias [9] and Hopp et al [6].

$$A_o^i(r_o^i, Q_o^i) = 1/Q_o^i \cdot \{\sum_{i=0}^{\infty} \sum_{y=r_o^i+1}^{r_o^i+Q_o^i} p(i+y)\} = 1/Q_o^i \cdot \{\sum_{y=r_o^i+1}^{r_o^i+Q_o^i} \sum_{i=0}^{\infty} p(i+y)\}$$
$$= 1/Q_o^i \cdot [\sum_{y=r_o^i+1}^{\infty} \{1 - P(y-1)\} - \sum_{y=r_o^i+Q_o^i+1}^{\infty} \{1 - P(y-1)\}]$$
$$= 1/Q_o^i \cdot [a(r_o^i) - a(r_o^i + Q_o^i)] \tag{24}$$

$$A_o^i(r_o^i, Q_o^i) = a(r_o^i) / Q_o^i \tag{25}$$

$a(r_o^i)$ in equation (25) represents the average backorder volume during lead time, and the probability density function assumes that the average and variance follow normal distribution, with average demand during lead time. This can be represented as $\Phi\{(u - \theta_o^i)/(\theta_o^i)^{1/2}\}$. Therefore, constraint equation (22) is equal to the following equation (26).

$$1 - \sum_{u=r_{oo}^i}^{\infty} [1 - \Phi\{(u - \theta_o^i)/(\theta_o^i)^{1/2}\}] / Q_o^i \;\ge\; S \tag{26}$$

Q_o^i and r_o^i are solved as equations (27) and (28) through the Lagrangian method used in the Hopp et al. [6], and ν and μ are Lagrangian coefficients. See Appendix for details.

$$Q_o^i = \max\,[\{2\nu\,(\lambda_o^i) / C_i\}^{1/2}, 1] \tag{27}$$

$$r_o^i = \begin{cases} \theta_o^i + \Phi^{-1}\{1 - (C_i Q_o^i)/\mu\} \cdot (\theta_o^i)^{1/2}, & if\ C_i Q_o^i \le \mu \\ 0 & , \ otherwise \end{cases} \tag{28}$$

The formula for the average backorder amount is equation (29). However, the previously defined average backorder volume during lead time, $a(r_o^i)$ is used in equation (30).

$$B_o^i(r_o^i,Q_o^i)=1/Q_o^i\cdot\{\sum_{i=0}^{\infty}i\cdot\sum_{y=r_o^i+1}^{r_o^i+Q_o^i}p(i+y)\}=1/Q_o^i\cdot\{\sum_{y=r_o^i+1}^{r_o^i+Q_o^i}\sum_{i=0}^{\infty}ip(i+y)\}$$

$$=1/Q_o^i\cdot[\sum_{y=r_o^i+1}^{\infty}\{y-(r_o^i+1)\}\{1-P(y-1)\}-\sum_{y=r_o^i+Q_o^i+1}^{\infty}\{y-(r_o^i+Q_o^i+1)\}\{1-P(y-1)\}]$$

$$=1/Q_o^i\cdot[\beta(r_o^i)-\beta(r_o^i+Q_o^i)] \quad (29)$$

$$B_o^i(r_o^i,Q_o^i)=a(r_o^i) \quad (30)$$

Finally, $h_o^i(r_o^i,Q_o^i)$ can be represented as equation (31) with the values, r_o^i,Q_o^i, and $B_o^i(r_o^i,Q_o^i)$.

$$h_o^i(r_o^i,Q_o^i)=r_o^i-\theta_o^i+Q_o^i/2+1/2+B_o^i(r_o^i,Q_o^i) \quad (31)$$

Heuristic procedure

Step 1 (DC): $Q_o^i,r_o^i,B_o^i(r_o^i,Q_o^i)$

(1-1) By selecting any value of ν,μ in equations (27) and (28), compute Q_o^i and r_o^i. $(i=1,...,N)$

(1-2) Substitute Q_o^i,r_o^i in equations (21) and (22), and ascertain whether it satisfies order frequency F and service level S.

(1-3) Repeat processes (1-1) and (1-2) by changing the values of ν and μ until F and S are satisfied.

(1-4) Find $B_o^i(r_o^i,Q_o^i)$ by using equation (30).

Step 2 (Customer) : $S_m^i,l_m^i,B_m^i(S_m^i)$

(2-1) Find S_m^i by choosing any value ρ in equation (12). $(i=1,...,N)$

(2-2) Substitute S_m^i in equation (10) and check whether it satisfies system utilization O_m.

(2-3) Repeat processes (2-1) and (2-2) by changing the value of ρ until O_m is satisfied.

(2-4) Find l_m^i by using the solved $B_o^i(r_o^i,Q_o^i)$ in step 1 and equation (14).

(2-5) Find $B_m^i(S_m^i)$ with the value S_m^i.

Step 3 (Integration) : Total cost

(3-1) Obtain solutions $h_o^i(r_o^i,Q_o^i)$ and $h_m^i(S_m^i)$ by using the solutions in steps 1 and step 2, and calculate the total cost.

EXPERIMENTS AND RESULTS

To test the performance of our combined DC-customer heuristic, we consider examples with ten items, two customers. The demand of each item follows Poisson distribution, and price is also considered according to the demand frequency of each item. A customer possesses 100 equal systems, and it is assumed that each customer requires system utilization greater than 96%. Table 1 shows the input data for the price of each product and the annual demand of customers, as well as the lead time from an external supplier to the DC respectively. The differences according to customers are not considered. The replenishment lead time from DC to the customers is set as 1 day, and the lead time between the external supplier and the DC is assumed to take from 10 to 100 days uniformly. Moreover, the order frequency at DC is fixed as four times.

Table 1: Input data for experiments

Items	1	2	3	4	5	6	7	8	9	10
C_i	40	36	32	28	24	20	16	12	8	4
λ_m^i	10	20	30	40	50	60	70	80	90	100
l_o^i	100	90	80	70	60	50	40	30	20	10

Based on the data above, the results of the DC heuristic and customer heuristic models are shown in table 2 and tables 3-5 respectively. At the DC, we found (r, Q) values where customer service rates are 70%, 80%, and 90%, and calculated their inventory holding cost. Moreover, as for the customers in cases where customer service rates are 70%, 80%, and 90%, we found out (S-1, S) values which maintain the system utilization at a customer as 96% ~ 100%, and calculated their inventory holding cost.

From results of the DC in table 2, Q_o^i is ascertained through the constraint conditions of the order frequency. Therefore, it has no relation with the customer service rate, and demand during lead time solved by the multiplication of annual demand and lead time is also irrelevant to the customer service rate.

If the customer service rate at DC increases, the backorder volume during lead time decreases. However, both the reorder point and inventory holding cost increase. According to the item in an equal customer service rate, if the demand during lead time is greater, reorder point increases. This is shown in Figure 3-4.

Table 2: DC results

Items		1	2	3	4	5	6	7	8	9	10
C_i		40	36	32	28	24	20	16	12	8	4
λ_o^i		20	40	60	80	100	120	140	160	180	200
l_o^i		100	90	80	70	60	50	40	30	20	10
Q_o^i		5	10	15	20	25	30	35	40	45	50
θ_o^i		5.479	9.863	13.151	15.342	16.438	16.438	15.342	13.151	9.863	5.479
Service level 90%	r_o^i	8	15	19	22	23	21	18	14	8	1
	B_o^i	0.436	0.803	1.476	1.821	2.165	2.913	3.374	3.524	3.953	4.634
	h_o^i	5.956	11.440	15.325	18.978	21.727	22.974	24.031	24.873	25.090	25.654
	$C_i h_o^i$	238.277	411.841	490.420	531.408	521.460	459.497	384.511	298.483	200.726	102.619
Service level 80%	r_o^i	7	12	16	17	17	15	12	7	2	0
	B_o^i	0.806	1.899	2.611	3.829	4.705	5.750	6.505	7.693	8.202	5.479
	h_o^i	5.328	9.536	13.460	15.987	18.266	19.812	21.162	22.042	23.339	25.500
	$C_i h_o^i$	213.123	343.299	430.742	447.626	438.386	396.241	338.599	264.510	186.716	102.000
Service level 70%	r_o^i	6	10	13	13	13	10	7	2	0	0
	B_o^i	1.260	2.841	4.028	5.917	6.895	8.791	9.817	11.485	9.863	5.479
	h_o^i	4.780	8.478	11.878	14.074	16.457	17.852	19.474	20.834	23.000	25.5
	$C_i h_o^i$	191.217	305.222	380.080	394.080	394.962	357.048	311.599	250.012	184.000	102.000

Table 3: Customer results (service level at DC = 70%)

Items		1	2	3	4	5	6	7	8	9	10
l_m^i		23.993	26.928	25.505	27.995	26.167	27.739	26.595	27.200	21.000	11.000
θ_m^i		0.657	1.476	2.096	3.068	3.585	4.560	5.100	5.962	5.178	3.014
utilization 100% (99.9)	S_m^i	2	4	6	8	10	13	15	17	15	8
	B_m^i	0.021	0.053	0.044	0.100	0.073	0.075	0.065	0.091	0.073	0.089
	h_m^i	1.363	2.578	3.948	5.032	6.488	8.515	9.964	11.130	9.895	5.075
	$C_i h_m^i$	54.536	92.796	126.326	140.897	155.717	170.296	159.428	133.565	79.161	20.300

utilization 99%	S_m^i	1	2	3	5	6	8	9	11	9	4
	B_m^i	0.322	0.565	0.642	0.634	0.668	0.712	0.768	0.765	0.813	0.969
	h_m^i	0.664	1.090	1.546	2.566	3.083	4.152	4.668	5.588	4.635	1.956
	$C_i h_m^i$	26.579	39.224	49.465	71.849	73.996	83.037	74.686	67.058	37.081	7.822
utilization 98%	S_m^i	0	1	1	3	4	5	6	8	6	3
	B_m^i	1.163	1.191	1.860	1.524	1.468	1.845	1.838	1.685	1.906	1.471
	h_m^i	0.506	0.716	0.764	1.456	1.884	2.286	2.738	3.774	2.727	1.457
	$C_i h_m^i$	20.233	25.773	24.435	40.755	45.204	45.714	43.801	45.290	21.820	5.830
utilization 97%	S_m^i	0	0	0	1	2	4	4	6	5	1
	B_m^i	1.163	2.033	2.701	2.909	2.704	2.394	2.931	2.613	2.419	2.851
	h_m^i	0.506	0.557	0.605	0.842	1.119	1.835	1.831	2.616	2.241	0.837
	$C_i h_m^i$	20.233	20.062	19.358	23.562	26.857	36.691	29.293	31.387	17.930	3.348
utilization 96%	S_m^i	0	0	0	0	1	2	3	4	3	0
	B_m^i	1.163	2.033	2.701	3.751	3.468	3.741	3.591	3.806	3.672	3.692
	h_m^i	0.506	0.557	0.605	0.683	0.884	1.181	1.491	0.616	1.494	0.678
	$C_i h_m^i$	20.233	20.062	19.358	19.120	21.206	23.623	23.849	7.387	11.953	2.714

Table 4: Customer results (service level at DC = 80%)

Items		1	2	3	4	5	6	7	8	9	10
l_m^i		15.737	18.329	16.886	18.470	18.171	18.491	17.959	18.550	17.633	11.000
θ_m^i		0.431	1.004	1.388	2.024	2.489	3.040	3.444	4.066	4.348	3.014
utilization 100% (99.9)	S_m^i	1	3	4	6	7	8	10	11	12	8
	B_m^i	0.094	0.025	0.035	0.034	0.055	0.094	0.054	0.097	0.090	0.089
	h_m^i	0.663	2.021	2.647	4.010	4.565	5.054	6.609	7.032	7.742	5.075
	$C_i h_m^i$	26.500	72.741	84.708	112.266	109.567	101.089	105.749	84.378	61.936	20.300
utilization 99%	S_m^i	0	1	2	3	3	4	5	7	7	4
	B_m^i	0.935	0.687	0.487	0.584	0.981	0.991	0.906	0.684	0.863	0.969
	h_m^i	0.504	0.683	1.099	1.560	1.492	1.951	2.461	3.618	3.516	1.956
	$C_i h_m^i$	20.154	24.590	35.182	43.669	35.802	39.026	39.382	43.416	28.124	7.822
utilization 98%	S_m^i	0	0	0	1	2	3	3	4	5	3
	B_m^i	0.935	1.529	1.939	1.782	1.559	1.496	1.893	1.917	1.656	1.471
	h_m^i	0.504	0.524	0.551	0.758	1.070	1.456	1.449	1.851	2.308	1.457
	$C_i h_m^i$	20.154	18.879	17.627	21.222	25.672	29.130	23.177	22.209	18.463	5.830
utilization 97%	S_m^i	0	0	0	0	1	1	2	3	3	1
	B_m^i	0.935	1.529	1.939	2.623	2.284	2.879	2.555	2.520	2.809	2.851
	h_m^i	0.504	0.524	0.551	0.599	0.795	0.839	1.111	1.454	1.461	0.837

	$C_i h_m^i$	20.154	18.879	17.627	16.780	19.076	16.784	17.778	17.450	11.692	3.348
utilization 96%	S_m^i	0	0	0	0	0	0	1	1	2	0
	B_m^i	0.935	1.529	1.939	2.623	3.125	3.720	3.316	3.989	3.515	3.692
	h_m^i	0.504	0.524	0.551	0.599	0.636	0.681	0.872	0.923	1.167	0.678
	$C_i h_m^i$	20.154	18.879	17.627	16.780	15.268	13.611	13.954	11.076	9.335	2.714

Table 5: Customer results (service rate at DC = 80%)

Items		1	2	3	4	5	6	7	8	9	10
l_m^i		8.964	8.328	9.981	9.310	8.905	9.861	9.798	9.040	9.017	9.457
θ_m^i		0.246	0.456	0.820	1.020	1.220	1.621	1.879	1.981	2.223	2.591
utilization 100% (99.9)	S_m^i	1	2	2	3	3	4	5	5	6	7
	B_m^i	0.001	0.000	0.079	0.028	0.085	0.094	0.066	0.092	0.067	0.072
	h_m^i	0.755	1.544	1.259	2.008	1.865	2.473	3.187	3.111	3.843	4.481
	$C_i h_m^i$	30.219	55.586	40.284	56.216	44.753	49.452	50.998	37.330	30.746	17.925
utilization 99%	S_m^i	0	0	1	1	1	2	2	3	3	4
	B_m^i	0.842	0.958	0.493	0.704	0.917	0.699	0.946	0.550	0.748	0.636
	h_m^i	0.597	0.502	0.672	0.684	0.697	1.078	1.067	1.569	1.525	2.045
	$C_i h_m^i$	23.873	18.076	21.510	19.154	16.740	21.553	17.065	18.822	12.198	8.180
utilization 98%	S_m^i	0	0	0	0	0	1	1	1	1	2
	B_m^i	0.842	0.958	1.334	1.546	1.759	1.348	1.626	1.736	1.997	1.664
	h_m^i	0.597	0.502	0.514	0.525	0.539	0.727	0.747	0.755	0.774	1.073
	$C_i h_m^i$	23.873	18.076	16.433	14.712	12.932	14.536	11.946	9.055	6.189	4.290
utilization 97%	S_m^i	0	0	0	0	0	0	0	0	0	1
	B_m^i	0.842	0.958	1.334	1.546	1.759	2.189	2.467	2.577	2.838	2.394
	h_m^i	0.597	0.502	0.514	0.525	0.539	0.568	0.588	0.596	0.615	0.803
	$C_i h_m^i$	23.873	18.076	16.433	14.712	12.932	11.363	9.407	7.151	4.920	3.212
utilization 96%	S_m^i	0	0	0	0	0	0	0	0	0	0
	B_m^i	0.842	0.958	1.334	1.546	1.759	2.189	2.467	2.577	2.838	3.235
	h_m^i	0.597	0.502	0.514	0.525	0.539	0.568	0.588	0.596	0.615	0.644
	$C_i h_m^i$	23.873	18.076	16.433	14.712	12.932	11.363	9.407	7.151	4.920	2.577

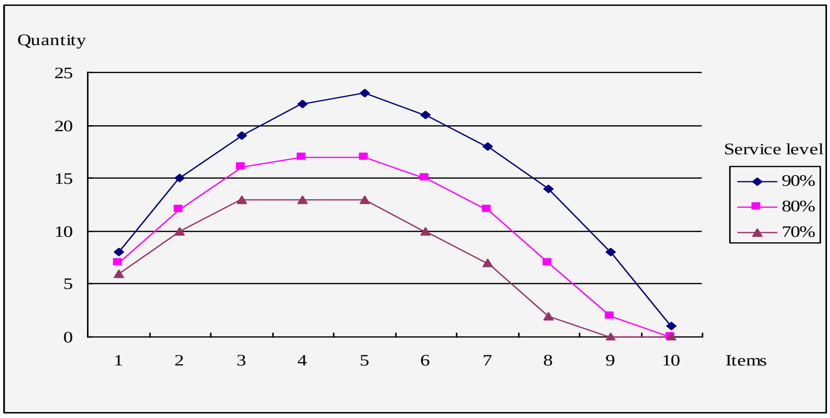

Figure 3: Reorder points for items according to service rate of customer at DC

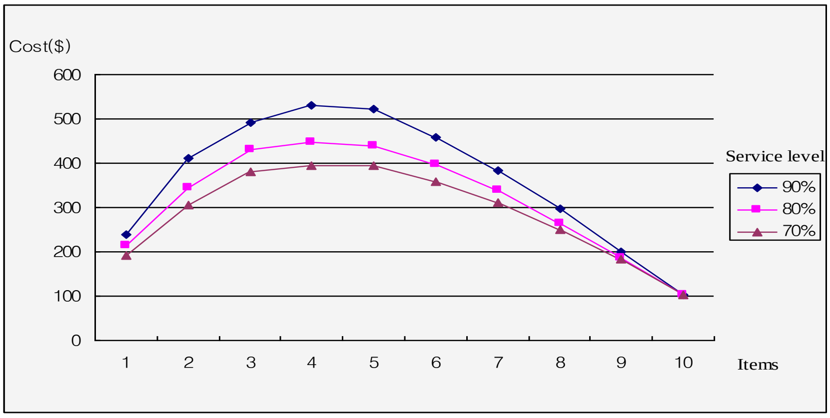

Figure 4: Inventory holding cost for items according to service rate of customer at DC

In the customer results, the lead time from DC to the customers includes both the shipping time and the time spent waiting for an item to be replaced when backorder occurs. Figure 5 shows that as service rate increases, lead-time decreases. When the service rate at a DC is 90%, the lead-time takes about 8~9 days, and when 80%, 11~19 days, and when 70%, more than 20 days. In the case where customer service rates are at 80% and 70%, the lead-time of 10^{th} item is equally 11 days. Because the reorder point of item 10 is 0, backorder occurs at the DC. The time spent waiting until the item is replaced becomes identically 10 days, and the shipping time from the DC to the customer becomes 1 day.

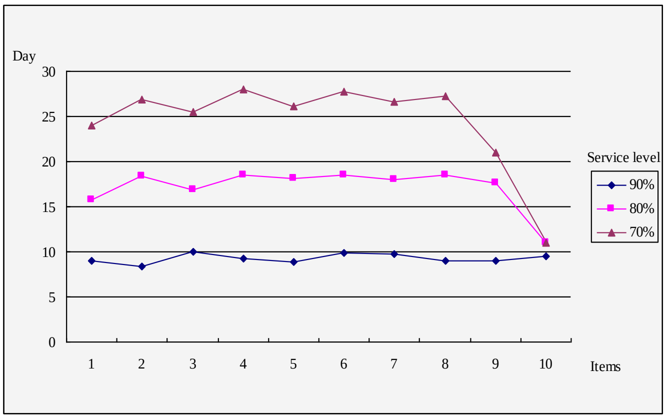

Figure 5: Lead-time for products according to service rate of customer at DC

In the case of order-up-to level which maintains a certain level of customer utilization, the more demand there is during lead-time per item, the more quantity is necessary. On other hand, as the customer service rate increases, the order-up-to level decreases. Also, as the customer service rate at DC increases, the demand during lead-time at the customers decreases.

Demand during the lead time at a customer can be calculated by multiplying the annual demand by lead time, and as the customer service rate increases, the demand during the lead time at a customer decreases. For a utilization of the same product, as the utilization increases, the target inventory to retain increases as well.

The base stock level per item according to utilization when the customer service rates are 70%, 80%, and 90% respectively is illustrated in figures 6-8.

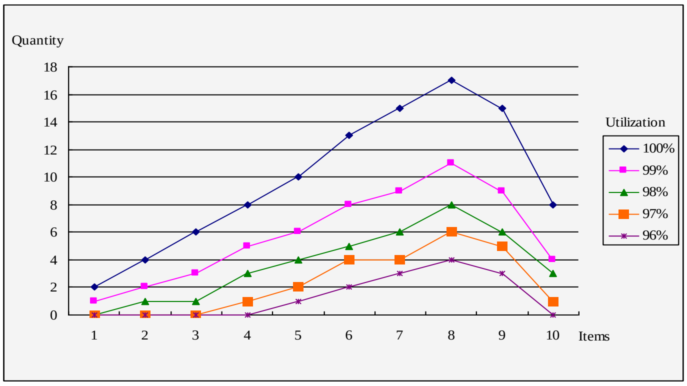

Figure 6: Order-up-to level per item according to utilization at the customers (DC service rate 70%)

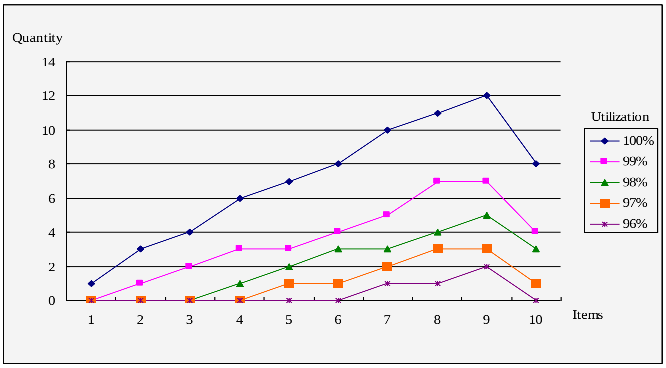

Figure 7: Order-up-to level per item according to utilization at the customers (DC service rate 80%)

Table 6 shows the total inventory holding cost of the DC and the customer at the DC. As the customer service rate increases, the inventory holding cost increases. Moreover, as the customer utilization increases, the inventory holding cost noticeably increases. According to the changes in the customer service rate at the DC, as the customer service rate increases, the inventory holding cost decreases at the customer. The inventory holding cost according to the customer utilization and the service rate at DC is shown in figure 9.

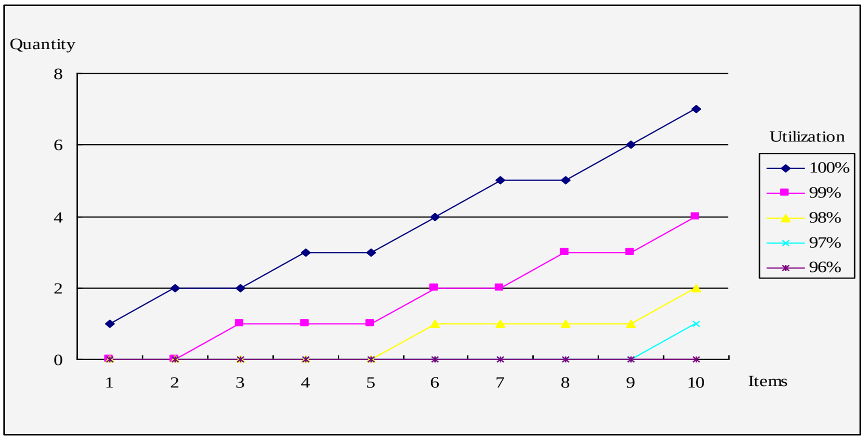

Figure 8: Order-up-to level per item according to utilization (DC service rate 90%)

Table 6: Total **i**nventory holding cost

DC Service rate	DC Inventory holding cost	Inventory holding cost of customer according to utilization				
		96%	97%	98%	99%	100%
70%	2870.22	169.51	228.72	318.86	530.80	1133.02
80%	3161.24	139.40	159.57	202.36	317.17	779.24
90%	3639.24	121.45	122.08	132.04	177.17	413.51

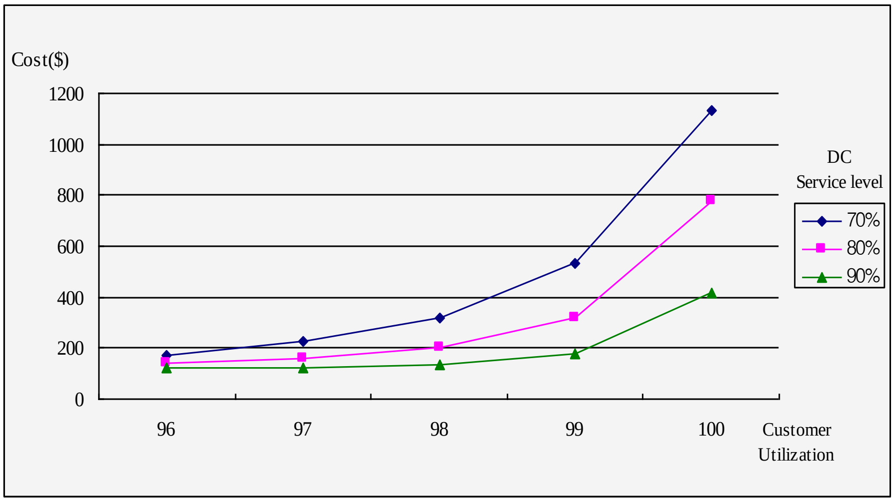

Figure 9: Inventory holding cost according to the customer utilization and service rate at DC

CONCLUSIONS AND FURTHER STUDIES

This paper focuses on the repair parts system in a two-echelon distribution supply chain. We determine the proper reorder points and batch quantity of size Q considering the customers service rate at a DC. For the customers, we decide the order-up-to level; this maintains a certain level of utilization by considering the relations existing between the system utilization and inventory volume. Also, by changing the utilization at a customer and the customer service rate at the DC, the cost for each is calculated. This model is expected to contribute greatly through its application in business fields operating and maintaining high-cost equipment and facilities.

In the future, studies examining an inventory volume according to the system utilization for various kinds of inventory policies will continue. Moreover, studies must be done concerning to determine whether it is more efficient to increase system utilization or increase the number of systems when the same amount of cost is invested.

ACKNOWLEDGEMENTS

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APPENDIX 1: Heuristic to find Q_o^i and r_o^i at DC

1. Common assumptions for heuristics

We can derive simple formulae for Q_o^i and r_o^i by using the following approximations for all heuristics:

(1) Inventory for item i is given by $h_o^i(r_o^i, Q_o^i) = r_o^i - \theta_o^i + Q_o^i / 2$.

(2) Demand for item i during the replenishment lead time is approximated by the normal distribution with mean θ_o^i and standard deviation $\sqrt{\theta_o^i}$ to match two moments with the Poisson distribution.

2. Type I heuristic

(1) We approximate inventory by assuming that $r_o^i - \theta_o^i \geq 0$.

(2) Service for item i is given by the Type I formula, $G_o^i(r_o^i)$, the cdf of lead time demand for item i.

$$Min \quad c_i \ (r_o^i - \theta_o^i + Q_o^i / 2) \tag{32}$$

subject to
$$\lambda_o^i / Q_o^i \leq F \tag{33}$$

$$1 - A_o^i(r_o^i, Q_o^i) \geq S \tag{34}$$

$$r_o^i, Q_o^i \geq 0 \qquad\qquad r_o^i, Q_o^i \ \text{: integer} \tag{35}$$

The Lagrangian for this problem is

$$L = c_i (r_o^i - \theta_o^i + \frac{Q_o^i}{2}) + \nu(\frac{\lambda_o^i}{Q_o^i} - F) - \mu(G_o^i(r_o^i) - S). \tag{36}$$

Differentiating L with respect to Q_o^i and solving for Q_o^i yields

$$\frac{\partial L}{\partial Q_o^i} = \frac{c_i}{2} - \frac{\nu \lambda_o^i}{(Q_o^i)^2} = 0, \qquad i = 1,2,..., N. \tag{37}$$

$$Q_o^i = \sqrt{\frac{2\nu\lambda_o^i}{c_i}} \tag{38}$$

Differentiating L with respect to r_o^i and solving for r_o^i yields

$$\frac{\partial L}{\partial r_o^i} = c_i - \mu\Phi\left(\frac{r_o^i - \theta_o^i}{\sqrt{\theta_o^i}}\right)\frac{1}{\sqrt{\theta_o^i}} = 0, \qquad i = 1,2,\dots,N. \tag{39}$$

$$r_o^i = \theta_o^i + \sqrt{-2\theta_o^i \ln\left(\sqrt{2\pi\theta_o^i}\,\frac{c_i}{\mu}\right)} \tag{40}$$

Since we want to restrict $Q_o^i \geq 1$ and $r_o^i \geq \underline{r_o^i}$ we modify these formulae to:

$$Q_o^i = \max\left\{\sqrt{\frac{2\nu\lambda_o^i}{c_i}}, 1\right\}, \qquad i = 1,2,\dots,N, \tag{41}$$

$$r_o^i = \begin{cases} \theta_i + \sqrt{-2\theta_o^i \ln\left(\sqrt{2\pi\theta_o^i}\,\frac{c_i}{\mu}\right)}, & \text{if } \sqrt{2\pi\theta_o^i}\,\frac{c_i}{\mu} \leq 1 \\ \underline{r_o^i}, & \text{otherwise.} \end{cases} \qquad i = 1,2,\dots,N. \tag{42}$$

3. Type II heuristic

(1) We approximate the average number of stockouts during the replenishment lead time, $a(r_o^i)$ is given by expression (43).

$$a(r_o^i) = \sum_{u=r_o^i}^{\infty}[1 - P(u-1)] = \sum_{u=r_o^i}^{\infty}(u - r_o^i)\,p(u), \qquad i = 1,2,\dots,N. \tag{43}$$

(2) Service for item i during the replenishment lead time is approximated by $1 - \{a(r_o^i)/Q_o^i\}$.

$$Min \quad c_i\,(r_o^i - \theta_o^i + Q_o^i/2) \tag{44}$$

Subject to: $\lambda_o^i / Q_o^i \leq F$ (45)

$$1 - \frac{\int_{r_o^i}^{\infty}[1 - \Phi((t - \theta_o^i)/\sqrt{\theta_o^i}\,]dt}{Q_o^i} \geq S \tag{46}$$

$$r_o^i, Q_o^i \geq 0 \qquad r_o^i, Q_o^i \ \text{: integer} \tag{47}$$

The Lagrangian for this problem is

$$L = \sum_{i=1}^{N} c_i (r_o^i - \theta_o^i + \frac{Q_o^i}{2}) + \nu(\frac{\lambda_o^i}{Q_o^i} - F) - \mu\left\{(1 - \frac{\int_{r_o^i}^{\infty}[1-\Phi((t-\theta_o^i)/\sqrt{\theta_o^i}]dt}{Q_o^i} - S)\right\}. \tag{48}$$

Differentiating L with respect to Q_o^i and solving for Q_o^i yields

$$\frac{\partial L}{\partial Q_o^i} = \frac{c_i}{2} - \frac{\mu\int_{r_o^i}^{\infty}[1-\Phi((t-\theta_o^i)/\sqrt{\theta_o^i}]dt}{(Q_o^i)^2} = 0, \qquad i = 1,2,...,N, \tag{49}$$

$$Q_o^i = \sqrt{\frac{2}{c_i}\left\{\nu\lambda_o^i + \mu\int_{r_o^i}^{\infty}[1-\Phi((t-\theta_o^i)/\sqrt{\theta_o^i}]dt\right\}}. \tag{50}$$

$$\frac{\partial L}{\partial r_o^i} = c_i - \frac{\mu}{Q_o^i}\left\{1 - \Phi\left(\frac{r_o^i - \theta_o^i}{\sqrt{\theta_o^i}}\right)\frac{1}{\sqrt{\theta_o^i}}\right\} = 0, \qquad i = 1,2,...,N, \tag{51}$$

$$r_o^i = \theta_o^i + \Phi^{-1}\left(1 - \frac{c_i Q_o^i}{\mu}\right)\sqrt{Q_o^i} \tag{52}$$

Since we want to restrict $Q_o^i \ge 1$ and $r_o^i \ge \underline{r_o^i}$ we modify these formulae to:

$$Q_o^i = \max\left\{\sqrt{\frac{2}{c_i}\left\{\nu\lambda_o^i + \mu\int_{r_o^i}^{\infty}[1-\Phi((t-\theta_o^i)/\sqrt{\theta_o^i}]dt\right\}}, 1\right\}, \qquad i = 1,2,...,N, \tag{53}$$

$$r_o^i = \begin{cases} \theta_o^i + \Phi^{-1}\left(1 - \frac{c_i Q_o^i}{\mu}\right)\sqrt{Q_o^i}, & \text{if } Q_o^i c_i \le \mu \\ \underline{r_o^i}, & \text{otherwise.} \end{cases} \qquad i = 1,2,...,N. \tag{54}$$

4. Type Ⅱ heuristic

We combine the two heuristics by first computing the order quantities from the Type Ⅰ model to compute reorder points. We call this the Hybrid heuristic since it combines formulae derived from both the Type Ⅰ and Type Ⅱ models.

$$Q_o^i = \max\left\{\sqrt{\frac{2\nu\lambda_o^i}{c_i}}, 1\right\}, \qquad i = 1,2,...,N. \tag{55}$$

$$r_o^i = \begin{cases} \theta_o^i + \Phi^{-1}\left(1 - \frac{c_i Q_o^i}{\mu}\right)\sqrt{Q_o^i}, & \text{if } Q_o^i c_i \le \mu \\ \underline{r_o^i}, & \text{otherwise.} \end{cases} \qquad i = 1,2,...,N. \tag{56}$$

APPENDIX Ⅱ: Heuristic to find S_m^i for customers

1. Common assumption for customer heuristic

We can derive simple formula for S_m^i by using the following approximations for customer heuristic:

(1) Inventory for item i is given by $h_m^i(S_m^i) = S_m^i - \theta_m^i$..

(2) Demand for item i during the replenishment lead time is approximated by the normal distribution with mean θ_m^i and standard deviation $\sqrt{\theta_i}$ to match two moments with the Poisson distribution.

2. Customer heuristic

(1) We approximate the average number of stockouts during the replenishment lead time, $a(r_o^i)$ is given by expression (57).

$$a(S_m^i) = \sum_{u=S_m^i}^{\infty}[1-P(u-1)] = \sum_{u=s_m^i}^{\infty}(u-S_m^i)p(u), \qquad i=1,2,...,N,\ m=1,2,...,M. \tag{57}$$

(2) Service for item i during the replenishment lead time is approximated by $1-\{a(r_o^i)/Q_o^i\}$.

$$Min \quad c_i\ (S_m^i - \theta_m^i) \tag{58}$$

Subject to:
$$1-\frac{B_m^i(S_m^i)}{K} \geq O_m \tag{59}$$

$$S_m^i \geq 0 \qquad S_m^i \text{ : integer} \tag{60}$$

The Lagrangian for this problem is

$$L = c_i(S_m^i - \theta_m^i) - \rho\left\{(1-\frac{\int_{S_m^i}^{\infty}[1-\Phi((t-\theta_m^i)/\sqrt{\theta_m^i}]dt}{K} - O_m)\right\}. \tag{61}$$

Differentiating L with respect to S_m^i and solving for S_m^i yields

$$\frac{\partial L}{\partial S_m^i} = c_i - \frac{\rho}{K}\left\{1-\Phi\left(\frac{S_m^i - \theta_m^i}{\sqrt{\theta_m^i}}\right)\right\} = 0, \qquad i=1,2,...,N,\ m=1,2,...,M. \tag{62}$$

$$S_m^i = \theta_o^i + \Phi^{-1}\left(1-\frac{c_i K}{\rho}\right)\sqrt{\theta_m^i} \tag{63}$$

Since we want to restrict $S_m^i \geq 0$ we modify these formulae to:

$$S_m^i = \begin{cases} \theta_m^i + \Phi^{-1}\left(1-\frac{c_i K}{\rho}\right)\sqrt{\theta_m^i}, & \text{if } c_i K \leq \rho \\ 0 & \text{otherwise.} \end{cases} \qquad i=1,2,...,N,\ m=1,2,...,M. \tag{64}$$