International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 21 (2017) pp. 11361-11364

Symmetry Identities of q-Changhee Polynomials

Se-Kyo Chung
Department of Control and Instrumentation Engineering and ERI,
Gyeongsang National University, Jinju, Gyeongsangnamdo, 52828, Korea.

Orcid: 0000 - 0002 - 3470 – 5020

Jongkyum Kwon*
Department of Mathematics Education and ERI,
Gyeongsang National University, Jinju, Gyeongsangnamdo, 52828, Korea.

**Corresponding author*

Abstract

In this paper, we give some identities of q-Changhee polynomials and investigate a new and interesting identities of q-Changhee polynomials arising from the symmetry properties of the p-adic invariant integral on $\mathbb{Z}_p$.

Keywords: symmetry identities, q-Changhee polynomials.

INTRODUCTION

Let p be an odd prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ will denote the ring of p-adic integer, the field of p-adic rational numbers and the complication of the algebraic closure of $\mathbb{Q}_p$. Let q be an indeterminate in $\mathbb{C}_p$ such that $|1-q|_p < p^{-\frac{1}{p-1}}$. The p-adic norm $|\cdot|_p$ is normally defined by $|p|_p = \frac{1}{p}$ and the q-analogue of the number x is defined as $[x]_q = \frac{1-q^x}{1-q}$. Note that $\lim_{q\to 1}[x]_q = x$.

As is known, the fermionic p-adic q-integral on $\mathbb{Z}_p$ is defined by Kim to be

$$\int_{\mathbb{Z}_p} f(x)du_{-q}(x) = \lim_{N\to\infty}\frac{1}{[p^N]_{-q}}\sum_{x=0}^{p^N-1} f(x)(-q)^x \quad (1)$$

$$= \lim_{N\to\infty}\frac{1+q}{1+q^{p^N}}\sum_{x=0}^{p^N-1} f(x)(-q)^x (see\,[1,2,3,9]),$$

where $f(x)$ is a continuous functions on $\mathbb{Z}_p$.

The Euler polynomials are defined by the generating function to be

$$\frac{2}{e^t+1}e^{xt} = \sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!} \qquad (see\,[6,7,10,11,12]). \quad (2)$$

When $x = 0$, $E_n = E_n(0), (n \geq 0)$, are called Euler numbers.

From (1.1), we derived the following formula for Carlitz's q-Euler polynomials:

$$\int_{\mathbb{Z}_p} [x+y]_q^n du_{-q}(y) = E_{n,q}(x),\ (n\geq 0), \quad (see\,[3,6,10,13,17]) \quad (3)$$

When $x = 0$, $E_{n,q} = E_{n,q}(0), (n\geq 0)$, are called Carlitz's q-Euler numbers.

From (3), we note that

$$\begin{aligned} E_{n,q}(x) &= \int_{\mathbb{Z}_p}[x+y]_q^n du_{-q}(y) \\ &= \sum_{l=0}^{n}\binom{n}{l} q^{lx}\int_{\mathbb{Z}_p}[y]_q^l du_{-q}(y)[x]_q^{n-l} \quad (4) \\ &= \sum_{l=0}^{n}\binom{n}{l} q^{lx} E_{l,q}[x]_q^{n-l}, \quad (see\ [10]) \end{aligned}$$

The Changhee polynomials are defined by the generating function to be

$$\frac{2}{2+t}(1+t)^x = \sum_{n=0}^{\infty} Ch_n(x)\frac{t^n}{n!}, \qquad (see\,[5,15,18]). \quad (5)$$

When $x = 0$, $Ch_n = Ch_n(0)$ are called Changhee numbers.

The stirling number of the first kind is defined by the generating function to be

$$\frac{1}{n!}(\log(1+t))^n = \sum_{m=n}^{\infty} S_1(m,n)\frac{t^m}{m!}, \qquad (see\,[7,8]). \quad (6)$$

The stirling number of the second kind is defined by the generating function to be

$$\frac{1}{n!}(e^t-1)^n=\sum_{m=n}^{\infty}S_2(m,n)\frac{t^m}{m!}, \qquad (see\,[7,8]). \tag{7}$$

From (2) and (5), we get

$$E_n(x)=\sum_{k=0}^{n}S_2(n,k)Ch_k(x),\ Ch_n(x)=\sum_{k=0}^{n}S_1(n,k)E_k(x),\ (n\geq 0), \quad (see\,[3]). \tag{8}$$

Recently, several authors have studies symmetric identities for Bernoulli, Euler and special polynomials arising from p-adic integral on $\mathbb{Z}_p$ (see [4, 8, 12, 14, 16, 19]).

In this paper, we give some identities of q-Changhee polynomials and investigate a new and interesting identities of q-Changhee polynomial arising from the symmetry properties of the p-adic invariant integral on $\mathbb{Z}_p$

Symmetry identities of q-Changhee polynomials

In this section, we will investigate interesting symmetry identities of the q-Changhee polynomials.

The q-Changhee polynomials are defined by the generating function to be

$$\int_{\mathbb{Z}_p}(1+t)^{[x+y]_q}\,d\mu_{-q}(y)=\sum_{n=0}^{\infty}Ch_{n,q}(x)\frac{t^n}{n!}, \qquad (see\,[3]). \tag{9}$$

When $x=0$, $Ch_{n,q}(0)=Ch_{n,q}$ are called q-Changhee numbers.

Throughout this section w_1, w_2, w_3 are odd positive integers.

Given integers k_1 and k_2, we consider symmetric identities for $Ch_{n,q}(x)$.

Let us observe that

$$\begin{aligned}
&\int_{\mathbb{Z}_p}(1+t)^{[w_1w_2y+w_1w_2w_3x+w_3w_2k_1+w_3w_1k_2]_q}\,d\mu_{-q^{w_1w_2}}(y)\\
&=\frac{[2]_{q^{w_1w_2}}}{2}\lim_{N\to\infty}\sum_{m=0}^{w_3-1}\sum_{y=0}^{p^N-1}(1+t)^{[w_2w_3y+w_1w_2w_3x+w_1w_3k_1+w_1w_2k_2]_q}(-q)^{w_1w_2(m+w_3y)}\\
&=\frac{[2]_{q^{w_1w_2}}}{2}\lim_{N\to\infty}\sum_{m=0}^{w_3-1}\sum_{y=0}^{p^N-1}(1+t)^{[w_2w_3(k+w_1y)+w_1w_2w_3x+w_1w_3k_1+w_1w_2k_2]_q}\\
&\times(-1)^{m+y}q^{w_1w_2m}q^{w_1w_2w_3y}\\
&=\frac{[2]_{q^{w_1w_2}}}{2}\sum_{m=0}^{w_3-1}(-1)^m q^{w_1w_2m}\lim_{N\to\infty}\sum_{y=0}^{p^N-1}(1+t)^{[w_2w_3(k+w_1y)+w_1w_2w_3x+w_1w_3k_1+w_1w_2k_2]_q}\\
&\times(-q)^{w_1w_2w_3y}.
\end{aligned} \tag{10}$$

Thus, we have

$$\begin{aligned}
&\frac{2}{[2]_{q^{w_1w_2}}}\int_{\mathbb{Z}_p}(1+t)^{[w_1w_2y+w_1w_2w_3x+w_2w_3k_1+w_1w_3k_2]_q}\,d\mu_{-q^{w_1w_2}}(y)\\
&=\sum_{m=0}^{w_3-1}(-1)^m q^{w_1w_2m}\lim_{N\to\infty}\sum_{y=0}^{p^N-1}(1+t)^{[w_2w_3(k+w_1y)+w_1w_2w_3x+w_1w_3k_1+w_1w_2k_2]_q}\\
&\times(-q)^{w_1w_2w_3y}.
\end{aligned} \tag{11}$$

Now, we set

$$\begin{aligned}
I(w_1,w_2,w_3)&=\frac{2}{[2]_{q^{w_1w_2}}}\sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}(-1)^{k_1+k_2}\\
&\times\int_{\mathbb{Z}_p}(1+t)^{[w_1w_2y+w_1w_2w_3x+w_2w_3k_1+w_1w_3k_2]_q}\,d\mu_{-q^{w_1w_2}}(y)\\
&=\lim_{N\to\infty}\sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}\sum_{m=0}^{w_3-1}\sum_{y=0}^{p^N-1}(1+t)^{[w_1w_2(m+w_3y)+w_1w_2w_3x+w_2w_3k_1+w_1w_3k_2]_q}\\
&\times(-1)^{k_1+k_2+m}q^{w_2w_3k_1+w_1w_3k_2+w_1w_2m}.
\end{aligned} \tag{12}$$

Thus we obtain the following theorem.

Theorem 2.1. Let w_1, w_2, w_3 be odd positive integers. Then, the following

$$\begin{aligned}
&\frac{2}{[2]_{q^{w_1w_2}}}\sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}(-1)^{k_1+k_2}\\
&\times\int_{\mathbb{Z}_p}(1+t)^{[w_1w_2y+w_1w_2w_3x+w_2w_3k_1+w_1w_3k_2]_q}\,d\mu_{-q^{w_1w_2}}(y)
\end{aligned} \tag{13}$$

is invariant under any permutation of w_1, w_2, w_3 in the symmetry group of degree 3.

Now, we consider the following equality.

$$\begin{aligned}
&[w_1w_2y+w_1w_2w_3x+w_3w_2k_1+w_3w_1k_2]_q\\
&=\frac{1-q^{w_1w_2}}{1-q}\,\frac{1-q^{w_1w_2(y+w_3x+\frac{w_3}{w_1}k_1+\frac{w_3}{w_2}k_2)}}{1-q^{w_1w_2}}\\
&=[w_1w_2]_q\left[y+w_3x+\frac{w_3}{w_1}k_1+\frac{w_3}{w_2}k_2\right]_{q^{w_1w_2}}.
\end{aligned} \tag{14}$$

Using (14), we obtain

$$\begin{aligned}
&\int_{\mathbb{Z}_p}(1+t)^{[w_1w_2y+w_1w_2w_3x+w_2w_3k_1+w_1w_3k_2]_q}\,d\mu_{-q^{w_1w_2}}(y)\\
&=\int_{\mathbb{Z}_p}e^{[w_1w_2]_q\left[y+w_3x+\frac{w_3}{w_1}k_1+\frac{w_3}{w_2}k_2\right]_{q^{\omega_1\omega_2}}\log(1+t)}\,d\mu_{-q^{w_1w_2}}(y)\\
&=\sum_{n=0}^{\infty}[w_1w_2]_q^n\int_{\mathbb{Z}_p}\left[y+w_3x+\frac{w_3}{w_1}k_1+\frac{w_3}{w_2}k_2\right]_{q^{w_2w_3}}^n d\mu_{-q^{w_1w_2}}(y)\frac{(\log(1+t))^n}{n!}\\
&=\sum_{m=0}^{\infty}\left(\sum_{n=0}^{\infty}[w_1w_2]_q^n E_{n,q^{w_1w_2}}(w_3x+\frac{w_3}{w_1}k_1+\frac{w_3}{w_2}k_2)S_1(m,n)\right)\frac{t^m}{m!}
\end{aligned} \tag{15}$$

From (12) and (15), we have

$$\begin{aligned} I(w_1,w_2,w_3) &= \sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}(-1)^{k_1+k_2} \\ &\times \int_{\mathbb{Z}_p}(1+t)^{[w_1w_2y+w_1w_2w_3x+w_2w_3k_1+w_1w_3k_2]_q}\,d\mu_{-q^{w_1w_2}}(y) \\ &= \sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}\sum_{m=0}^{\infty}\Big(\sum_{n=0}^{\infty}(-1)^{k_1+k_2}\left[w_1w_2\right]_q^n \\ &\times E_{n,q^{w_1w_2}}(w_3x+\frac{w_3}{w_1}k_1+\frac{w_3}{w_2}k_2)S_1(m,n)\Big)\frac{t^m}{m!}. \end{aligned} \tag{16}$$

Thus we obtain the following theorem.

Theorem 2.2. Let w_1, w_2, w_3 be odd positive integers.

$$\sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}\sum_{m=0}^{\infty}\Big(\sum_{n=0}^{\infty}(-1)^{k_1+k_2}\left[w_1w_2\right]_q^n \times E_{n,q^{w_1w_2}}(w_3x+\frac{w_3}{w_1}k_1+\frac{w_3}{w_2}k_2)S_1(m,n)\Big)\frac{t^m}{m!} \tag{17}$$

is invariant under any permutation of w_1, w_2, w_3 in the symmetry group of degree 3.

Now, we consider the following equality.

$$\begin{aligned} \left[y+w_1x+\frac{w_1}{w_2}i+\frac{w_1}{w_3}j\right]_{q^{w_2w_3}} &= \frac{1-q^{w_2w_3(y+w_1x+\frac{w_1}{w_2}i+\frac{w_1}{w_3}j)}}{1-q^{w_2w_3}} \\ &= \frac{1-q^{w_1}}{1-q^{w_2w_3}}\frac{1-q^{w_1(w_3i+w_2j)}}{1-q^{w_1}}+q^{w_1w_3i+w_1w_2j}\frac{1-q^{w_2w_3(y+w_1x)}}{1-q^{w_2w_3}} \\ &= \frac{[w_1]_q}{[w_2w_3]_q}[w_3i+w_2j]_{q^{w_1}}+q^{w_1w_3i+w_1w_2j}[y+w_1x]_{q^{w_2w_3}}. \end{aligned} \tag{18}$$

Using (18), we obtain

$$\begin{aligned} &\int_{\mathbb{Z}_p}\left[y+w_1x+\frac{w_1}{w_2}i+\frac{w_1}{w_3}j\right]_{q^{w_2w_3}}^n d\mu_{-q^{w_2w_3}}(y) \\ &= \sum_{k=0}^{n}\binom{n}{k}\Big(\frac{[w_1]_q}{[w_2w_3]_q}\Big)^{n-k}[w_3i+w_2j]_{q^{w_1}}^{n-k}q^{k(w_1w_3i+w_1w_2j)}E_{k,q^{w_2w_3}}(w_1x). \end{aligned} \tag{19}$$

From (17) and (19), we have

$$\begin{aligned} I(w_1,w_2,w_3) &= \sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}(-1)^{k_1+k_2} \\ &\times \int_{\mathbb{Z}_p}(1+t)^{[w_1w_2y+w_1w_2w_3x+w_2w_3k_1+w_1w_3k_2]_q}\,d\mu_{-q^{w_1w_2}}(y) \\ &= \sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}\sum_{m=0}^{\infty}\Big(\sum_{n=0}^{\infty}(-1)^{k_1+k_2}\left[w_1w_2\right]_q^n \\ &\times\binom{n}{k}\Big(\frac{[w_1]_q}{[w_2w_3]_q}\Big)^{n-k}[w_3i+w_2j]_{q^{w_1}}^{n-k}q^{k(w_1w_3i+w_1w_2j)}E_{k,q^{w_2w_3}}(w_1x) \\ &\times S_1(m,n)\Big)\frac{t^m}{m!}. \end{aligned} \tag{20}$$

By (20), we obtain the following theorem.

Theorem 2.3. Let w_1, w_2, w_3 be odd positive integers.

$$\begin{aligned} &\sum_{k_1=0}^{w_1-1}\sum_{k_2=0}^{w_2-1}\sum_{m=0}^{\infty}\Big(\sum_{n=0}^{\infty}(-1)^{k_1+k_2}\left[w_1w_2\right]_q^n\binom{n}{k}\Big(\frac{[w_1]_q}{[w_2w_3]_q}\Big)^{n-k} \\ &\times[w_3i+w_2j]_{q^{w_1}}^{n-k}q^{k(w_1w_3i+w_1w_2j)}E_{k,q^{w_2w_3}}(w_1x)S_1(m,n)\Big)\frac{t^m}{m!} \end{aligned} \tag{21}$$

is invariant under any permutation of w_1, w_2, w_3 in the symmetry group of degree 3.

REFERENCES

[1] L. Carlitz, "q-Bernoulli numbers and polynomials", Duke Math. J., 15 (1948), 987-1000.

[2] Y.-K. Cho, T. Kim, T. Mansour, S.-H. Rim, "On a (r; s)-analogue of Changhee and Daehee numbers and polynomials", Kyungpook Math. J., 55 (2015), no. 2, 225-232.

[3] D. V. Dolgy, G. -W. Jang, H. -I. Kwon, T. Kim, "A note on Carlitz's type q-Changhee numbers and polynomials", Adv. Stud. Contemp. Math., (Kyungshang) 27 (2017), no. 4, 451-459.

[4] D. S. Kim, T. Kim, "Some identities of symmetry for q-Bernoulli polynomials under symmetric group of degree n", Ars Combin., 126 (2016), 435-441.

[5] D. S. Kim, T. Kim, J. -J. Seo, "A note on Changhee polynomials and numbers", Adv. Studies Theor. Phys., 7 (2013), no. 20, 993-1003.

[6] D. S. Kim, T. Kim, "Some symmetric identites for the higher-order q-Euler polynomials related to symmetry group S_3 arising from p-adic q-fermionic integral on $\mathbb{Z}_p$", Filomat, 30 (2016), no. 7, 1717-1721.

[7] D. S. Kim, N. Lee, J. Na, K. H. Park, "Identities of symmetry for higher-order Euler polynomials in three variables (I)", Adv. Stud. Contemp. Math., (Kyungshang) 22 (2012), no. 1, 51-74.

[8] D. S. Kim, N. Lee, J. Na, K. H. Park, "Abundant symmetry for higher-order Bernoulli polynomials (I)", Adv. Stud. Contemp. Math., (Kyungshang) 23 (2013), no. 3, 461-482.

[9] T. Kim, "q-Volkenborn integration", Russ. J. Math. Phys., 9 (2002), no. 3, 288-299.

[10] T. Kim, "q-Euler numbers and polynomials associated with p-adic q-integrals", J. Nonlinear Math. Phys., 14 (2007), 15-27.

[11] T. Kim, "Symmetry p-adic invariant integral on $\mathbb{Z}_p$ for

Bernoulli and Euler polynomials", J. Difference Equ. Appl., 14 (2008), no. 12, 1267-1277.

[12] T. Kim, "Symmetry of power sum polynomials and multivariate fermionic p-adic invariant integral on $\mathbb{Z}_p$", Russ. J. Math. Phys., 16 (2009), no. 1, 93-96

[13] T. Kim, D. V. Dolgy, L.-C. Jang, H.-I. Kwon, "Some identities of degenerate q-Euler polynomials under the symmetry group of degree n", J. Nonlinear Sci. Appl., 9 (2016), no. 6, 4707-4712.

[14] T. Kim, D. V. Dolgy, D. S. Kim, "Some identities of q-Bernoulli polynomials under symmetry group S_3", J. Nonlinear Convex Anal., 16 (2015), no. 9, 1869-1880.

[15] T. Kim, T. Mansour, S. -H. Rim, J. -J. Seo, "A note on q-Changhee polynomials and numbers, Adv. Studies Theor. Phys., 8 (2014), no. 1, 35-41.

[16] T. Kim, D. S. Kim, "Identites of symmetry for degenerate Euler polynomials and alternating generalized falling factorial sums", Iran J. Sci. Tech. Trans. Sci., 2017, 10. 1007.

[17] N. I. Mahmudov, "On a class of q-Bernoulli and q-Euler polynomials", Adv. Diff. Equ., 2013, 2013:108.

[18] E. -J. Moon, J. -w. Park, "A note on the generalized q-Changhee numbers of higher order", J. Comput. Anal. Appl., 20 (2016), no. 3, 470-479.

[19] J. J. Seo, T. Kim, Some identities of symmetry for Daehee polynomials arising from p-adic invariant integral on $\mathbb{Z}_p$", Proc. Jangjeon Math. Soc., 19 (2016), no. 2, 285-292.