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On Effect of Delayed Differentiation on a Multiproduct Vendor– Buyer Integrated Inventory System with Rework

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Abstract

Effect of delayed differentiation on a multiproduct vendor-buyer integrated inventory system with rework was investigated [1], using mathematical modeling and differential calculus. The optimal solution to this multiproduct problem was derived to minimize overall system cost and shorten fabrication cycle time, and effects of variation in system parameters to optimal solution and to the system were explored. This paper uses an algebraic approach in lieu of differential calculus in [1] to reexamine the problem and demonstrate that the optimal solution is obtainable using this alternative approach. The objective of this paper is to provide a simplified method for managers/practitioners in production planning and control, with a means to effectively resolve the problem without derivatives.

Keywords: Operations research, multiproduct fabrication system, delayed differentiation, vendor-buyer system, algebraic approach, multi-delivery, rework

INTRODUCTION

Managers of production planning constantly seek ways to increase machine utilization, assure quality of products, and reduce relevant operating costs. The most economical batch size was first introduced by Taft [2] to minimize total production-inventory cost, wherein some simple assumptions of his model include perfect production process, fabrication of single product, and continuous end item issuing policy. However, production of defective items is inevitable, due to various unexpected or uncontrollable factors. Many studies have been performed to address different aspects of imperfect production situations and their subsequent matters [3-12]. Certain defective/nonconforming features of products can sometimes be reworked, so they can keep hold of the acceptable quality [13-18].Also, in real supply-chain systems, the distribution of end products are often handled by diverse discontinuous multi-shipment policies [19-25].

In order to increase machine utilization, fabrication of multiproduct on a single machine can be an effective operating strategy. During past decades, studies relating to various aspects of multi- product fabrication systems have been extensively explored [26-32]. Further, when multiple products sharing a common part, the delayed differentiation strategy is often considered with the aim of shortening production cycle length and reducing total relevant system costs [33-38]. Chiu et al. [1] derived the optimal production-shipment solutions and investigated the effect of delayed differentiation on a multiproduct vendor-buyer integrated inventory system with rework, using mathematical modeling and conventional differential calculus method.

Unlike the needs of using differential calculus, Grubbstrom and Erdem [39] introduced an algebraic approach to solve a specific economic order quantity (EOQ) problem without using derivatives. A few studies employed the same/similar approach to resolve various aspects of lot sizing problems in fabrication and/or vendor-buyer integrated supply-chain systems [40-42]. Likewise, this paper extends such a simplified approach to reexamine the multiproduct problem described in [1]. and demonstrate that the optimal solutions can be derived without derivatives.

THE PROBLEM, MODELING AND PROPOSED METHOD

The problem as in [1] is a multi-product single-machine vendor–buyer integrated system with features of delayed differentiation and rework process. Annual product demand is λ_i for L different products (where i = 1, 2, …, L) and these products share a *common part*. Two-stage fabrication process is used, wherein the first stage only produces common parts, at a rate of $P_{1,0}$, and upon completion of stage 1, L different lots of *common parts* are made ready for the production in the second stage, where L different end products are fabricated at a rate of $P_{1,i}$ (where i = 1, 2, …, L) in sequence under the common production cycle time policy (Figure 1). The objectives are to increase utilization, shorten production cycle time, and minimize total production- inventory-delivery costs.

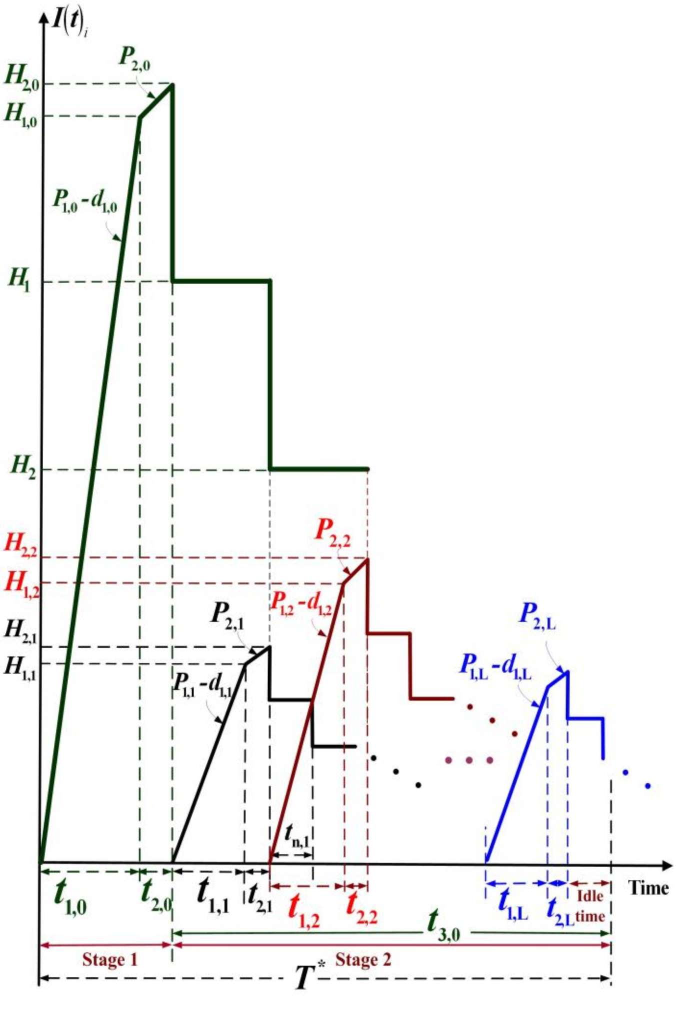

Figure 1: Inventory status of common parts and end products in the proposed multiproduct vendor-buyer integrated system [1]

During the production processes in each stage, x_i portion of nonconforming items may randomly be produced at a rate of $d_{1,i}$ (where $d_{1,i} = P_{1,i}\, x_i$ and i = 0, 1, 2, …, L; with i = 0 denotes it is in stage 1). All nonconforming items are assumed to be repairable, thru rework processes at a rate of $P_{2,i}$ (where i = 0, 1, 2, …, L) in the end of regular production processes in each stage (see Figures 1 and 2). No shortages are allowed, so we have $(P_{1,i} - d_{1,i} - \lambda_i) > 0$ for i = 0, 1, 2, …, L.

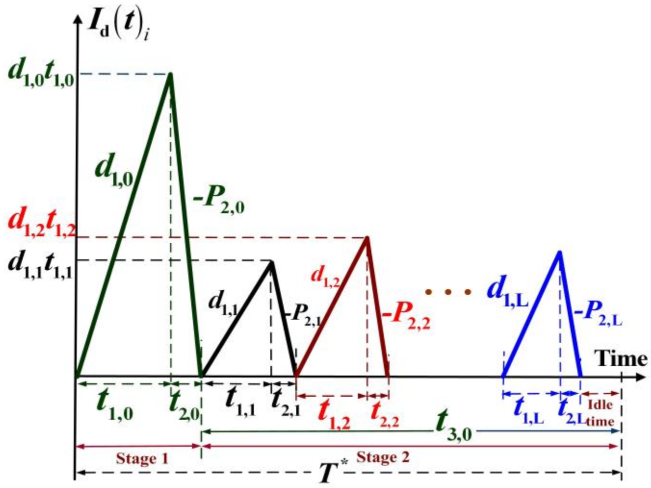

Figure 2: Inventory status of nonconforming common parts and nonconforming end products in the proposed multiproduct vendor-buyer integrated system [1]

In the end of rework process $t_{2,i}$, fixed quantity n installments of the finished lot are distributed to buyers, at a fixed interval of time in delivery time $t_{3,i}$ (see Fig. 1). Additional notation used in this study includes the following (where i = 1, 2, …, L stands for L different products in stage 2; and i = 0 represents the *common part* in stage 1):

Q_i – fabrication lot size for product i,

K_i – fabrication setup cost for product i,

C_i – unit production cost for product i,

$C_{R,i}$ – unit reworking cost for product i,

$h_{1,i}$ – unit holding cost for product i,

$h_{2,i}$ – unit holding cost per reworked item for product i,

$h_{3,i}$ – unit holding cost for stocks stored at buyer's side,

$h_{4,i}$ – unit holding cost for safety stocks stored at producer's side,

$K_{1,i}$ – fixed delivery cost per shipment for product i,

$C_{T,i}$ – unit delivery cost for product i,

$t_{1,i}$ – fabrication uptime for product i,

$t_{2,i}$ – rework time for product i,

$t_{3,i}$ – delivery time for product i,

T – common production cycle length - the decision variable,

n – number of fixed quantity installments of finished lot to be distributed to buyers in each cycle, the other decision variable,

α – completion rate of common part as compared to the finished product,

H_i – inventory level of common part at the time for end product i,

$H_{1,i}$ – inventory level of perfect quality items i in the end of regular production,

$H_{2,i}$ – inventory level of perfect quality items i in the end of rework process,

$t_{n,i}$ – a fixed interval of time between two consecutive deliveries in $t_{3,i}$,

$I(t)_i$ – on-hand inventory level of perfect quality items i at time t,

$I_d(t)_i$ – on-hand inventory level of defective items i at time t,

I_i – the left-over number of finished items of product i in each $t_{n,i}$ at buyer's side,

D_i – number of finished items of product i to be distributed to customer in each shipment,

$TC(T, n)$ – total relevant system cost per cycle,

E[T] – the expected common production cycle length,

E[$TC(T, n)$] – the expected total relevant system cost per cycle,

E[$TCU(T, n)$] – the expected total relevant system cost per unit time.

From Figure 1, we observe the common production cycle time and lot sizes as follows:

$$T = t_{1,i} + t_{2,i} + t_{3,i} = \frac{Q_i}{\lambda_i} \quad for\ i = 0,\ 1,\ 2,\ \cdots, L \tag{1}$$

$$Q_i = \lambda_i T \quad for\ i = 1,\ 2, \cdots, L \tag{2}$$

$$Q_0 = \sum_{i=1}^{L} Q_i = \lambda_0 T \tag{3}$$

The prerequisite condition of the proposed system [1] is

$$\left[\left(t_{1,0} + t_{2,0} \right) + \sum_{i=1}^{L} \left(t_{1,i} + t_{2,i} \right) \right] < T \tag{4}$$

or

$$\left[Q_0 \left(\frac{1}{P_{1,0}} + \frac{E[x_0]}{P_{2,0}} \right) + \sum_{i=1}^{L} Q_i \left(\frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}} \right) \right] < T$$

The following total relevant system cost per cycle $TC(T, n)$ consists of fabrication setup cost; variable fabrication cost; rework cost; and holding costs for reworked items, safety stock, and perfect quality stocks in both stages; fixed and variable delivery costs; and holding costs for stocks stored at buyers' side in stage two [1]:

$$TC(T,\ n) = \left\{ \begin{array}{l} K_0 + C_0 Q_0 + C_{R,0} x_0 Q_0 + h_{2,0} \left(\frac{d_{1,0} t_{1,0}}{2} \right) \left(t_{2,0} \right) + h_{4,0} \left(x_0 Q_0 \right) T \\ + h_{1,0} \left[\frac{H_{1,0} t_{1,0}}{2} + \frac{H_{2,0} + H_{1,0}}{2} \left(t_{2,0} \right) + \frac{d_{1,0} t_{1,0}}{2} \left(t_{1,0} \right) + \sum_{i=1}^{L} H_i \left(t_{1i} + t_{2i} \right) \right] \end{array} \right\}$$
$$+ \sum_{i=1}^{L} \left\{ \begin{array}{l} K_i + C_i Q_i + C_{R,i} x_i Q_i + n K_{1,i} + C_{T,i} Q_i + h_{2,i} \left(\frac{P_{2,i} t_{2,i}}{2} \right) \left(t_{2,i} \right) \\ + h_{1,i} \left[\frac{Q_i}{2} \left(t_{1,i} \right) + \frac{H_{1,i} t_{1,i}}{2} + \frac{H_{2,i} + H_{1,i}}{2} \left(t_{2,i} \right) + \left(\frac{n-1}{2n} \right) H_{2,i} t_{3,i} + \frac{d_{1,i} t_{1,i}}{2} \left(t_{1,i} \right) \right] \\ + h_{3,i} \left[\frac{n \left(D_i - I_i \right) t_{n,i}}{2} + \frac{n(n+1)}{2} I_i t_{n,i} + \frac{n I_i \left(t_{1,i} + t_{2,i} \right)}{2} \right] + h_{4,i} \left(x_i Q_i \right) T \end{array} \right\} \tag{5}$$

Substituting relevant system parameters [1] in Eq. (5) along with and using the expected value of x_i (to take the randomness of defective rate into account), and with extra derivations we obtain E[$TCU(T, n)$] as follows [1]:

$$E\left[TCU(T,\ n) \right] = E\left[TC(T,\ n) \right] / E[T] = \left\{ \frac{K_0}{T} + C_0 \lambda_0 + C_{R,0} \lambda_0 E[x_0] + z_0 T \right\}$$
$$+ \sum_{i=1}^{L} \left\{ \begin{array}{l} \left[\frac{K_i}{T} + C_i \lambda_i + C_{R,i} \lambda_i E[x_i] + \frac{n K_{1,i}}{T} + C_{T,i} \lambda_i \right] + \frac{h_{1,i} T \lambda_i^2}{2} \left\{ \delta_{2,i} - \frac{\delta_{1,i}}{n} \right\} \\ + \frac{h_{2,i} T \lambda_i^2 E[x_i]^2}{2 P_{2,i}} + \frac{h_{3,i} T \lambda_i^2}{2} \left[\frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}} + \frac{\delta_{1,i}}{n} \right] + T h_{4,i} \lambda_i E[x_i] \end{array} \right\} \tag{6}$$

where

$$z_0 = \left\{ \begin{array}{l} \frac{h_{1,0} \lambda_0^2}{2} \left[\frac{1}{P_{1,0}} + \frac{2E[x_0]}{P_{2,0}} - \frac{E[x_0]^2}{P_{2,0}} \right] + \frac{h_{2,0} \lambda_0^2 E[x_0]^2}{2 P_{2,0}} \\ + h_{1,0} \sum_{i=1}^{L} \left\{ \left(\frac{\lambda_i}{P_{1,i}} + \frac{\lambda_i E[x_i]}{P_{2,i}} \right) \left[\sum_{i=1}^{L} (\lambda_i) - \sum_{j=1}^{i} (\lambda_j) \right] \right\} + h_{4,0} \lambda_0 E[x_0] \end{array} \right\};$$

$$\delta_{1,i} = \left[\frac{1}{\lambda_i} - \frac{1}{P_{1,i}} - \frac{E[x_i]}{P_{2,i}} \right];\ \text{and}\ \delta_{2,i} = \left[\frac{1}{\lambda_i} - \frac{E[x_i]^2}{P_{2,i}} + \frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}} \right] \quad for\ i = 1,\ 2,\ \cdots, L$$

The proposed algebraic method

It can be seen that the decision variables in E[$TCU(T, n)$] (i.e., Eq. (6)) are in the forms of T, T^{-1}, nT^{-1}, and $n^{-1}T$. Let ω_0, ω_1, ω_2, ω_3, and ω_4 stand for the following:

$$\omega_0 = \left[C_0 \lambda_0 + C_{R,0} \lambda_0 E[x_0] \right] + \sum_{i=1}^{L} \left[C_i \lambda_i + C_{R,i} \lambda_i E[x_i] + C_{T,i} \lambda_i \right] \tag{7}$$

$$\omega_1 = K_0 + \sum_{i=1}^{L} K_i \tag{8}$$

$$\omega_2 = \sum_{i=1}^{L} \left(K_{1,i} \right) \tag{9}$$

$$\omega_3 = z_0 + \sum_{i=1}^{L}\left\{\frac{h_{1,i}\lambda_i^2}{2}\left(\delta_{2,i}\right) + \frac{h_{2,i}\lambda_i^2 E[x_i]^2}{2P_{2,i}} + \frac{h_{3,i}\lambda_i^2}{2}\left[\frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}}\right] + h_{4,i}\lambda_i E[x_i]\right\} \tag{10}$$

$$\omega_4 = \sum_{i=1}^{L}\left\{\frac{h_{1,i}\lambda_i^2}{2}\left(-\delta_{1,i}\right) + \frac{h_{3,i}\lambda_i^2}{2}\left(\delta_{1,i}\right)\right\} \tag{11}$$

Substitute ω_i into Eq. (6), E[$TCU(T, n)$] becomes the following:

$$E\left[TCU(T,\ n)\right] = \omega_0 + \omega_1\left(T^{-1}\right) + \omega_2\left(nT^{-1}\right) + \omega_3\left(T\right) + \omega_4\left(Tn^{-1}\right) \tag{12}$$

We can rearrange Eq. (12) as follows:

$$E\left[TCU(T,\ n)\right] = \omega_0 + \left(\sqrt{\omega_1} - \sqrt{\omega_3}T\right)^2 T^{-1} + \left(\sqrt{\omega_2} - \sqrt{\omega_4}Tn^{-1}\right)^2 nT^{-1} + 2\sqrt{\omega_1}\sqrt{\omega_3} + 2\sqrt{\omega_2}\sqrt{\omega_4} \tag{13}$$

It is noted that E[$TCU(T, n)$] will be minimized if the second and third terms of Eq. (13) equal to zeros. That is

$$T = \sqrt{\frac{\omega_1}{\omega_3}} \tag{14}$$

and

$$n = T\sqrt{\frac{\omega_4}{\omega_2}} = \sqrt{\frac{\omega_1}{\omega_3}}\sqrt{\frac{\omega_4}{\omega_2}} \tag{15}$$

Substitute ω_i into Eq. (15), we obtain the following:

$$n^* = \sqrt{\frac{\left(K_0 + \sum_{i=1}^{L} K_i\right)\sum_{i=1}^{L}\left[\frac{\lambda_i^2}{2}\left(h_{3,i} - h_{1,i}\right)\left(\delta_{1,i}\right)\right]}{\left(\sum_{i=1}^{L} K_{1,i}\right)\left\{z_0 + \sum_{i=1}^{L}\left\{\frac{h_{1,i}\lambda_i^2}{2}\left(\delta_{2,i}\right) + \frac{h_{2,i}\lambda_i^2 E[x_i]^2}{2P_{2,i}} + \frac{h_{3,i}\lambda_i^2}{2}\left[\frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}}\right] + h_{4,i}\lambda_i E[x_i]\right\}\right\}}} \tag{16}$$

Once n is found, E[$TCU(T, n)$] can be reconsidered as a function with single decision variable. So, we rearrange Eq. (12) as follows:

$$E\left[TCU(T,\ n)\right] = \omega_0 + \left(\omega_1 + \omega_2 n\right)\left(T^{-1}\right) + \left(\omega_3 + \omega_4 n^{-1}\right)\left(T\right) \tag{17}$$

or

$$E\left[TCU(T,\ n)\right] = \omega_0 + \left(\sqrt{\omega_1 + \omega_2 n} - T\sqrt{\omega_3 + \omega_4 n^{-1}}\right)^2 T^{-1} + 2\sqrt{\omega_1 + \omega_2 n}\sqrt{\omega_3 + \omega_4 n^{-1}} \tag{18}$$

It is noted that E[$TCU(T, n)$] can be minimized if the second term of Eq. (19) equals to zero. Therefore, we have the following:

$$T* = \sqrt{\frac{\omega_1 + \omega_2 n}{\omega_3 + \omega_4 n^{-1}}} \tag{19}$$

Substitute ω_i into Eq. (19), we obtain the following:

$$T^* = \sqrt{\frac{K_0 + \sum_{i=1}^{L}\left(K_i + nK_{1,i}\right)}{z_0 + \sum_{i=1}^{L}\left\{\frac{h_{1,i}\lambda_i^2}{2}\left\{\delta_{2,i} - \frac{\delta_{1,i}}{n}\right\} + \frac{h_{2,i}\lambda_i^2 E[x_i]^2}{2P_{2,i}} + \frac{h_{3,i}\lambda_i^2}{2}\left[\frac{1}{P_{1,i}} + \frac{E[x_i]}{P_{2,i}} + \frac{\delta_{1,i}}{n}\right] + h_{4,i}\lambda_i E[x_i]\right\}}} \tag{20}$$

The resulting optimal solutions (equations (16) and (20) to the problem are identical to what were obtained in [1]. The similar procedure for seeking an integer value for n can be applied by using n^- and n^+ as shown in [1], and the effects of variation in system parameters to the optimal solution and to this specific system can be fully explored accordingly.

CONCLUSIONS

Chiu et al. [1] derived the optimal solution and investigated the effect of delayed differentiation on a multiproduct vendor-buyer integrated inventory system with rework using mathematical modeling along with differential calculus. Unlike the needs of using differential calculus, an algebraic approach is employed in this paper to reexamine their proposed problem [1] and show that optimal solutions are obtainable by a simplified method. We provide a means for managers/ practitioners in the field of production planning and control (who may only have basic algebra background) to successfully resolve the problem without derivatives.

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