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Radial Plain Bearings Operating on Viscoelastic Lubricant Caused by the Melt, Taking into Account the Dependence of the Viscosity of the Lubricant and the Shear Modulus on the Pressure

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Abstract

The method of forming an exact self-similar solution for the hydrodynamic calculation of a radial plain bearing operating on a viscoelastic liquid lubricant due to the melt of the guide is given in the paper, taking into account the dependence of the viscosity of the lubricant and the shear modulus on pressure.

Based on the system of equations for the motion of the incompressible fluid with viscoelastic properties, the analytical dependence for the profile of the molten surface of the guide is obtained for the case of a "thin layer", taking into account the dependence of the viscosity of a viscoelastic lubricant and the shear modulus on pressure, and the continuity equation, expressions for the energy dissipation rate. In addition, the main operating characteristics of the friction pair under consideration are determined.

The effect of the parameter due to the melt of the guide and the Deborah number on the components of the supporting force vector and the friction force are estimated.

Keywords: hydrodynamics, radial plain bearing, viscous incompressible liquid viscoelastic lubricant, molten surface of a guide.

INTRODUCTION

Tribosystems of modern machines operate under high load-speed regimes. In these conditions, the use of liquid friction provided by liquid lubricants is the most promising. And the absolute advantage in this mode of lubrication belongs to the hydrodynamic process.

The wide application of hydrodynamic lubrication in a wide variety of machines and mechanisms required the development of a significant range of the new highly effective lubricating fluids, such as micro-polar, viscoelastic, viscoplastic compressible and incompressible. The practice of using these liquid lubricants significantly outstrips the development of models and theoretical calculations necessary for their wide application in practice.

The attempt to develop a general methodology for calculating the hydrodynamic lubrication regime for different bearing designs, lubricated with liquids with different physical and mechanical properties is of particular importance and scientific interest.

Signs

μ_0 – characteristic viscosity, Ns/m^2 ;

p' – hydrodynamic pressure, Pa;

α' – constant experimental value,

μ' – coefficient of dynamic viscosity of the lubricant, Ns/m^2;

r_0 – shaft radius, m;

r_1 – bearing radius, m;

e – eccentricity;

δ – radial clearance, m;

ω, a – parameters characterizing the adapted bearing profile;

u, v – components of the velocity vector of the lubricating medium;

α – parameter characterizing the dependence of viscosity on pressure,

β – Deborah number,

G_0 – characteristic value of the shear modulus, Pa;

G' – shear modulus, Pa.

h – oil film thickness, m;

Ω – angular rate, c^{-1};

η – relative eccentricity of the bearing bush.

It should also be noted that currently there are a number of promising directions for new research in the field of hydrodynamics, the further development of which will substantially accelerate their bringing to industrial application. Such areas include smearing of low-melting metal alloys with a melt.

TASK SETTING

The steady flow of an incompressible viscoelastic lubricant in the gap of an infinite radial plain bearing of a melt-coated low-melting coating is under consideration.

The shaft is rotating with the angular speed Ω (Fig. 1), and the bearing bushing is stationary. It is assumed that the space between the eccentrically located shaft and the bearing is completely filled with a viscoelastic lubricant, and the bearing bush is made of a material with a low melting point.

In the polar coordinate system r, θ, pole of which is located in the center of the bearing bush, the equation of the contour of the shaft of the molten surface of the bearing bush and the surface of the bearing bush covered with a metallic melt will be written as follows:

$$r' = r_0\left(1+H\right), \quad r' = r_1, \qquad r' = r_1 + \lambda' f\left(\theta\right), \qquad (1)$$

where $H = \varepsilon\cos\theta - \frac{1}{2}\varepsilon^2\sin^2\theta + ...,\quad \varepsilon = \frac{e}{r_0};$ r_0 is the shaft radius; r_1 is radius of the bearing bush covered with a metal melt; e is eccentricity; ε is relative eccentricity; $\lambda' f(\theta)$ is bounded function with $\theta \in \left[0 \div 2\pi\right]$ is subject to defining.

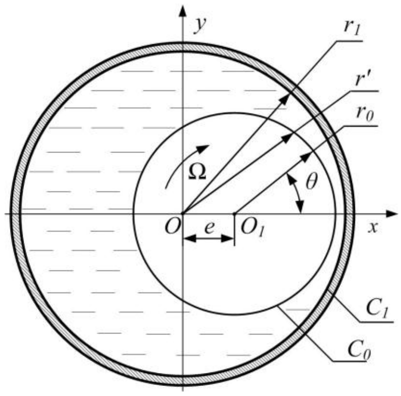

Figure 1: Design Scheme

The conditions for the motion of the infinite radial plain bearing are considered under the following assumptions:

1. All the heat released in the lubricating film goes to the melting surface of the material of the bearing bush.
2. The radial component v_r the speed is slightly less than its circumferential component v_θ.
3. The pressure is constant throughout the thickness of the lubricating film.

The dependence of the viscosity of the lubricant and the shear modulus on the pressure is expressed by the dependences:

$$\mu' = \mu_0 e^{\alpha' p'},\ G' = G_0 e^{\alpha' p'}, \qquad (2)$$

where μ_0 is characteristic viscosity, μ' is coefficient of dynamic viscosity of the lubricant, p' is hydrodynamic pressure in the lubricating layer, α' is constant, G_0 is characteristic value of the shear modulus, G' is the shear modulus.

INITIAL EQUATIONS AND BOUNDARY CONDITIONS

The system of dimensionless equations of motion of a lubricant having viscoelastic properties (the Maxwell liquid) is taken as initial equations for the case of a "thin layer", and also the continuity equation with allowance for (2):

$$\frac{\partial^2 v}{\partial r^2} = e^{-\alpha p}\frac{dp}{d\theta} + \beta e^{-\alpha p}\frac{d^2 p}{d\theta^2}, \quad \frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta} = 0, \qquad (3)$$

where $\beta = \frac{\mu_0 \Omega}{G_0}$ is the Deborah number, u, v are components of the velocity vector of the lubricating medium; p is hydrodynamic pressure in the lubricating layer.

In order to define the function $\lambda' f\left(\theta\right) = \Phi\left(\theta\right)$, caused by the molten surface of the bearing, we use the formula for the rate of energy dissipation.

$$-\frac{d\Phi\left(\theta\right)}{d\theta} = K\int\limits_{-\Phi(\theta)}^{h(\theta)}\left(\frac{\partial v}{\partial r}\right)^2 dr, \qquad (4)$$

where $K = \frac{2\mu_0 \Omega r_0}{\delta L}$ is parameter characterizing the rate of dissipation of mechanical energy, $h\left(\theta\right) = 1 - \eta\cos\theta$, L is specific heat of fusion per unit volume.

The boundary conditions for the system of equations (3) and (4) within the accuracy of $O\left(\varepsilon^2\right)$ will be written as:

$u=1, \quad v=-\eta\sin\theta$ at $r=1-\eta\cos\theta=h(\theta);$

$v=0, \quad u=0$ at

$$r=0-\Phi(\theta); \quad p(0)=p(2\pi)=\frac{p_a}{p^*}, \qquad (5)$$

where $\eta=\frac{e}{\delta}; \quad \eta_1=\frac{\lambda'}{\delta}; \quad \Phi(\theta)=\eta_1 f(\theta).$

When forming the analytical expression for hydrodynamic pressure, we transform the boundary conditions, assuming that the lubricant undergoes a shift at the entrance to the loaded zone. We believe that the lubricant enters the zone of hydrodynamic flow with complete relaxation. Then the corresponding boundary conditions for the hydrodynamic pressure can be written in the form:

$$\frac{dc}{d\theta}=0, \quad \frac{\beta}{\mu}\frac{d^2p}{d\theta^2}=0 \text{ at } \theta=0. \qquad (6)$$

If the shift occurs quickly enough compared to relaxation, the lubricant will be in a relaxed state and its shift will occur at the time it enters the loaded bearing zone. Suppose that this is the case. This is equivalent to assuming:

$$\tau_0=0 \text{ at } \theta=0. \qquad (7)$$

or, applying this condition to the entire liquid at the time it enters the bearing,

$$c'=0, \quad \frac{dp'}{d\theta}=0 \text{ at } \theta=0. \qquad (8)$$

The corresponding boundary conditions are defined as:

$\frac{dp}{d\theta}=0, \quad c=0$ at

$$\theta=0; \quad p(0)=p(2\pi)=\frac{p_a}{p^*}. \qquad (9)$$

Relations between dimensionless and dimensional variables are given in the form:

$$r'=r_0+\delta r, \ \delta=r_1-r_0; \quad v'_\theta=\Omega r_0 v;$$

$$v'_{r'}=\Omega\delta u; \quad p'=p^*p; \quad p^*=\frac{\mu_0\Omega r_0^2}{\delta^2};$$

$$\mu'=\mu_0\mu; \quad G'=G_0G; \quad c'=c^*c;$$

$$c^*=\frac{\mu_0\Omega r_0}{\delta}; \quad \alpha'=\frac{\alpha}{p^*}. \qquad (10)$$

Let's introduce the signs, let $Z = e^{-\alpha p}$. Having differentiated both sides of the equation within the accuracy of the members $O(\alpha\beta)$ the first equation of the system (3) takes the form:

$$\beta\frac{d^2Z}{d\theta^2}+\frac{dZ}{d\theta}=-\alpha\left(\frac{\tilde{C}_1}{h^2(\theta)}+\frac{\tilde{C}_2}{h^3(\theta)}\right).$$

Then the system of equations (3) and (4) takes the following form:

$$\beta\frac{d^2Z}{d\theta^2}+\frac{dZ}{d\theta}=-\alpha\left(\frac{\tilde{C}_1}{h^2(\theta)}+\frac{\tilde{C}_2}{h^3(\theta)}\right); \qquad \frac{\partial u}{\partial r}+\frac{\partial v}{\partial\theta}=0;$$

$$Z\frac{d\Phi(\theta)}{d\theta}=-K\int_0^{1-\eta\cos\theta}\left(\frac{\partial v}{\partial r}\right)^2 dr \qquad (11)$$

With correspondent boundary conditions

$u=1, \quad v=-\eta\sin\theta$ at $r=1-\eta\cos\theta;$

$v=0, \quad u=0$ at

$$r=0-\Phi(\theta); \quad Z(0)=Z(2\pi)=e^{-\alpha\frac{p_a}{p^*}};$$

$$Z''(0)=0; \quad c'(0)=0. \qquad (12)$$

Taking K as a small parameter due to the melt and the rate of energy dissipation, we seek the function $\Phi(\theta)$ as:

$$\Phi(\theta)=-K\Phi_1(\theta)-K^2\Phi_2(\theta)-K^3\Phi_3(\theta)-\ldots=H, \quad (13)$$

where $H=-K\Phi_1(\theta)-K^2\Phi_2(\theta)-K^3\Phi_3(\theta)-\ldots.$

Boundary conditions for dimensionless velocity components u and v on the contour $r=-\Phi(\theta)$ can be written as follows:

$$v(0-H(\theta))=v(0)-\left(\frac{\partial v}{\partial r}\right)_{r=0}\cdot H(\theta)-\left(\frac{\partial^2 v}{\partial r^2}\right)_{r=0}\cdot H^2(\theta)-\ldots=0;$$

$$u(0-H(\theta))=u(0)-\left(\frac{\partial u}{\partial r}\right)_{r=0}\cdot H(\theta)-\left(\frac{\partial^2 u}{\partial r^2}\right)_{r=0}\cdot H^2(\theta)-\ldots=0. \qquad (14)$$

We seek the asymptotic solution of the system of differential equations (11) with allowance for the boundary conditions (12) and (14) in the form of series under degrees of the small parameter K:

$$v=v_0(r,\theta)+Kv_1(r,\theta)+K^2v_2(r,\theta)+\ldots;$$

$$u = u_0(r,\theta) + Ku_1(r,\theta) + K^2 u_2(r,\theta) + ...;$$

$$\Phi(\theta) = -K\Phi_1(\theta) - K^2\Phi_2(\theta) - K^3\Phi_3(\theta) - \ldots;$$

$$Z = Z_0 + KZ_1(\theta) + K^2 Z_2(\theta) + K^3 Z_3(\theta)... \quad (15)$$

Performing the substitution (15) into the system of differential equations (11), taking into account the boundary conditions (12) and (14), we obtain the following equations:

- for the zeroth approximation:

$$\frac{\partial^2 v_0}{\partial r^2} = \beta \frac{d^2 Z_0}{d\theta^2} + \frac{dZ_0}{d\theta}, \quad \frac{\partial v_0}{\partial \theta} + \frac{\partial u_0}{\partial r} = 0 \quad (16)$$

With boundary conditions:

$$u_0 = 1, \quad v_0 = -\eta \sin\theta \text{ at } r = 1 - \eta\cos\theta;$$

$$v_0 = 0, \quad u_0 = 0 \text{ at } r = 0; \quad K\mathit{\Phi}_0(0) = Kg_0;$$

$$Z_0(0) = Z_0(2\pi) = \frac{p_a}{p^*}; \quad c_0'(0) = 0; \quad Z_0''(0) = 0. \quad (17)$$

– for first approximation:

$$\frac{\partial^2 v_1}{\partial r^2} = \beta \frac{d^2 Z_1}{d\theta^2} + \frac{dZ_1}{d\theta}; \quad \frac{\partial v_1}{\partial \theta} + \frac{\partial u_1}{\partial r} = 0;$$
$$Z_0 \frac{d\Phi_1(\theta)}{d\theta} = -K \int_0^{1-\eta\cos\theta} \left(\frac{\partial v_0}{\partial r}\right)^2 dr \quad (18)$$

With boundary conditions:

$$v_1 = \left(\frac{\partial v_0}{\partial r}\right)_{r=0} \cdot \Phi_1(\theta); \quad u_1 = \left(\frac{\partial u_0}{\partial r}\right)_{r=0} \cdot \Phi_1(\theta);$$

$$c_1'(0) = 0; \quad Z_1''(0) = 0;$$

$$v_1 = 0; \quad u_1 = 0 \text{ at } r = 1 - \eta\cos\theta;$$

$$Z_1(0) = Z_1(2\pi) = 0; \quad K\Phi_1(0) = K\tilde{\alpha}, \quad \Phi(0) = \Phi(2\pi) = \tilde{\alpha}. \quad (19)$$

EXACT SELF-SIMILAR SOLUTION

The exact self-similar solution of the problem for the zeroth approximation will be sought in the form:

$$v_0 = \frac{\partial \psi_0}{\partial r} + V_0(r,\theta); \quad u_0 = -\frac{\partial \psi_0}{\partial \theta} + U_0(r,\theta);$$
$$\psi_0(r,\theta) = \tilde{\psi}_0(\xi); \quad \xi = \frac{r}{1 - \eta\cos\theta}; \quad (20)$$
$$V_0(r,\theta) = \tilde{v}(\xi); \quad U_0(r,\theta) = -\tilde{u}_0(\xi) \cdot h'(\theta);$$
$$\beta \frac{d^2 Z_0}{d\theta^2} + \frac{dZ_0}{d\theta} = -\alpha\left(\frac{\tilde{C}_1}{h^2(\theta)} + \frac{\tilde{C}_2}{h^3(\theta)}\right); \quad h(\theta) = 1 - \eta\cos\theta.$$

Substituting (20) into the system of differential equations (16), taking into account the boundary conditions (17), we obtain the following system of differential equations:

$$\tilde{\psi}_0'''(\xi) = \tilde{C}_2; \quad \tilde{v}_0''(\xi) = \tilde{C}_1; \quad \tilde{u}_0'(\xi) + \varepsilon \tilde{v}_0'(\xi) = 0;$$
(21)

And boundary conditions:

$$\tilde{\psi}_0'(0) = 0, \quad \tilde{\psi}_0'(1) = 0, \quad \tilde{u}_0(1) = -\eta\sin\theta, \quad \tilde{v}_0(1) = 0;$$

$$Z_0(0) = Z_0(2\pi) = \frac{p_a}{p^*};$$

$$\tilde{u}_0(0) = 0, \quad \tilde{v}_0(0) = 1, \quad \int_0^1 \tilde{v}_0(\xi)\, d\xi = 0.$$

(22)

By the direct integration we get:

$$\tilde{\psi}_0'(\xi) = \frac{\tilde{C}_2}{2}(\xi^2 - \xi), \quad \tilde{v}_0(\xi) = \tilde{C}_1 \frac{\xi^2}{2} - \left(1 + \frac{\tilde{C}_1}{2}\right)\xi + 1,$$

$$\tilde{C}_1 = 6, \quad \tilde{C}_2 = -6\alpha \frac{2 - 3\eta^2}{2 - 6\eta^2}. \quad (23)$$

5. Determination of hydrodynamic pressure

Solving the equation for hydrodynamic pressure $\beta \frac{d^2 Z_0}{d\theta^2} + \frac{dZ_0}{d\theta} = -\alpha\left(\frac{\tilde{C}_1}{h^2(\theta)} + \frac{\tilde{C}_2}{h^3(\theta)}\right);$ taking into account the boundary condition $Z_0(0) = Z_0(2\pi) = e^{-\alpha \frac{p_a}{p^*}}$ a $Z_0''(0) = 0$ we get:

$$Z_0 = -6\alpha\left[\theta + \frac{2\eta(-\beta\cos\theta + \sin\theta)}{\beta^2 + 1} - \frac{3\eta^2\theta}{2} - \frac{3\eta^2\left(-\beta\cos 2\theta + \frac{1}{2}\sin 2\theta\right)}{2(4\beta^2 + 1)}\right] +$$

$$+6\alpha \cdot \frac{2 - 3\eta^2}{2 - 6\eta^2}\left[\theta + \frac{3\eta(-\beta\cos\theta + \sin\theta)}{\beta^2 + 1} - 3\eta^2\theta - \frac{3\eta^2\left(-\beta\cos 2\theta + \frac{1}{2}\sin 2\theta\right)}{4\beta^2 + 1}\right] +$$

$$+e^{-\alpha\frac{p_a}{p^*}}-6\alpha\frac{6\eta^2\beta\left(\beta^2+1\right)-4\eta\beta\left(4\beta^2+1\right)}{2\left(2-6\eta^2\right)\left(4\beta^2+1\right)\left(\beta^2+1\right)}. \qquad (24)$$

In order to define $\Phi_1(\theta)$ taking into account the equation (23), we come at the following equation:

$$\frac{d\Phi_1(\theta)}{d\theta}=h(\theta)\int_0^1\left(\frac{\tilde{\psi}_0''(\xi)}{h^2(\theta)}+\frac{\tilde{v}_0'(\xi)}{h(\theta)}\right)^2 d\xi. \qquad (25)$$

Integrating the equation (25), we obtain:

$$\Phi_1(\theta)=\int_0^\theta\frac{\Delta_1 d\theta}{h^3(\theta)}+\int_0^\theta\frac{\Delta_2 d\theta}{h^2(\theta)}+\int_0^\theta\frac{\Delta_3 d\theta}{h(\theta)}. \qquad (26)$$

where

$$\Delta_1=\int_0^1\left(\tilde{\psi}''(\xi)\right)^2 d\xi=\frac{\tilde{C}_2^2}{12};\ \Delta_2=\int_0^1 2\tilde{\psi}''(\xi)\cdot\tilde{v}'(\xi)d\xi=\tilde{C}_2;\ \Delta_3=\int_0^1\left(\tilde{v}'(\xi)\right)^2 d\xi=4. \qquad (27)$$

Solving equations (26) taking into account (27) and condition $K\Phi_1(0)=K\tilde{\alpha}$, we obtain:

$$\Phi_1(\theta)=\frac{\tilde{C}_2^2}{12}\left[\theta+3\eta\sin\theta-3\eta^2\left(\theta+\frac{1}{2}\sin 2\theta\right)\right]+\tilde{C}_2\left[\theta+2\eta\sin\theta-\frac{3}{2}\eta^2\left(\theta+\frac{1}{2}\sin 2\theta\right)\right]+$$
$$+4\left[\theta+\eta\sin\theta-\frac{\eta}{2}\left(\theta+\frac{1}{2}\sin 2\theta\right)\right]+\tilde{\alpha}. \qquad (28)$$

Then for the first approximation we get:

$$v_1=\frac{\partial\psi_1}{\partial r}+V_1(r,\theta);\quad u_1=-\frac{\partial\psi_1}{\partial\theta}+U_1(r,\theta);$$
$$\psi_1(r,\theta)=\tilde{\psi}_1(\xi);\quad \xi=\frac{r}{1-\eta\cos\theta};$$
$$V_1(r,\theta)=\tilde{v}(\xi);\quad U_1(r,\theta)=-\tilde{u}_1(\xi)\cdot h'(\theta); \qquad (29)$$
$$\beta\frac{d^2Z_1}{d\theta^2}+\frac{dZ_1}{d\theta}=-\alpha\left(\frac{\tilde{\tilde{C}}_1}{h^2(\theta)}+\frac{\tilde{\tilde{C}}_2}{h^3(\theta)}\right);\quad h(\theta)=1-\eta\cos\theta.$$

Substituting (29) into the system of differential equations (18), taking into account the boundary conditions (19), we obtain the following system of differential equations:

$$\tilde{\psi}_1'''(\xi)=\tilde{\tilde{C}}_2;\quad \tilde{v}_1''(\xi)=\tilde{\tilde{C}}_1;\ \tilde{u}_1'(\xi)+\xi\tilde{v}_1'(\xi)=0; \qquad (30)$$

And boundary conditions:

$$\tilde{\psi}_1'(0)=0,\quad \tilde{\psi}_1'(1)=0,\quad \tilde{u}_1(1)=0,\quad \tilde{v}_1(1)=0;$$

$$Z_1(0) = Z_1(2\pi) = \frac{p_a}{p^*}; \quad Z_1''(0) = 0;$$

$$\tilde{u}_1(0) = 0, \quad \tilde{v}_1(0) = M, \quad \int_0^1 \tilde{v}_1(\xi)\, d\xi = 0. \tag{31}$$

From conditions $Z_1(0) = Z_1(2\pi) = \frac{p_a}{p^*}; \quad Z_1''(0) = 0$, we obtain

$$Z_1 = -6\alpha M \left[\theta + \frac{2\eta(-\beta\cos\theta + \sin\theta)}{\beta^2+1} - \frac{3\eta^2\theta}{2} - \frac{3\eta^2\left(-\beta\cos 2\theta + \frac{1}{2}\sin 2\theta\right)}{2\left(4\beta^2+1\right)} + \right.$$

$$+ \frac{2-3\eta^2}{2-6\eta^2}\left[\theta + \frac{3\eta(-\beta\cos\theta + \sin\theta)}{\beta^2+1} - 3\eta^2\theta - \frac{3\eta^2\left(-\beta\cos 2\theta + \frac{1}{2}\sin 2\theta\right)}{4\beta^2+1} \right] - \tag{33}$$

$$\left. - \frac{6\eta^2\beta\left(\beta^2+1\right) - 4\eta\beta\left(4\beta^2+1\right)}{2\left(2-6\eta^2\right)\left(4\beta^2+1\right)\left(\beta^2+1\right)} \right],$$

where $\tilde{\tilde{C}}_2 = -6\alpha M \frac{2-3\eta^2}{2-6\eta^2}$,

$$M = \sup_{\theta\in[0:2\pi]} \left| \left.\frac{\partial v_0}{\partial r}\right|_{r=0} \cdot \Phi_1(\theta) \right| = \sup_{\theta\in[0:2\pi]} \left| \left\{ \frac{-\eta\sin\theta}{1-\eta\cos\theta} - 3\left[1 - \eta\cos\theta + \frac{2\eta}{\beta^2+1}(\beta\sin\theta + \cos\theta) - \right.\right.\right.$$

$$- \frac{3\eta^2}{2} - \frac{3\eta^2(2\beta\sin 2\theta + \cos 2\theta)}{2\left(4\beta^2+1\right)} - \frac{2-3\eta^2}{2-6\eta^2}\left(1 - \eta\cos\theta + \frac{3\eta}{\beta^2+1}(\beta\sin\theta + \cos\theta) - 3\eta^2 - \right.$$

$$\left.\left. - \frac{3\eta^2(2\beta\sin 2\theta + \cos 2\theta)}{\left(4\beta^2+1\right)} \right) \right] - 3\beta\left[\frac{24}{\beta^2+1}(\beta\cos\theta - \sin\theta) - \frac{3\eta^2(4\beta\cos 2\theta - 2\sin 2\theta)}{2\left(4\beta^2+1\right)} - \right.$$

$$\left.\left. - \frac{2-3\eta^2}{2-6\eta^2}\left(\frac{3\eta}{\beta^2+1}(\beta\cos\theta - \sin\theta) - \frac{3\eta^2(4\beta\cos 2\theta - 2\sin 2\theta)}{\left(4\beta^2+1\right)} \right) \right] \right\} \times$$

$$\left. \times \left\{ \tilde{C}_2^2\left(\frac{13\theta}{12} + \frac{9\eta\sin\theta}{4} - \frac{7\eta^2}{4}\left(\theta + \frac{1}{2}\sin 2\theta\right) \right) + 4(\theta + \eta\sin\theta) - \frac{\eta}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + \tilde{\alpha} \right\} \right|. \tag{34}$$

Then for $Z = Z_0 + KZ_1$ we get the following expression

$$Z = -6\alpha(1+KM)\Phi(x) + e^{-\alpha\frac{p_a}{p^*}}, \tag{35}$$

where

$$\Phi(x) = \theta + \frac{2\eta(-\beta\cos\theta + \sin\theta)}{\beta^2+1} - \frac{3\eta^2\theta}{2} - \frac{3\eta^2\left(-\beta\cos2\theta + \frac{1}{2}\sin2\theta\right)}{2(4\beta^2+1)} -$$

$$-\frac{2-3\eta^2}{2-6\eta^2}\left[\theta + \frac{3\eta(-\beta\cos\theta+\sin\theta)}{\beta^2+1} - 3\eta^2\theta - \frac{3\eta^2\left(-\beta\cos2\theta+\frac{1}{2}\sin2\theta\right)}{4\beta^2+1}\right] +$$

$$+\frac{6\eta^2\beta(\beta^2+1) - 4\eta\beta(4\beta^2+1)}{2(2-6\eta^2)(4\beta^2+1)(\beta^2+1)}.$$

Then applying Taylor series expansions $e^{-\alpha p}$ and $e^{-\alpha\frac{p_a}{p^*}}$, we get

$$e^{-\alpha p} = -6\alpha(1+KM)\Phi(x) + e^{-\alpha\frac{p_a}{p^*}} \tag{36}$$

$$1 - \alpha p + \frac{\alpha^2 p^2}{2} - 1 + \alpha\frac{p_a}{p^*} - \frac{\alpha^2}{2}\left(\frac{p_a}{p^*}\right)^2 = -6\alpha(1+KM)\Phi(x). \tag{37}$$

Solving the equation (37) within the accuracy of $O(\alpha^3)$, $O\left(\left(\frac{p_a}{p^*}\right)^3\right)$ for hydrodynamic pressure we obtain

$$p = \frac{p_a}{p^*} - 6(1+KM)\Phi(x)\left(1 + \alpha\frac{p_a}{p^*} - \frac{\alpha^2}{2}\left(\frac{p_a}{p^*}\right)^2\right). \tag{38}$$

RESULTS OF THE RESEARCH AND THEIR DISCUSSION

Let us now turn to the determination of the basic operating characteristics of the bearing.

Taking into account (16), (18), and (38) for the component of the supporting force vector and the frictional force, we obtain:

$$R_x = \frac{\mu\Omega r_0^3}{\delta^2}\int_0^{2\pi}\left(p - \frac{p_a}{p^*}\right)\cos\theta\, d\theta =$$

$$= \frac{6\mu\Omega r_0^3\pi}{\delta^2}\left(1 + \alpha\frac{p_a}{p^*} - \frac{\alpha^2}{2}\left(\frac{p_a}{p^*}\right)^2\right)\left(\frac{2\eta}{\beta^2+1} - \frac{2-3\eta^2}{2-6\eta^2}\cdot\frac{3\eta}{\beta^2+1}\right)(1+KM),$$

$$R_y = \frac{\mu\Omega r_0^3}{\delta^2}\int_0^{2\pi}\left(p - \frac{p_a}{p^*}\right)\sin\theta d\theta =$$
$$= \frac{6\mu\Omega r_0^3\pi}{\delta^2}\left(1 + \alpha\frac{p_a}{p^*} - \frac{\alpha^2}{2}\left(\frac{p_a}{p^*}\right)^2\right)\left(\frac{-2\eta}{\beta^2+1} + \frac{2-3\eta^2}{2-6\eta^2}\cdot\frac{3\eta}{\beta^2+1}\right)(1+KM), \quad (38)$$

$$L_{\text{тр}} = \frac{\mu\Omega r_0^2 e^{-\alpha p}}{\delta}\int_0^{2\pi}\left[\left.\frac{\partial v_0}{\partial r}\right|_{r=0} + K\left.\frac{\partial v_1}{\partial r}\right|_{r=0}\right]d\theta =$$
$$= \frac{\mu\Omega r_0^2}{\delta}\left(1 - \alpha p + \frac{\alpha^2 p^2}{2}\right)\left[-3\alpha\pi\left(2 - 9\eta^2 - \frac{4-6\eta^2}{2-6\eta^2}\right)(1+KM) +\right.$$
$$\left. +2M\sqrt{\frac{1+\eta}{1-\eta}}arctg\,2\pi\sqrt{\frac{1+\eta}{1-\eta}}\right].$$

For the verification calculations, the following values are used based on the theoretical models obtained:

μ_0 = 0,085 Нс/м^2; η = 0,3…1 м; r_0 = 0,019995…0,0493 м;

Ω = 100…1800 с$^{-1}$; $\delta = 0{,}05 \cdot 10^{-3}\ldots 0{,}07 \cdot 10^{-3}$; K = 0,0000022…0,00052;

$p_a = 0{,}08 \div 0{,}101325$ МПа; α = 0...1; L' = 35,33…38,1 N/m^2; M = 26,5…100.

Based on the results of numerical calculations, the graphs shown in Fig. 2-5 were built.

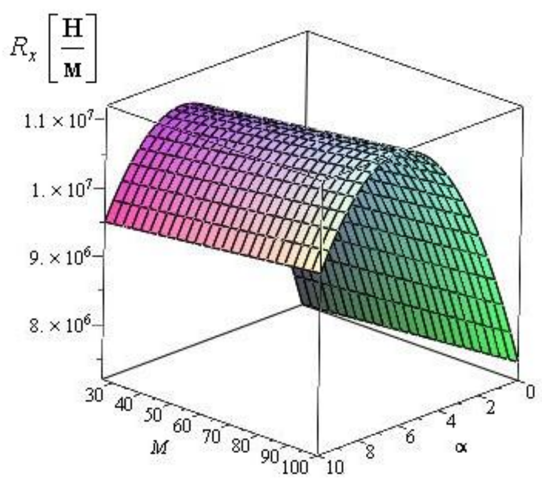

Figure 2: Dependence of the component of the supporting force (R_x) on the parameter α, which characterizes the dependence of viscosity on pressure, and on the parameter M, which characterizes the thickness of the molten film**.**

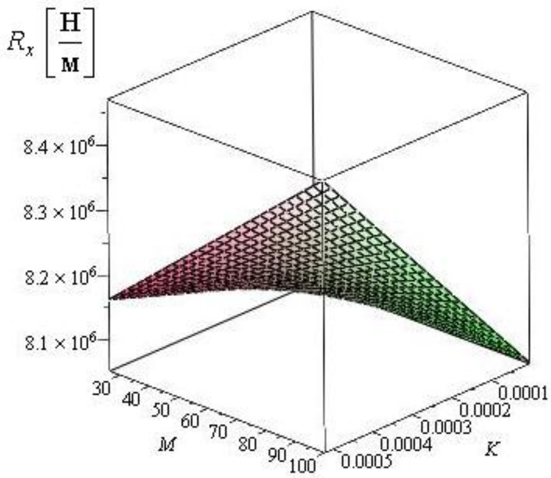

Figure 3: Dependence of the component of the supporting force (R_x) on the parameters: M, characterizing the thickness of the melted film, and K, due to the melt and the rate of energy dissipation.

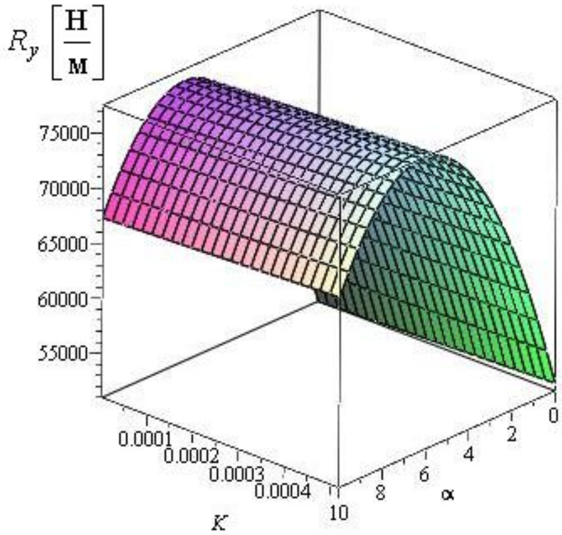

Figure 4: Dependence of the component of the supporting force (R_x) on the parameter α, characterizing the dependence of viscosity on pressure, and on the parameter K, due to the melt and the rate of energy dissipation.

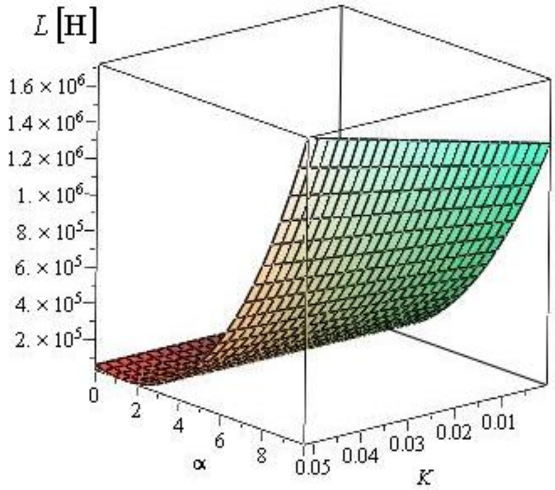

Figure 5: The dependence of the frictional force on the parameter α, which characterizes the dependence of viscosity on pressure, and on the parameter K, due to the melt and the energy dissipation rate.

CONCLUSIONS

The analysis of the calculated models and graphs allows us to draw a number of the following conclusions:

1. A refined design model of a radial plain bearing operating under conditions of hydrodynamic lubrication with a melt of a low-melting coating is obtained, taking into account the dependence of the viscosity of the lubricant and the shear modulus on pressure.
2. A significant contribution of the constructive parameter K due to the melt is shown. As the design parameter K increases, the frictional force decreases, and the bearing capacity increases.

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