

# Wedge-Shaped Sliding Supports Operating on Viscoelastic Lubricant Material Due to the Melt, Taking Into Account the Dependence of Viscosity and Shear Modulus on Pressure

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## Abstract

The method of forming an exact self-similar solution of the problem of hydrodynamic calculation of a wedge-shaped support (slider, guide) working on viscoelastic liquid lubricant caused by the melt of the guide is given in the paper, taking into account the viscosity dependence of the viscoelastic lubricant and the shear modulus from the pressure.

Based on the system of equations of motion of an incompressible fluid with viscoelastic properties for the case of a "thin layer", taking into account the viscosity dependence of the viscosity of a viscoelastic lubricant and the shear modulus from pressure, and the continuity equation, expressions for the energy dissipation rate, an analytical dependence for the profile of the molten surface guide. In addition, the main operating characteristics of the friction pair under consideration are determined.

The effect of the parameter due to the melt of the guide and the Deborah number on the bearing capacity and the friction force are estimated.

**Keywords:** Hydrodynamics, sliding support (slider, guide), viscous incompressible liquid viscoelastic lubricant, molten surface of a guide.

## INTRODUCTION

Continuous development and improvement of piston and rotary machines, as well as the desire of modern engineers and designers to create energy-efficient plants for various purposes, led to the development of new structural and lubricating materials with improved tribological properties. For example, various kinds of motor oils with high anti-wear properties are promising, providing serviceability under extreme conditions (at elevated loads, speeds and temperatures) and allowing to reduce power losses due to friction in tribosystems. Such oils are called energy-saving, but their rheological behavior is significantly different from ordinary Newtonian fluids. Many of these engine oils, which are a mixture of base components and an additive package for

various purposes, are non-Newtonian fluids. The most known non-Newtonian properties of these oils include the dependence of their viscosity on the shear rate (pseudoplasticity, dilatancy), viscoelastic effects (relaxation of tangential stresses, appearance of normal shear stresses), as well as their structural heterogeneity (suspensions, gas content, etc.).

Thus, in many works, the researchers used the power law of Ostwald-de Vela to describe the dependence of the viscosity of a lubricant on shear rate. This law has a fairly simple mathematical form, although its scope of application is limited. There are also approaches that more realistically reflect the dependence of the viscosity of the lubricant on the shear rate. These include the models of Ghezime and Cross, but methods for determining the parameters characterizing the behavior of the lubricant, both at reduced and at elevated shear rates, are not indicated in the melt of the surface of the guide coating by the melt of the low-melting coating [1-11].

Thus, the development of theoretical bases for the calculation of tribosystems lubricated by a metallic melt, taking into account the rheological properties of a viscoelastic lubricant, the rationale for the dynamics and lubrication model that outstrips the real processes occurring in the lubricating layer, and the creation of algorithms and software for solving practical problems, and mechanisms determines the relevance of this article.

## TASK SETTING.

A wedge-shaped support consisting of a "slider-guide" system is considered. It is assumed that the surfaces of the slider and the guide are separated by a layer of lubricant having viscoelastic properties, the slider is stationary, and the guide made of a material with a low melting point moves toward the narrowing of the gap at a speed  $u^*$  (Fig. 1).

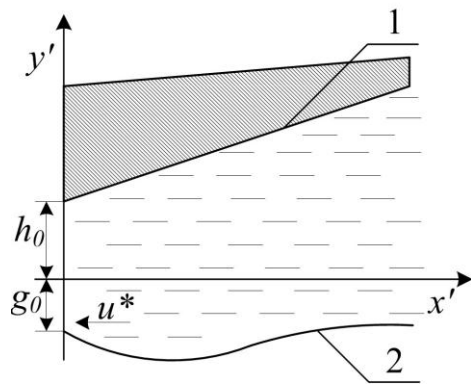


Fig. 1. Working Scheme

In a Cartesian coordinate system the  $xOy$  equation of the contour of the slider and the molten surface of the guide can be written in the form.

$$y = h_0 + l \operatorname{tg} \alpha^*, \quad y = -\Phi(x). \quad (1)$$

where  $\Phi(x)$  is thickness of the molten film in the initial section;  $\alpha^*$  is angle of slope of the linear contour of the slider

to the axis  $Ox$ ;  $l \frac{\operatorname{tg} \alpha^*}{h_0}$  is small dimensionless value;  $h_0$

is thickness of the lubricating film in the initial section;  $l$  is length of the fixed working surface of the bearing (slider).

We assume that the dependence of the viscosity and shear modulus on pressure is expressed by formulas

$$\mu' = \mu_0 e^{\tilde{\alpha} p'}, \quad G' = G_0 e^{\tilde{\alpha} p'}. \quad (2)$$

where  $\mu'$  is coefficient of dynamic viscosity of the lubricant;  $\mu_0$  is the intrinsic viscosity of the Newtonian lubricant;  $p'$  is hydrodynamic pressure in the lubricating layer;  $\tilde{\alpha}$  is the experimental constant;  $G_0$  is characteristic value of the shear modulus,  $G'$  is shear modulus.

When forming the analytical solution of the problem under consideration, the following assumptions are made:

1. The pressure  $p$  is constant at the lubricating film surface, given by the equation (1).
2. The liquid medium is a viscous incompressible fluid.
3. All the heat released in the lubricating film goes to the melting surface of the material of the guide.

## INITIAL EQUATIONS AND BOUNDARY CONDITIONS

We consider a system of dimensionless equations of motion of a lubricant having viscoelastic properties for the case of a "thin layer" and the continuity equation as initial equations.

$$\frac{\partial^2 v}{\partial y^2} = \frac{1}{\mu} \frac{dp}{dx} + \frac{\beta}{\mu} \frac{d^2 p}{dx^2}, \quad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0, \quad (3)$$

Here  $u, v$  are components of the velocity vector of the lubricating medium;  $p$  is hydrodynamic pressure in the lubricating layer;  $\mu$  is coefficient of dynamic viscosity,

$$\beta = \frac{\mu_0 u^*}{GLh}$$

is the Deborah number.

In order to define functions  $\Phi(x)$ , caused by a molten guide, we use the formula for the rate of energy dissipation

$$\frac{d\Phi(x)}{dx} = -K \int_{-\Phi(x)}^{h(x)} \left( \frac{\partial v}{\partial y} \right)^2 dy, \quad K = \frac{2\mu_0 u^*}{h_0 L}. \quad (4)$$

where  $L$  is specific heat of fusion per unit volume.

## Signs

$\mu_0$  – characteristic viscosity,  $\text{Ns/m}^2$ ;

$p'$  – hydrodynamic pressure, Pa;

$\alpha^*$  – constant experimental value,

$\mu'$  – coefficient of dynamic viscosity of the lubricant,  $\text{Ns/m}^2$ ;

$\alpha^*$  – angle of slope of the slider with linear contour to the axis  $Ox$ ;

$l$  – slider length, m;

$\omega$  – parameter characterizing the adapted profile of the slider;

$h_0$  – thickness of the lubricating film at the entrance to the zone of the hydrodynamic wedge, m;

$\omega, a$  – parameters characterizing the adapted bearing profile;

$u, v$  – components of the velocity vector of the lubricating medium;

$\alpha$  – parameter characterizing the dependence of viscosity on pressure,

$u^*$  – speed of the guide, m/s;

$h$  – thickness of oil film, m;

$\eta$  – relative eccentricity of the bearing bush;

$\beta$  – Deborah number,

$G_0$  – characteristic value of the shear modulus, Pa;

$G'$  – shear modulus, Pa.

The system of equations (3) and (4) is solved using the following boundary conditions:

1. Equivalence to zero of the components of the velocity field of the lubricating layer in the neighborhood of the working surface (the slider)

$$u = 0, \quad v = 0 \quad \text{at} \quad y = 1 + \eta x = h(x);$$

$$\eta = \frac{tg\alpha^*}{h_0}; \quad (5)$$

2. Equivalences of the components of the velocity field in the lubricating layer to the components of the velocity of the guiding surface

$$u = 0, \quad v = -1 \quad \text{at} \quad y = -\Phi(x); \quad (6)$$

3. The pressure at the entrance to the zone of the hydrodynamic contact and at its exit from it is considered

$$\text{equal to } \frac{P_a}{P^*}$$

$$p(0) = p(1) = \frac{P_a}{P^*}; \quad (7)$$

4. The appearance of viscoelastic properties of the lubricant is associated with the prediction of its special state at the entrance to the region of hydrodynamic pressure. Assuming that the lubricant enters the bearing in the absence of an elastic deformation component, the characteristic of the special state of the lubricating medium will be written as follows:

$$\frac{dc}{dx} = 0, \quad \frac{d^2p}{dx^2} = 0 \quad \text{at} \quad x = 0; \quad (8)$$

5. If the lubricant in an unloaded state is subjected to shear at a certain speed when entering the loaded bearing zone, then the following assumption:

$$\tilde{c} = 0 \quad \text{at} \quad x = 0$$

following

$$c = 0, \quad \frac{dp}{dx} = 0 \quad \text{at} \quad x = 0; \quad (9)$$

$$6. \quad \Phi(x) = \tilde{g}_0 = Kg_0 \quad \text{at} \quad x = 0. \quad (10)$$

The transition to dimensionless variables is carried out by the formulas:

$$u' = u^* u; \quad v' = u^* \varepsilon v; \quad x' = lx; \quad y' = h_0 y; \quad p' = p^* p; \\ \varepsilon = \frac{h_0}{l}; \quad \mu' = \mu_0 \mu; \quad G' = G_0 G; \quad C' = C^* C; \quad (11)$$

$$C^* = \frac{\mu_0 u^*}{h_0}; \quad p^* = \frac{\mu_0 u^* l}{h_0^2}.$$

Taking into account (2), the system of equations (3) and (4) will be as follows:

$$\frac{\partial^2 v}{\partial y^2} = e^{-\alpha y} \frac{dp}{dx} + \beta e^{-\alpha y} \frac{d^2 p}{dx^2}, \quad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0; \\ \frac{d\Phi(x)}{dx} = -K \int_{-\Phi(x)}^{1+\eta x} \left( \frac{\partial v}{\partial y} \right)^2 dy. \quad (12)$$

Let's introduce the sign

$$Z = e^{-\alpha y}. \quad (13)$$

Taking into account (13) within the accuracy of members  $O(\alpha\beta)$  the system of equations (12) with the corresponding boundary conditions takes the form:

$$\beta \frac{d^2 Z}{dx^2} + \frac{dZ}{dx} = -\alpha \left( \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} \right), \quad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0; \\ Z \frac{d\Phi(x)}{dx} = -K \int_{-\Phi(x)}^{1+\eta x} \left( \frac{\partial v}{\partial y} \right)^2 dy. \quad (14)$$

$$u = 0, \quad v = 0 \quad \text{at} \quad y = 1 + \eta x,$$

$$u = 0, \quad v = -1 \quad \text{at} \quad y = -\Phi(x), \quad (15)$$

$$Z(0) = Z(1) = e^{-\alpha \frac{P_a}{P^*}}, \quad C'(0) = 0, \quad Z''(0) = 0.$$

Taking  $K$  as a small parameter, due to the melt and the energy dissipation rate, we seek the function  $\Phi(x)$  as:

$$\Phi(x) = -K\Phi_1 - K^2\Phi_2 - K^3\Phi_3 - \dots = H, \quad (16)$$

where  $H = -K\Phi_1 - K^2\Phi_2 - K^3\Phi_3 - \dots$

The boundary conditions for dimensionless velocity components  $u$  and  $v$  on contour  $y = 0 - \Phi(x)$  can be written as:

$$v(0 - H(x)) = v(0) - \left(\frac{\partial v}{\partial y}\right)_{y=0} H(x) - \left(\frac{\partial^2 v}{\partial y^2}\right)_{y=-g_0} H^2(x) - \dots = -1;$$

$$u(0 - H(x)) = u(0) - \left(\frac{\partial u}{\partial y}\right)_{y=g_0} H(x) - \left(\frac{\partial^2 u}{\partial y^2}\right)_{y=-g_0} H^2(x) - \dots = 0. \quad (17)$$

We seek the asymptotic solution of the system of differential equations (14) with allowance for the boundary conditions (15) in the form of series in powers of the small parameter  $K$ :

$$v = v_0(x, y) + K v_1(x, y) + K^2 v_2(x, y) + \dots,$$

$$u = u_0(x, y) + K u_1(x, y) + K^2 u_2(x, y) + \dots,$$

$$\Phi(x) = -K \Phi_1(x) - K^2 \Phi_2(x) - K^3 \Phi_3(x) - \dots,$$

$$z = z_0 + K z_1(x) + K^2 z_2(x) + K^3 z_3(x) \dots \quad (18)$$

Performing the substitution (18) into the system of differential equations (14), taking into account the boundary conditions (15), we obtain the following equations:

– for the zeroth approximation:

$$\frac{\partial^2 v_0}{\partial y^2} = \frac{dZ_0}{dx} + \beta \frac{d^2 Z_0}{dx^2}, \quad \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} = 0. \quad (19)$$

With boundary conditions:

$$v_0 = 0, \quad u_0 = 0 \text{ at } y = 1 + \eta x;$$

$$v_0 = -1, \quad u_0 = 0 \text{ at } y = 0;$$

$$C'_0(0) = 0; \quad Z'_0(0) = 0; \quad Z_0(0) = Z_0(1) = e^{-\frac{\alpha p_0}{p}}; \quad K \Phi_0(0) = K g_0; \quad (20)$$

– for first approximation:

$$\frac{\partial^2 v_1}{\partial y^2} = \frac{dZ_1}{dx} + \beta \frac{d^2 Z_1}{dx^2}, \quad \frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} = 0,$$

$$-Z_0 \frac{d\Phi_1(x)}{dx} = K \int_{-\Phi_0}^{1+\eta x} \left(\frac{\partial v_0}{\partial y}\right)^2 dy; \quad (21)$$

с граничными условиями:

$$v_1 = \left(\frac{\partial v_0}{\partial y}\right)_{y=0} \cdot \Phi_1(x);$$

$$u_1 = \left(\frac{\partial u_0}{\partial y}\right)_{y=0} \cdot \Phi_1(x);$$

$$v_1 = 0, \quad u_1 = 0 \text{ at } h(x) = 1 + \eta x;$$

$$C'_0(0) = 0; \quad Z'_1(0) = 0, \quad Z_1(0) = Z_1(1) = 0,$$

$$K \Phi_1(0) = K \tilde{\alpha}^*, \quad \Phi(0) = \Phi(1) = \tilde{\alpha}^*. \quad (22)$$

### EXACT SELF-SIMILAR SOLUTION

The exact self-similar solution of the problem for the zeroth approximation will be sought in the form:

$$u_0 = -\frac{\partial \Psi_0}{\partial x} + U_0(x, y); \quad v_0 = \frac{\partial \Psi_0}{\partial y} + V_0(x, y);$$

$$\Psi_0(x, y) = \tilde{\Psi}_0(\xi); \quad \xi = \frac{y}{1 + \eta x}; \quad h(x) = 1 + \eta x;$$

$$V_0(x, y) = \tilde{v}(\xi); \quad U_0(x, y) = -\tilde{u}_0(\xi) \cdot h'(x);$$

$$\beta \frac{d^2 Z_0}{dx^2} + \frac{dZ_0}{dx} = -\alpha \left( \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} \right). \quad (23)$$

Substituting (23) into the system of differential equations (19), taking into account the boundary conditions (20), we obtain the following system of differential equations:

$$\tilde{\Psi}_0''' = \tilde{C}_2; \quad \tilde{v}_0'' = \tilde{C}_2; \quad \tilde{u}_0' + \xi \tilde{v}_0' = 0; \quad (24)$$

And boundary conditions:

$$\tilde{\Psi}_0'(0) = 0, \quad \tilde{\Psi}_0'(1) = 0; \quad \tilde{u}_0(1) = 0, \quad \tilde{v}_0(1) = 0;$$

$$\tilde{u}_0(0) = 0, \quad \tilde{v}_0(0) = -1, \quad \int_0^1 \tilde{v}_0(\xi) d\xi = 0, \quad Z_0(0) = Z_0(1) = e^{-\frac{\alpha p_0}{p}}. \quad (25)$$

By direct integration we obtain:

$$\tilde{\Psi}_0'(\xi) = \frac{\tilde{C}_2}{2} (\xi^2 - \xi), \quad \tilde{C}_1 = 6,$$

$$\tilde{v}_0(\xi) = \tilde{C}_1 \frac{\xi^2}{2} + \left(1 - \frac{\tilde{C}_1}{2}\right) \xi - 1. \quad (26)$$

Let's define  $Z_0$ . Solving the equation

$$\beta \frac{d^2 Z_0}{dx^2} + \frac{dZ_0}{dx} = -\alpha \left( \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} \right) \text{ taking into account}$$

the boundary conditions  $Z_0(0) = Z_0(1) = e^{-\frac{\alpha p_0}{p}}$  and  $Z_0''(0) = 0$  we obtain:

$$Z_0 = -\frac{\alpha}{\beta} \left[ 6 \left( x + 2\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 3\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) + \tilde{C}^2 \left( x + 3\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 6\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) \right] + \frac{C_1}{\beta} \left( e^{\frac{x}{\beta}} - 1 \right) + e^{\frac{x}{\beta}}, \quad (27)$$

where

$$C_1 = \alpha\beta \left( 6 \left( 2\eta + 6\eta^2 - \frac{2}{\beta} \right) + \tilde{C}_2 \left( 3\eta + 12\eta^2 - \frac{2}{\beta} \right) \right),$$

$$\tilde{C}_2 = \frac{\left( e^{\frac{1}{\beta}} - 1 \right) \left( 12\eta + 36\eta^2 - \frac{12}{\beta} \right) - \frac{6}{\beta^2} \left( 1 + 2\eta \left( \frac{1}{2} - \frac{1}{\beta} \right) + 3\eta^2 \left( \frac{1}{3} - \frac{1}{\beta} - \frac{2}{\beta^2} \right) \right)}{\left( e^{\frac{1}{\beta}} - 1 \right) \left( -3\eta - 12\eta^2 + \frac{2}{\beta} \right) + \frac{1}{\beta^2} \left( 1 + 3\eta \left( \frac{1}{2} - \frac{1}{\beta} \right) + 6\eta^2 \left( \frac{1}{3} - \frac{1}{\beta} - \frac{2}{\beta^2} \right) \right)}$$

In order to define  $\Phi_1(x)$  with allowance for equation (26), we arrive at the following equation:

$$\frac{d\Phi_1}{dx} = \frac{h(x)}{Z_0} \int_0^1 \left( \frac{\Psi_0''(\xi)}{h^2(x)} + \frac{\tilde{v}'_0(\xi)}{h(x)} \right)^2 d\xi. \quad (28)$$

Integrating the equation (28), we get:

$$\Phi_1(x) = \frac{1}{Z_0} \left[ \int_0^x \frac{\Delta_1 dx}{h^3(x)} + \int_0^x \frac{\Delta_2 dx}{h^2(x)} + \int_0^x \frac{\Delta_3 dx}{h(x)} \right], \quad (29)$$

where

$$\Delta_1 = \int_0^1 (\Psi''(\xi))^2 d\xi = \frac{\tilde{C}_2^2}{12}; \quad \Delta_2 = \int_0^1 2\Psi''(\xi) \cdot \tilde{v}'(\xi) d\xi = \tilde{C}_2;$$

$$\Delta_3 = \int_0^1 (\tilde{v}'(\xi))^2 d\xi = 4. \quad (30)$$

$$\sup_{(0;1]} Z_0 = \sup_{(0;1]} \left[ -\frac{\alpha}{\beta} \left[ 6 \left( x + 2\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 3\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) + \tilde{C}^2 \left( x + 3\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 6\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) \right] + \frac{C_1}{\beta} \left( e^{\frac{x}{\beta}} - 1 \right) + e^{\frac{x}{\beta}} \right] = 0.85$$

Solving the equation (29) taking into account (30) and  $K\Phi_1(0) = K\tilde{\alpha}^*$ , we get:

$$\Phi_1(x) = \frac{1}{Z_0} \left[ \left( x - \frac{3}{2}\eta x^2 + 2\eta^2 x^3 \right) \frac{\tilde{C}_2^2}{12} + \tilde{C}_2 \left( x - \eta x^2 + \eta^2 x^3 \right) + 4 \left( x - \frac{\eta}{2} x^2 + \frac{1}{3} \eta^2 x^3 \right) + \tilde{\alpha}^* \right]. \quad (31)$$

The exact self-similar solution for the first approximation will be sought in the form:

$$u_1 = -\frac{\partial \Psi_1}{\partial x} + U_1(x, y); \quad v_1 = \frac{\partial \Psi_1}{\partial y} + V_1(x, y);$$

$$\Psi_1(x, y) = \tilde{\Psi}_1(\xi); \quad \xi = \frac{y}{1 + \eta x}; \quad h(x) = 1 + \eta x;$$

$$V_1(x, y) = \tilde{v}(\xi); \quad U_1(x, y) = -\tilde{u}_1(\xi) \cdot h'(x);$$

$$\beta \frac{d^2 Z_1}{dx^2} + \frac{dZ_1}{dx} = -\alpha \left[ \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} \right]. \quad (32)$$

Substituting (32) into the system of differential equations (21), taking into account the boundary conditions (22), we obtain the following system of differential equations:

$$\tilde{\Psi}_1'''(\xi) = \tilde{C}_2, \quad \tilde{v}_1'' = \tilde{C}_1, \quad \tilde{u}_1' + \xi \tilde{v}_1' = 0,$$

$$\beta \frac{d^2 Z_1}{dx^2} + \frac{dZ_1}{dx} = -\alpha \left[ \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} \right], \quad (33)$$

And boundary conditions

$$\tilde{\Psi}_1'(0) = 0, \quad \tilde{\Psi}_1'(1) = 0; \quad \tilde{u}_1(1) = 0, \quad \tilde{v}_1(1) = 0;$$

$$\tilde{v}_1(0) = M, \quad \tilde{u}_1(0) = 0, \quad \int_0^1 \tilde{v}_1(\xi) d\xi = 0, \quad Z_1(0) = Z_1(1) = 0. \quad (34)$$

By the direct integration we obtain:

$$\tilde{\Psi}_1'(\xi) = \frac{\tilde{C}_2}{2} (\xi^2 - \xi), \quad \tilde{C}_1 = 6M,$$

$$\tilde{v}_1(\xi) = \tilde{C}_1 \frac{\xi^2}{2} - \left( \frac{\tilde{C}_1}{2} + M \right) \xi + M. \quad (35)$$

From the condition  $Z_1(0) = Z_1(1) = 0$  for  $Z_1$  we get:

$$Z_1 = -\frac{\alpha}{\beta} \left[ 6M \left( x + 2\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 3\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) + \tilde{C}_2 \left( x + 3\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 6\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) \right] + \frac{C_1}{\beta} \left( e^{\frac{x}{\beta}} - 1 \right), \quad (36)$$

where

$$C_1^* = \alpha\beta \left( 6M \left( 2\eta + 6\eta^2 - \frac{2}{\beta} \right) + \tilde{C}_2 \left( 3\eta + 12\eta^2 - \frac{2}{\beta} \right) \right),$$

$$\tilde{C}_2 = M\tilde{C}_2 = \frac{\left( e^{\frac{1}{\beta}} - 1 \right) \left( 6M \left( 2\eta + 6\eta^2 - \frac{2}{\beta} \right) - \frac{6M}{\beta^2} \left( 1 + 2\eta \left( \frac{1}{2} - \frac{1}{\beta} \right) + 3\eta^2 \left( \frac{1}{3} - \frac{1}{\beta} - \frac{2}{\beta^2} \right) \right) \right)}{\left( e^{\frac{1}{\beta}} - 1 \right) \left( -3\eta - 12\eta^2 + \frac{2}{\beta} \right) + \frac{1}{\beta^2} \left( 1 + 3\eta \left( \frac{1}{2} - \frac{1}{\beta} \right) + 6\eta^2 \left( \frac{1}{3} - \frac{1}{\beta} - \frac{2}{\beta^2} \right) \right)}$$

$$M = \sup_{x \in [0;1]} \left( \frac{\partial v_0}{\partial y} \right)_{y=0} \cdot \Phi_1(x) =$$

$$\begin{aligned}
 &= \sup_{x \in [0,1]} \left\{ \left[ 1 - \eta x + \frac{\alpha}{\beta^2} \left[ 3 \left[ 1 + 2\eta \left( x - \frac{1}{\beta} \right) + 3\eta^2 \left( x^2 - \frac{2x}{\beta} - \frac{2}{\beta^2} \right) + \eta x + 2\eta^2 x \left( x - \frac{1}{\beta} \right) \right] + \right. \right. \\
 &+ \frac{\tilde{C}_2}{2} \left[ 1 + 3\eta \left( x - \frac{1}{\beta} \right) + 6\eta^2 \left( x^2 - \frac{2x}{\beta} - \frac{2}{\beta^2} \right) + \eta x + 3\eta^2 x \left( x - \frac{1}{\beta} \right) \right] + \frac{C_1(1+\eta x)}{2\beta^2} e^{\frac{x}{\beta}} \left. \right] + \\
 &+ \frac{\alpha}{\beta} \left\{ 3 \left[ 2\eta + 3\eta^2 \left( 2x - \frac{2}{\beta} \right) + 2\eta^2 x \right] + \frac{\tilde{C}_2}{2} \left[ 3\eta + 6\eta^2 \left( 2x - \frac{2}{\beta} \right) + 3\eta^2 x \right] + \frac{C_1(1+\eta x)}{2\beta^2} e^{\frac{x}{\beta}} \right\} \left. \right\} \times \\
 &\times \frac{1}{Z_0} \left[ \left( x - \frac{3}{2}\eta x^2 + 2\eta^2 x^3 \right) \frac{\tilde{C}_2^2}{12} + \tilde{C}_2 \left( x - \eta x^2 + \eta^2 x^3 \right) + 4 \left( x - \frac{1}{2}\eta x^2 + \frac{1}{3}\eta^2 x^3 \right) + \tilde{\alpha} \right].
 \end{aligned}$$

Taking into account (36) for  $Z = Z_0 + KZ_1$  we get:

$$\begin{aligned}
 Z &= -\frac{\alpha}{\beta^2} (1 + KM) \left[ 6 \left( x + 2\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 3\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) + \right. \\
 &+ \tilde{C}_2 \left( x + 3\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 6\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) + \frac{C_1}{\beta} \left( e^{\frac{x}{\beta}} - 1 \right) \left. \right] + e^{-\alpha \frac{p_a}{p^*}}, \quad (37)
 \end{aligned}$$

$$\text{or } e^{-\alpha p} = -\frac{\alpha}{\beta^2} (1 + KM) \Phi(x) + e^{-\alpha \frac{p_a}{p^*}},$$

where

$$\begin{aligned}
 \Phi(x) &= \left[ 6 \left( x + 2\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 3\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) + \right. \\
 &+ \tilde{C}_2 \left( x + 3\eta \left( \frac{x^2}{2} - \frac{x}{\beta} \right) + 6\eta^2 \left( \frac{x^3}{3} - \frac{x^2}{\beta} - \frac{2x}{\beta^2} \right) \right) + \frac{C_1}{\beta} \left( e^{\frac{x}{\beta}} - 1 \right) \left. \right]. \quad (38)
 \end{aligned}$$

Applying the Taylor expansion for functions  $e^{-\alpha p}$ ,  $e^{-\alpha \frac{p_a}{p^*}}$ , we get:

$$1 - \alpha p + \frac{\alpha^2 p^2}{2} - 1 + \alpha \frac{p_a}{p^*} - \frac{\alpha^2}{2} \left( \frac{p_a}{p^*} \right)^2 = -\frac{\alpha}{\beta^2} (1 + KM) \Phi(x). \quad (39)$$

Solving the equation (39) to within the accuracy of  $O(\alpha)^3$ ,  $O\left(\left(\frac{p_a}{p^*}\right)^2\right)$  for hydrodynamic pressure we obtain:

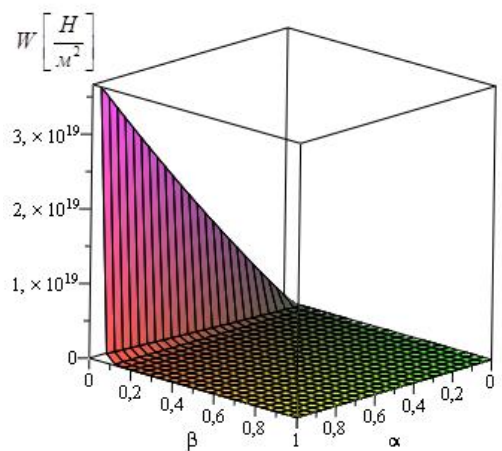
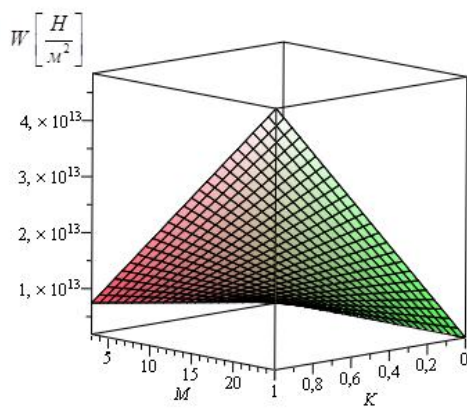
$$p = \frac{p_a}{p^*} - \Phi(x) (1 + KM) \left( 1 + \alpha \frac{p_a}{p^*} - \frac{\alpha^2}{2} \left( \frac{p_a}{p^*} \right)^2 \right). \quad (40)$$

**5. Results of the study and their discussion.** Let us now turn to the determination of the basic operating characteristics of the bearing. Taking into account (19), (21) and (39) for the bearing capacity and the friction force, we obtain:

$$\begin{aligned}
 W &= \frac{\mu_0 l u^*}{h_0^2} \int_0^1 \left( p_0 - \frac{p_a}{p^*} \right) dx = \frac{\mu_0 l u^*}{h_0^2} \left( 1 + \alpha \frac{p_a}{p^*} - \frac{\alpha^2}{2} \left( \frac{p_a}{p^*} \right)^2 \right) (1 + KM) \left( 6 \left( \frac{1}{2} + 2\eta \left( \frac{1}{6} - \frac{1}{2\beta} \right) \right) + \right. \\
 &+ 3\eta^2 \left( \frac{1}{12} - \frac{1}{3\beta} - \frac{1}{\beta^2} \right) + \tilde{C}_2 \left( \frac{1}{2} + 3\eta \left( \frac{1}{6} - \frac{1}{2\beta} \right) + 6\eta^2 \left( \frac{1}{12} - \frac{1}{3\beta} - \frac{1}{\beta^2} \right) \right) + \frac{C_1}{\beta} \left( \frac{1}{\beta} e^{\frac{1}{\beta}} - 2 \right) \left. \right)
 \end{aligned}$$

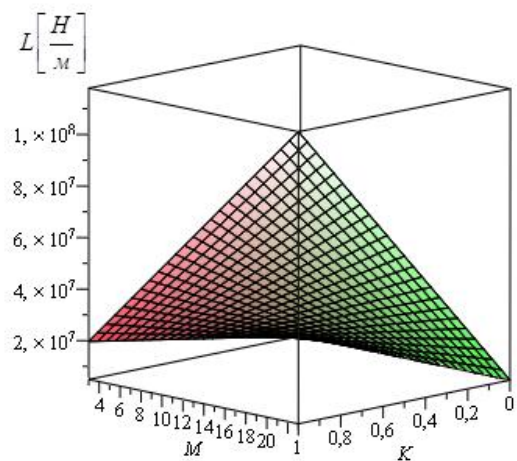
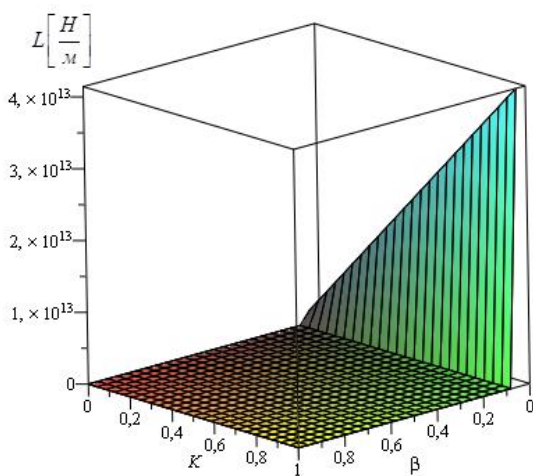
$$L_{mp} = \frac{\mu_0 l u^*}{h_0} e^{-\alpha p} \int_0^1 \left[ \frac{\partial v_0}{\partial y} \right]_{y=0} + K \left[ \frac{\partial v_1}{\partial y} \right]_{y=0} dx =$$

$$\begin{aligned}
 &= \frac{\mu_0 l u^*}{h_0} \left( 1 - \alpha p + \frac{\alpha^2 p^2}{2} \right) \left[ 1 - \frac{\eta}{2} + \frac{\alpha}{\beta^2} \left( 1 + 2\eta \left( \frac{1}{2} - \frac{1}{\beta} \right) + 3\eta^2 \left( \frac{1}{3} - \frac{1}{\beta} - \frac{2}{\beta^2} \right) + \frac{\eta}{2} + \frac{2}{3} \eta^2 + \eta^2 \frac{1}{\beta} \right) \right. \\
 &+ \frac{\tilde{C}_2}{2} \left( 1 + 3\eta \left( \frac{1}{2} - \frac{1}{\beta} \right) + 6\eta^2 \left( \frac{1}{3} - \frac{1}{\beta} - \frac{2}{\beta^2} \right) + \frac{\eta}{2} + \eta^2 - \frac{3}{2} \eta^2 \frac{1}{\beta} \right) + \frac{C_1}{2\beta^3} \left( e^{\frac{1}{\beta}} - 1 \right) + \\
 &+ \frac{C_1 \eta}{2\beta^2} \left[ e^{\frac{1}{\beta}} \left( \frac{1}{\beta} - \frac{1}{\beta^2} \right) + \frac{\alpha}{\beta} \left( 3 \left( 2\eta + 3\eta^2 \left( 1 - \frac{2}{\beta} \right) + \eta^2 \right) + \frac{\tilde{C}_2}{2} \left( 3\eta + 6\eta^2 \left( 1 - \frac{2}{\beta} \right) + \frac{3}{2} \eta^2 \right) + \right. \\
 &\left. \left. + \frac{C_1}{2\beta^3} \left( e^{\frac{1}{\beta}} - 1 \right) + \frac{C_1 \eta}{2\beta^2} \left( e^{\frac{1}{\beta}} \left( \frac{1}{\beta} - \frac{1}{\beta^2} \right) + \frac{1}{\beta^2} \right) \right) \right] \right] (1 + KM). \tag{42}
 \end{aligned}$$



**Figure 2:** The dependence of the load-carrying capacity on the parameter  $K$ , due to the melt and the energy dissipation rate, and the parameter  $M$ , which characterizes the thickness of the molten film.

**Figure 3:** Dependence of the bearing capacity on the Deborah number  $\beta$  and on the parameter  $\alpha$ , which characterizes the dependence of the viscosity of the lubricant on pressure



**Figure 4:** The dependence of the frictional force on the parameter  $K$ , due to the melt and the energy dissipation rate, and on the Deborah number  $\beta$ .

**Figure 5:** The dependence of the frictional force on the parameter  $K$ , due to the melt and the energy dissipation rate, and the parameter  $M$ , which characterizes the thickness of the molten film.

The input parameters for calculating the bearing capacity and the friction force, determined by expressions (42):

$$\eta = 0,3 \div 1; \quad \omega = 0 \div 1; \quad K = 0,000022 \div 0,0052; \quad u^* = 1 \div 3 \text{ m/c};$$
$$\beta = 0 \div 1; \quad \alpha = 0 \div 1; \quad \mu_0 = 0,085 \frac{\text{H} \cdot \text{c}}{\text{M}^2}; \quad p = 0,5 \div 16 \text{ МПа};$$
$$p_a = 0,08 \div 0,101325 \text{ МПа}; \quad h_0 = 10^{-7} \div 2 \cdot 10^{-6} \text{ м}.$$

Based on the results of numerical analysis, graphs are constructed (Figures 2-5), which allow us to draw the following conclusions.

1. A refined calculation model of a wedge-shaped sliding support operating under hydrodynamic lubrication conditions on a viscoelastic liquid lubricant due to the melt surface of a low-melting surface of the guide surface.
2. The significant contribution of the parameters is shown: Deborah number  $\beta$ , parameter  $K$ , caused by the melt of the surface of the low-melting surface of the guide surface, the parameter  $M$  characterizing the thickness of the molten film, and the parameter  $\alpha$ , characterizing the dependence of the viscosity of the lubricant on pressure.
2. It has been established that a significant increase in the bearing capacity and a decrease in the frictional force occurs with an increase in the parameter  $K$  caused by the melt of the surface of the low-melting surface of the guide surface, the parameter  $M$  characterizing the thickness of the molten film, and parameter  $\alpha$ , characterizing the dependence of the viscosity of the lubricant on pressure.

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