Improving the Performance of Current Controller in a Grid Connected Renewable PV System using Fractional Order Controller

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Abstract
This paper focuses on the implementation of fractional order control techniques in real world control system for improving the performance of current controller used in grid connected PV system. Two types of controller are usually used in the grid connected system they are Linear and non-linear controller. PI-controller being a linear controller is basically used in the grid connected PV system. Therefore Fractional order controller with Linear PI controller is used in this paper to enhance the performance of current controller. MATLAB Simulink software is used to implement and test the controller. Fractional order operator function and codes are used to optimise and tuning of the controller.

Keywords Fractional order Operator; PI-controller; Fractional Order PI; Bounded system

INTRODUCTION
Today’s in digital era, Fractional order controller has many applications such as in Digital Signal Processing, Control engineering, Instrumental applications and in Biological applications too. Fractional order calculus are the advanced application version of Integer and derivative function which allows the user to describe the real world system more accurately. A varying solar radiation can be treated as a Fractional Order system which clearly shows the application of controller to a variety of engineering applications.

Fractional Order system is more flexible because most of the real world applications possess some degree of fractionality in its characteristics and operation [1]. However in most of the cases fractionality is not enough to affect the behaviour of the system. Therefore fractional order controller based on Integration & derivative is applied to the real world system for changing its characteristics. There are many definitions for fractional order controller however Grunwald-Letnikov, Cauchy Integral Formula, Caputo and Riemann-Liouville are more commonly used definitions. Some special functions used in the fractional order controller are represented below.

A. Gamma Function
One of the most important function of fractional order controller is the Gamma Function. The function is represented as
\[
\Gamma(n) = \int_{0}^{\infty} t^{n-1} e^{-t} dt
\]
This is the general function for fractional order controller [4,5].

B. Mittag-Leffler Function
MittagLeffler function [6] plays an important role in the exponential function in integer order calculus. Mathematically it can be represented as
\[
E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}
\]
Where R(\alpha)>0 and R(\beta)>0

Some of the important Fractional order Mittag-Leffler function are as follows
\[
E_{1,1}(z) = e^z
\]
\[
E_{0,1}(z) = \frac{1}{1-z}
\]
\[
E_{2,1}(z) = \cosh(z)
\]
\[
E_{0.5,1}(z) = e^{\sqrt{z}} \text{ erfc}(\sqrt{z})
\]
All the Mittag-Leffler function are bounded by [1, 1].
C. Grunwald-Letnikov Fractional Order Integral & Derivative Controller

Real world system consisting of higher order derivatives can be written as

\[ f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]  

Or in more compact manner

\[ \int_{a}^{t} \frac{df(x)}{dt} \, dt = \int_{a}^{t} f(x) \, dx \]

Grunwald-Letnikov Fractional Order Integral and Derivative for real world system can be represented as

\[ D^\alpha_{a} f(t) = \lim_{h \to 0} \frac{1}{\Gamma(n+1)} \sum_{j=0}^{n} \frac{(-1)^j}{j!} \alpha^{j}(\frac{t-a}{h})^j f(t-jh) \]  

Where \( \alpha, j \) represents

\[ \begin{pmatrix} \alpha \\ j \end{pmatrix} = \frac{\Gamma(\alpha + 1)}{\Gamma(j+1)\Gamma(\alpha - j + 1)} \]  

An alternative way of representing the Grunwald-Letnikov Fractional Order Integral and Derivative [7] is as follows

\[ D^\alpha_{a} f(t) = \sum_{j=0}^{n} \frac{f^{(j)}(t-a)}{\Gamma(n+1-j)} \frac{(-1)^j}{j!} \alpha^{j}(\frac{t-a}{h})^j \]  

for \( n+1 \geq \alpha \geq n \)

D. Reimann-Liouville Fractional Order Integral and Derivative

Reimann-Liouville Fractional Order Integral and Derivative [7,8] for real world system can be represented as

\[ I^\alpha_{a} f(t) = \frac{1}{\Gamma(n+1)} \int_{a}^{t} (t-x)^{n-\alpha} f(x) \, dx \]  

Derivative order function for Reimann-Liouville Fractional Order system can be represented as

\[ D^\alpha_{a} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_{a}^{t} (t-x)^{n-\alpha} f(x) \, dx \]  

Fractional order equation for Reimann-Liouville Fractional Order system can be written as

\[ D^\alpha_{a} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} f(t) \]  

E. Caputo Fractional Order Derivatives

Caputo Fractional order derivatives can be modelled by modifying the Reimann-Liouville Fractional Order Integral and Derivative [10] for real world system. For a real world system it can be written as

\[ D^\alpha_{a} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{a}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha}} \, d\tau \]  

One of the greatest advantages of Caputo Fractional order derivatives is that initial condition for integer order differential equation holds good for Caputo Fractional order derivatives. Both Caputo Fractional order derivatives and Reimann-Liouville Fractional Order Integral and Derivative becomes equal for homogeneous condition of real world system. Relationship between Caputo Fractional order derivatives and Reimann-Liouville Fractional Order Integral and Derivative under homogeneous condition becomes

\[ RL_{a} \frac{D^\alpha_{a} f(t)}{\alpha} = \frac{C}{a} D^\alpha_{a} f(t) = \frac{C}{a} \sum_{k=0}^{\infty} \frac{(t-a)^{k-\alpha}}{\Gamma(k+1)} f^{(k)}(t) \]  

Where \( RL \) and \( \alpha D \) represents the derivative function for Reimann-Liouville Fractional Order and Caputo Fractional order system respectively.

**PROPERTIES OF FRACTIONAL ORDER CONTROLLER**

A. Property-1

From the geometrical Interpretation point of view for a fractional order control function \((g,f)\), \(g\) always scales \(f\) based on the following equation

\[ g(t) = \frac{1}{\Gamma(\alpha+1)} \left\{ t^{\alpha} - (t-t)^{\alpha} \right\} \]  

B. Property-2

Based on short memory principle of fractional order controller for \( t > a+L \),

\[ D^\alpha_{a} f(t) = \sum_{l=1}^{\infty} D^\alpha_{a} f(t) \]  

Where \( L \) represents the length of memory or in general fractional order controller works on the recent past values such as on function \( f(t) \). Error associated with Fractional order controller based on short memory principle can be written as

\[ D^\alpha_{a} f(t) - \sum D^\alpha_{a} f(t) \leq \frac{M L^{\alpha}}{\Gamma(1-\alpha)} \]  

Here \( M \) represents the Upper boundary of the function.

C. Property-3

For an analytical function of \( f(t) \), \( D^\alpha_{a} f(t) \) is also analytical for all \( t \) and \( \alpha \).
**D. Property -4**
Fractional order derivatives are the common form of integer order derivatives. Therefore for a fractional order integer and derivatives the output result is also same as integer and derivative operator.

**E. Property-5**
Fractional order operators are linear operators for all constant a and b.
\[ aD^\alpha f(t) + bg(t) = aD^\alpha f(t) + wD^\alpha g(t) \]
(19)

**F. Property-6**
With reference to Caputo definition of system, both fractional order and integer order derivatives are interchangeable i.e.,
\[ \frac{c}{a}D^\alpha f(t) = \frac{c}{a}D^\alpha f(t) = \frac{c}{a}D^\alpha f(t) \]
(20)
Similarly for both mixed and integer order derivative
\[ aD^\alpha(I^\beta f(t)) = \begin{cases} aD^\gamma f(t) & \alpha > \beta \\ I^\gamma f(t) & \beta > \alpha \end{cases} \]
(21)
for condition-1 and \( \beta > \alpha \) for condition-2

**G. Property-7**
For all continuous function of \( f(t) \) and \( g(t) \) in the boundary \([a,t]\), Leibniz’s rule becomes
\[ aD^\alpha f(t) + g(t) = \sum_{k=0}^{r} f^{(k)}(t) \cdot D^\alpha g(t) \]
(22)

**H. Property-8**
Caputo derivative for a constant term is always Zero, however Riemann-Liouville derivative of a constant results into a nonzero term.
\[ aD^\alpha K = \frac{K(t-a)^\alpha}{\Gamma(1-\alpha)} \]
(23)

**FRACTIONAL ORDER DYNAMIC SYSTEM**
At the real world phenomena possess some degree of fractionality in their behavior some of these fractionality are dominant and some are negligible in their behavior and recognition. Hence Integer Order differential equation is the closet approximated value for fractional order dynamics. The general form of fractional order differential equation can be written as
\[ aD^\alpha y(t) + aD^\beta y(t) + \ldots + aD^\alpha y(t) = bD^\alpha g(t) + bD^\beta g(t) + \ldots + bD^\alpha g(t) \]
(24)
Where \( a_k \) and \( b_k \) are the constants and \( y(t) \) and \( g(t) \) represents the input and output of the system. Applying Laplace transformation to the above differential equation results into a transfer function as follows
\[ G(s) = \frac{aD^\alpha g(t) + aD^\beta g(t) + \ldots + aD^\alpha g(t)}{aD^\alpha y(t) + aD^\beta y(t) + \ldots + aD^\alpha y(t)} \]
(25)
In general equation (25) can be written as
\[ G(s) = \frac{\sum b_k s^{-\alpha}}{\sum a_k s^{-\alpha}} \]
(26)
For pseudo rational function equation (26) can be written as
\[ G(s) = \frac{\sum b_k s^{-\alpha}}{\sum a_k s^{-\alpha}} \]
(27)
Above transfer function can be written as
\[ G(s) = K \sum_{k=1}^{\infty} A_i s^{-\alpha} e^{-\alpha} \]
(28)

**A. Fractional Order PI-Controller**
In the real world system First order system with time delay is wisely used to model the physical system. A generalized model with Fractional order pole plus time delay can be represented as
\[ P(s) = \frac{K}{T s^\alpha + 1} e^{-\alpha} \]
(29)
Where \( K \), \( L \) and \( T \) represents the constant for fractional order system. Transfer function for Fractional order open PI and closed bounded [PI] controller can be written as
\[ C_s(s) = K \left( 1 + \frac{K_s}{S^\alpha} \right) \]
(30)
\[ C_s(s) = K \left( 1 + \frac{K_s}{S^\alpha} \right) \]
(31)
Here it can be assumed that gain cross over frequency and phase cross over frequency are the two constraints for designing and tuning the controller parameters. These constraints are as follows
- **Constraint for Phase Margin**
\[ \angle G(j\omega_c) = \angle C(j\omega_c)P(j\omega_c) = \omega C(j\omega_c) + \omega P(j\omega_c) = \omega + \phi \]
(32)
Where \( G(j\omega_c) \) is the open loop transfer function and \( C(j\omega_c) \) is the controller transfer function.
Similarly constraint for gain cross over frequency

\[ \left[ G(j \omega_c) \right] = \left[ C(j \omega_c) P(j \omega_c) \right] = \left[ C(j \omega_c) \right] P(j \omega_c) \right] = 0 \]

(33)

B. Fractional Order PI Controller, Unbounded System

Transfer function for open loop system based on Fractional Order PI Controller can be written as

\[ G_i(s) = C_i(s)P(s) = K_i \left( 1 + \frac{K_i}{s^\alpha} \right) \left( \frac{K}{T s^\alpha + 1} \right) e^{-t} \]

(35)

Or

\[ C_i(s) = K_i\left( 1 + \frac{K_i}{s^\alpha} \right) = K_i \left( 1 + \frac{K_i}{(j \omega)^\alpha} \right) = K_i \left( 1 + \frac{K_i (\omega^\alpha)^{-1}}{(j \omega)^\alpha} \right) \]

(36)

Where \( C_i \) represents the Fractional order controller. Fractional order controller can be written as

\[ C_i(s) = K_i \left( 1 + \frac{\sqrt{2 \pi \lambda}}{\cos(\frac{\lambda \pi}{2}) + j \sin(\frac{\lambda \pi}{2})} \right) \]

(37)

Now open loop phase at gain cross over frequency can be written as

\[ \arg \left[ G_i(j \omega_c) \right] = -\tan^{-1} \left[ \frac{K_i \omega_c^{-\alpha} \sin \left( \frac{\lambda \pi}{2} \right)}{1 + K_i \omega_c^{-\alpha} \cos \left( \frac{\lambda \pi}{2} \right)} \right] - \tan^{-1} \left[ \frac{B}{A} \right] - \omega_c \]

(38)

Where \( A = 1 + T \omega_c \cos \left( \frac{\alpha \pi}{2} \right) \) and \( B = 1 + T \omega_c \sin \left( \frac{\alpha \pi}{2} \right) \).

Phase margin constraint for the above transfer function can be written as

\[ \tan^{-1} \left[ \frac{K_i \omega_c^{-\alpha} \sin \left( \frac{\lambda \pi}{2} \right)}{1 + K_i \omega_c^{-\alpha} \cos \left( \frac{\lambda \pi}{2} \right)} \right] - \tan^{-1} \left[ \frac{B}{A} \right] - \omega_c = -\pi + \phi_u \]

(39)

\[ \frac{K_i \omega_c^{-\alpha} \sin \left( \frac{\lambda \pi}{2} \right)}{1 + K_i \omega_c^{-\alpha} \cos \left( \frac{\lambda \pi}{2} \right)} = \tan \left[ \frac{1}{A} \right] + L \omega_c + \phi_u \]

(40)

Hence the relationship between \( K_i \) and \( \lambda \) can be written as

\[ K_i = \frac{\omega_c^{-\alpha} \sin \left( \frac{\lambda \pi}{2} \right)}{\omega_c^{-\alpha} \cos \left( \frac{\lambda \pi}{2} \right)} \]

(41)

Where \( D_2 \) becomes \( D_2 = \tan \left[ \tan^{-1} \left( \frac{B}{A} \right) + \phi_u + L \right] \)

Gain of fractional order controller at open loop frequency becomes

\[ G_i(j \omega) = \left( \frac{1 + K_i \omega_c^{-\alpha} \sin \left( \frac{\lambda \pi}{2} \right)}{\sqrt{A^2 + B^2}} \right) \]

(42)

Now with reference to the third constraint for checking the robustness of the Fractional Order Controller

\[ K_i = \sqrt{ \left( 1 + K_i \omega_c^{-\alpha} \cos \left( \frac{\lambda \pi}{2} \right) \right)^2 + \left( K_i \omega_c^{-\alpha} \sin \left( \frac{\lambda \pi}{2} \right) \right)^2 } \]

(43)

Now with reference to the third constraint for checking the robustness of the Fractional Order Controller

\[ \frac{K_i \omega_c^{-\alpha} \sin \left( \frac{\lambda \pi}{2} \right)}{\omega_c^{-\alpha} \cos \left( \frac{\lambda \pi}{2} \right) + K_i} \]

(44)

Where \( E_z = \frac{\alpha T \omega_c^{-\alpha} \sin \left( \frac{\alpha \pi}{2} \right) - B \cos \left( \frac{\alpha \pi}{2} \right)}{A^2 + B^2} + L \)

(45)

Now relation between \( K_i \) and \( \lambda \) can be written as

\[ K_i = \frac{\sqrt{E_z^2 - 4 E_z \omega_c^{-2 \alpha}}}{2 \omega_c^{2 \alpha}} \]

(46)

From the above equation \( K_i, \lambda \) and \( K_P \) can be found out either by using graphical method or by minimum search algorithm.
C. Fractional Order PI Controller, Bounded System

Transfer function for bounded Fractional order PI-controller can be written as

\[ G_i(s) = \left( K_i + \frac{K_p}{s} \right) \left( \frac{K}{T s^\mu + 1} e^{-s \tau} \right) \]  

(47)

Open loop phase gain at cross over frequency can be written as

\[ \text{Arg} [G_s(j\omega_c)] = -\lambda \tan^{-1} \left( \frac{K_i}{K_p \omega_c} \right) - \tan^{-1} \left( \frac{B}{A} \right) - L \omega_c \]  

(48)

Now with reference to 1\textsuperscript{st} constraint of stability

\[ -\lambda \tan^{-1} \left( \frac{K_i}{K_p \omega_c} \right) - \tan^{-1} \left( \frac{B}{A} \right) - L \omega_c = -\pi + \phi_s \]  

(49)

Or

\[ \frac{K_i}{K_p \omega_c} = D_1 \]  

(50)

Or

\[ D_1 = \tan \left[ \left( \pi - \phi_s - \tan^{-1}(B/A) - L \omega_c \right) / \lambda \right] \]  

(51)

Again with reference to 2\textsuperscript{nd} constraint of Stability the gain cross over frequency must satisfy the stability criteria and under such condition

\[ K \left[ \frac{K_i^2}{K_p \omega_c} + K_i^2 \left( \frac{K_i}{K_p \omega_c} \right)^2 \right] \frac{1}{\sqrt{A^2 + B^2}} = 1 \]  

(52)

Similarly with respect to the 3\textsuperscript{rd} constraint of stability gain variation can be written as

\[ \frac{\lambda K_i K_p}{(K_p \omega_c) + K_i^2} = E_i \]  

(53)

Or

\[ E_i = \frac{\alpha T \omega_c}{A^2 + B^2} \left[ A \sin \left( \frac{\omega \pi}{2} \right) - B \cos \left( \frac{\omega \pi}{2} \right) \right] + L \]  

(54)

From the above three constraints of equations \( K_i \) and \( K_p \) can be written as

\[ K_i = \frac{E_i \omega_c}{\lambda D_1} \left( A \omega_c \right)^{-\frac{2}{\mu}} + B' \omega_c \]  

(55)

\[ K_p = \frac{E_i \omega_c \left( A \omega_c \right)^{-\frac{2}{\mu}} + B' \omega_c}{\lambda D_1} \]  

(56)

DESIGNING AND TUNING OF CURRENT CONTROLLER

Let us consider a plant system with transfer function

\[ G(s) = \frac{a_1 S^{n_1} + a_0}{b_2 S^{n_2-2} + b_1 S^{n_2-1} + b_0} \]  

(57)

Where \( a_1, a_0 \) and \( b_2, b_1, b_0 \) are the coefficients. The fractional order transfer function shown in (57) corresponds to time domain representation of PI controller used in current control strategies of a voltage source inverter. The corresponding differential equation to equation (57) can be written as

\[ b_2 y^{(n_2)}(t) + b_1 y^{(n_2-1)}(t) + b_0 y(t) = u(t) \]  

(58)

The validity of equation (57) holds good only when all the initial condition sets to zero i.e.

\[ Y(0)=0, Y'(0)=0 \text{ and } Y''(0)=0 \]  

(59)

Unit step response to equation (58) can be found out by transferring the equation (58) as

\[ y(t) = \frac{1}{b_2} \sum_{i=0}^{k-1} \left( \frac{b_2}{b_1} \right)^i + \frac{1}{b_1} \sum_{i=0}^{k-1} \left( \frac{b_2}{b_1} \right)^i \varepsilon_{t-1} \]  

(59)

Real time transfer function for PI-controller can be written as (extracted from MATLAB signal Processor)

\[ G(s) = \frac{0.001093s - 1.264e-06}{s^2 + 0.001376s + 4.945e-07} \]  

(60)

Controlling parameters for equation (60) are extracted using frequency domain analysis tools. Bode plot for equation (60) is shown in fig-1. Stability criteria parameters extracted from bode plot is shown in table-1

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Parameter</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Proportional Gain (Kp)</td>
<td>-0.10717</td>
</tr>
<tr>
<td>02</td>
<td>Integral Gain (Ki)</td>
<td>-0.0019911</td>
</tr>
<tr>
<td>03</td>
<td>Rise Time</td>
<td>2.35e03sec</td>
</tr>
<tr>
<td>04</td>
<td>Settling Time</td>
<td>4.31e04sec</td>
</tr>
<tr>
<td>05</td>
<td>Over Shoot</td>
<td>5.81%</td>
</tr>
<tr>
<td>06</td>
<td>Gain Margin</td>
<td>4.47dB @ 0.00597 rad/sec</td>
</tr>
<tr>
<td>07</td>
<td>Phase Margin</td>
<td>22° @ 0.00417 rad/sec</td>
</tr>
<tr>
<td>08</td>
<td>Closed loop Stability</td>
<td>Stable</td>
</tr>
</tbody>
</table>
In the above transfer function and its corresponding frequency domain bode plot analysis it can be seen that the cross over frequency is maintained at 1000 rad/sec for $\lambda=0.637$. Applying tuning condition to the controller transfer function under the three different constraint as discussed above normalised open loop transfer function becomes

$$G(s) = \frac{1}{0.01S + 1}$$  \hspace{1cm} (61)$$

And the corresponding fractional order transfer function after pass-1 with $q=0.002$ becomes

$$G(s) = \frac{s^{3.5} + s^{2.5} + s + 1}{s^{2.395} + s^{3.1104} + 0.0049978s^{1.92} + 1.7824e-06s^{0.47} + 1.569e-11}$$  \hspace{1cm} (62)$$

Stability Criteria parameters for Fractional order transfer function (62) can be found out from superimpose graphical analysis of (62) as shown in fig-2

<table>
<thead>
<tr>
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<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Proportional Gain ($K_p$)</td>
<td>-0.10717</td>
</tr>
<tr>
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<td>Integral Gain ($K_i$)</td>
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</tr>
<tr>
<td>08</td>
<td>Closed loop Stability</td>
<td>Stable</td>
</tr>
</tbody>
</table>

Table 2: Stability Criteria parameters for Fraction order transfer function after pass-1

After applying the auto tuner for Pass-2 with $q=0.0047$

$$G(s) = \frac{s^{3.5} + s^{2.5} + s + 1}{s^{2.395} + s^{3.1104} + 0.0049978s^{1.92} + 1.7824e-06s^{0.47} + 1.569e-11}$$  \hspace{1cm} (63)$$
\[ G(s) = \frac{s^{3.538} + s^{3.17} + 0.0039471s^{2.413} + s^{2.47} + s^{2.01} + 0.00015671s^{1.57} + s^{1.62} + s + 1.3487 \times 10^{-6}s^{0.58} + 1.4581e^{-06}s^{0.58} + 1.4581e^{-08}}{0.018635s^{2.613} + 2.458e^{-07}s^{2.37} + 0.000187307s^{1.528} + 1.8745e^{-08}s + 1.25993e^{-04}s^{0.59} + 1.8295e^{-06}} \]

(64)

\[ G(s) = \frac{s^{3.2} + s^{2.7} + s^{2.2} + s^{1.7} + 0.0051913s^{1.33} + s^{1.2} + s^{0.7} + 1.557e^{-06}s^{0.63} + 1.245e^{-08}}{0.0049956s^{2.53} + 3.9948e^{-05}s^{1.9} + 0.00022897s^{1.33} + 1.831e^{-06}s^{0.7} + 2.0707e^{-06}s^{0.63} + 1.6558e^{-08}} \]

(65)

\[ G(s) = \frac{s^{3.2} + s^{2.7} + s^{2.2} + s^{1.7} + 0.0051913s^{1.33} + s^{1.2} + s^{0.7} + 1.557e^{-06}s^{0.63} + 1.245e^{-08}}{0.0049956s^{2.53} + 3.9948e^{-05}s^{1.9} + 0.00022897s^{1.33} + 1.831e^{-06}s^{0.7} + 2.0707e^{-06}s^{0.63} + 1.6558e^{-08}} \]

(66)

\[ G(s) = \frac{s^{3.2} + s^{2.7} + s^{2.2} + s^{1.7} + 0.0051913s^{1.33} + s^{1.2} + s^{0.7} + 1.557e^{-06}s^{0.63} + 1.245e^{-08}}{0.0049956s^{2.53} + 3.9948e^{-05}s^{1.9} + 0.00022897s^{1.33} + 1.831e^{-06}s^{0.7} + 2.0707e^{-06}s^{0.63} + 1.6558e^{-08}} \]

(67)

\[ G(s) = \frac{s^{3.2} + s^{2.7} + s^{2.2} + s^{1.7} + 0.0051913s^{1.33} + s^{1.2} + s^{0.7} + 1.557e^{-06}s^{0.63} + 1.245e^{-08}}{0.0049956s^{2.53} + 3.9948e^{-05}s^{1.9} + 0.00022897s^{1.33} + 1.831e^{-06}s^{0.7} + 2.0707e^{-06}s^{0.63} + 1.6558e^{-08}} \]

(68)

\[ G(s) = \frac{s^{3.2} + s^{2.7} + s^{2.2} + s^{1.7} + 0.0051913s^{1.33} + s^{1.2} + s^{0.7} + 1.557e^{-06}s^{0.63} + 1.245e^{-08}}{0.0049956s^{2.53} + 3.9948e^{-05}s^{1.9} + 0.00022897s^{1.33} + 1.831e^{-06}s^{0.7} + 2.0707e^{-06}s^{0.63} + 1.6558e^{-08}} \]

(69)

\[ G(s) = \frac{s^{3.2} + s^{2.7} + s^{2.2} + s^{1.7} + 0.0051913s^{1.33} + s^{1.2} + s^{0.7} + 1.557e^{-06}s^{0.63} + 1.245e^{-08}}{0.0049956s^{2.53} + 3.9948e^{-05}s^{1.9} + 0.00022897s^{1.33} + 1.831e^{-06}s^{0.7} + 2.0707e^{-06}s^{0.63} + 1.6558e^{-08}} \]

(70)

\[ G(s) = \frac{s^{3.2} + s^{2.7} + s^{2.2} + s^{1.7} + 0.0051913s^{1.33} + s^{1.2} + s^{0.7} + 1.557e^{-06}s^{0.63} + 1.245e^{-08}}{0.0049956s^{2.53} + 3.9948e^{-05}s^{1.9} + 0.00022897s^{1.33} + 1.831e^{-06}s^{0.7} + 2.0707e^{-06}s^{0.63} + 1.6558e^{-08}} \]

(71)

\[ G(s) = \frac{s^{3.2} + s^{2.7} + s^{2.2} + s^{1.7} + 0.0051913s^{1.33} + s^{1.2} + s^{0.7} + 1.557e^{-06}s^{0.63} + 1.245e^{-08}}{0.0049956s^{2.53} + 3.9948e^{-05}s^{1.9} + 0.00022897s^{1.33} + 1.831e^{-06}s^{0.7} + 2.0707e^{-06}s^{0.63} + 1.6558e^{-08}} \]

(72)

\[ G(s) = \frac{s^{3.2} + s^{2.7} + s^{2.2} + s^{1.7} + 0.0051913s^{1.33} + s^{1.2} + s^{0.7} + 1.557e^{-06}s^{0.63} + 1.245e^{-08}}{0.0049956s^{2.53} + 3.9948e^{-05}s^{1.9} + 0.00022897s^{1.33} + 1.831e^{-06}s^{0.7} + 2.0707e^{-06}s^{0.63} + 1.6558e^{-08}} \]

(73)
The solver have reached a local minimum, but cannot be certain because the first-order optimality measure is not less than the TolFun tolerance (1e-4*TolFun for the Levenberg-Marquardt algorithm).

Table 3: Successful Iteration Function Parameters after Pass-1

<table>
<thead>
<tr>
<th>Iteration No.</th>
<th>Function Count</th>
<th>F(x)</th>
<th>Norm of Step</th>
<th>1st Order Optimality</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>4</td>
<td>4.821e11</td>
<td>0</td>
<td>9.64e09</td>
</tr>
<tr>
<td>01</td>
<td>8</td>
<td>3.05e11</td>
<td>10</td>
<td>7.56e09</td>
</tr>
<tr>
<td>02</td>
<td>12</td>
<td>1.744e10</td>
<td>20</td>
<td>3.72e09</td>
</tr>
<tr>
<td>03</td>
<td>16</td>
<td>0.0066017</td>
<td>20.82</td>
<td>74.6</td>
</tr>
<tr>
<td>04</td>
<td>20</td>
<td>0.006183</td>
<td>3.91</td>
<td>4.33</td>
</tr>
<tr>
<td>05</td>
<td>24</td>
<td>0.005965</td>
<td>7.89</td>
<td>34.4</td>
</tr>
<tr>
<td>06</td>
<td>28</td>
<td>00005919</td>
<td>1.31</td>
<td>1.16</td>
</tr>
<tr>
<td>07</td>
<td>32</td>
<td>0.005911</td>
<td>1.07</td>
<td>0.626</td>
</tr>
<tr>
<td>08</td>
<td>36</td>
<td>0.005910</td>
<td>0.9807</td>
<td>0.622387</td>
</tr>
<tr>
<td>09</td>
<td>40</td>
<td>0.005908</td>
<td>0.9784</td>
<td>0.6214</td>
</tr>
<tr>
<td>10</td>
<td>44</td>
<td>0.005908</td>
<td>0.9781</td>
<td>0.6214</td>
</tr>
</tbody>
</table>

For tuning the Current controller with fractional order controller Result obtained from equation (73) is plotted with integer order controller as obtained in chapter-3 of PI-controller in a frequency domain plot shown in fig:-3,4 & 5. Intersection of the two plot clearly determines the order and gain of the controller. Putting this value in equation 58 will determine the proportional gain of the controller.

Table 4: Extracted Parameters from Stability Tuner

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Parameter</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Proportional Gain (K_p)</td>
<td>-1.386</td>
</tr>
<tr>
<td>02</td>
<td>Integral Gain (K_i)</td>
<td>-3.84e-05</td>
</tr>
<tr>
<td>03</td>
<td>Rise Time</td>
<td>1.19e03</td>
</tr>
<tr>
<td>04</td>
<td>Settling Time</td>
<td>1.29e-05</td>
</tr>
<tr>
<td>05</td>
<td>Over Shoot</td>
<td>9.99%</td>
</tr>
<tr>
<td>06</td>
<td>Gain Margin</td>
<td>1.17dB@ 0.0756 rad/sec</td>
</tr>
<tr>
<td>07</td>
<td>Phase Margin</td>
<td>51.3° @ 0.0692 rad/sec</td>
</tr>
<tr>
<td>08</td>
<td>Closed loop Stability</td>
<td>Marginally Stable</td>
</tr>
</tbody>
</table>
CONCLUSION
Fractional order controller shows better performance as compared to linear PI-controller. This can be easily understood from the Bode plot characteristics. Fractional order controller limits the maximum signal thereby increasing the efficiency by reducing the error. It signifies that higher order controller can be easily realised with the help of lower order controllers.

REFERENCES