International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 17 (2017) pp. 6954-6961

Construction of Control Systems with High Potential of Robust Stability in the Case of Catastrophe Elliptical Umbilic

Mamyrbek A. Beisenbi
L.N Gumilyov Eurasion National University, Department of System Analysis and Control, 2 Satpayev St., Astana, 010008, Republic of Kazakhstan.

Salamat T. Suleimenova
L.N Gumilyov Eurasion National University, Department of System Analysis and Control, 2 Satpayev St., Astana, 010008, Republic of Kazakhstan.

Orcid ID: 0000-0002-9494-1391

Vladimir V. Nikulin
State University of New York, Department of Electrical and Computer Engineering, Binghamton, New York 13902-600, USA.

Dana K. Satybaldina
L.N Gumilyov Eurasion National University, Department of System Analysis and Control, 2 Satpayev St., Astana, 010008, Republic of Kazakhstan.

Abstract

This article is cover the study of control systems with a high potential of robust stability is formed in class of three-parameter structurally stable mapping (elliptical umbilic) for objects with m inputs and n outputs.

Research of robust stability of control systems based on the construction of Lyapunov function. Lyapunov’s function is formed as vector-function, antigradient of which is given by velocity vector’s components. Stability region of steady states of the system is obtained in the form of simple inequations according to uncertain parameters of control object and selected controller parameters.

Keywords: Lyapunov’s function, robust stability, elliptical umbilic, three-parameter structurally stable mapping.

Introduction

The development process of some technical device pose a deep problem in science and technology. System can declined from given program motion and execute other motion under real performance. One of the reasons for these actions may be the availability of small deviations from the initial state of a given movement or the availability of small forces unaccounted under modeling systems. One of the main problems that take place already during the construction of the system is stabilization problem of their movements.

This research focuses on current issues to the construction of robust stability of dynamic objects control systems with uncertain parameters with approach to the construction of the control system in the class of structurally stable mappings allowing increase of the potential of robust stability of the system. Research of robust stability is based on a new approach resulting from the geometric interpretation of Lyapunov's theorem [1, 2]. Lyapunov’s function is synthesized as vector-function antigradient of which is given by velocity vector’s components in the form of tensor.

Consider the control system with m input and n output, described by the equation of state

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

Where $x(t) \in R^n$ – control object’s state vector; $\mathrm{u}(t) \in R^m$ –scalar function of control actions; $A \in R^{n\times n}$ - control object’s matrix with uncertain parameters, $B \in R^{n\times m}$ – control matrix.

Law of control is given in the form of a sum of three-parameter structurally stable mappings (catastrophe elliptical umbilic)

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 17 (2017) pp. 6954-6961

$$\begin{aligned} u(x) = & -x_2^3 + 3x_2x_1^2 - k_{12}(x_1^2 + x_2^2) + k_2x_2 + k_1x_1 - \\ & -x_4^3 + 3x_4x_3^2 - k_{34}(x_3^2 + x_4^2) + k_4x_4 + k_3x_3 -, \ldots, - \\ & -x_n^3 + 3x_nx_{n-1}^2 - k_{n-1,\text{n}}(x_{n-1}^2 + x_n^2) + k_nx_n + k_{n-1}x_{n-1} \end{aligned} \quad (2)$$

Matrices A and B have the following form:

$$A = \begin{vmatrix} a_{11} & a_{21} & \ldots & a_{n1} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix}, \quad B = \begin{vmatrix} b_{11} & 0 & \ldots & 0 \\ 0 & b_{22} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & b_{nn} \end{vmatrix}$$

Research of control systems with a high potential for robust stability in the catastrophe class of elliptical umbilical for objects with one input and one output are described in the paper [3].

System (1) is written in the expanded form as:

$$\begin{cases} \dot{x}_1 = -b_{11}x_2^3 + 3b_{11}x_2x_1^2 - b_{11}k_{12}(x_1^2 + x_2^2) + (a_{11} + b_{11}k_1)x_1 + \\ +(a_{12} + b_{11}k_2)x_2 + a_{13}x_3 + \ldots + a_{1n}x_n \\ \dot{x}_2 = -b_{22}x_2^3 + 3b_{22}x_2x_1^2 - b_{22}k_{12}(x_1^2 + x_2^2) + (a_{21} + b_{22}k_1)x_1 + \\ +(a_{22} + b_{22}k_2)x_2 + a_{23}x_3 + \ldots + a_{2n}x_n \\ \dot{x}_3 = -b_{33}x_4^3 + 3b_{33}x_4x_3^2 - b_{33}k_{34}(x_3^2 + x_4^2) + a_{31}x_1 + a_{32}x_2 + \\ +(a_{33} + b_{33}k_3)x_3 + (a_{34} + b_{33}k_4)x_4 + a_{35}x_5 \ldots + a_{3n}x_n \\ \dot{x}_4 = -b_{44}x_4^3 + 3b_{44}x_4x_3^2 - b_{44}k_{34}(x_3^2 + x_4^2) + a_{41}x_1 + a_{42}x_2 + \\ +(a_{43} + b_{44}k_3)x_3 + \ldots + (a_{44} + b_{44}k_4)x_4 + a_{45}x_5 \ldots + a_{4n}x_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{x}_{n-1} = -b_{n-1,n-1}x_n^3 + 3b_{n-1,n-1}x_nx_{n-1}^2 - b_{n-1,n-1}k_{n.-1,\text{n}}(x_{n-1}^2 + x_n^2) + \\ +a_{n-1,1}x_1 + a_{n-1,2}x_2 + \ldots + (a_{n-1,n-1} + b_{n-1,n-1}k_{n-1})x_{n-1} + \\ +(a_{n-1,n} + b_{n-1,n-1}k_n)x_n \\ \dot{x}_n = -b_{n,n}x_n^3 + 3b_{n,n}x_nx_{n-1}^2 - b_{n,n}k_{n.-1,\text{n}}(x_{n-1}^2 + x_n^2) + a_{n,1}x_1 + \\ +a_{n,2}x_2 + \ldots + (a_{n,n-1} + b_{n,n}k_{n-1})x_{n-1} + (a_{n,n} + b_{n,n}k_n)x_n \end{cases} \quad (3)$$

Steady states condition of the system (3) are defined by solution of the equations:

$$\begin{cases} -b_{11}x_{2s}^3 + 3b_{11}x_{2s}x_{1s}^2 - b_{11}k_{12}(x_{1s}^2 + x_{2s}^2) + (a_{11} + b_{11}k_1)x_{1s} + \\ +(a_{12} + b_{11}k_2)x_{2s} + a_{13}x_{3s} + \ldots + a_{1n}x_{ns} = 0 \\ -b_{22}x_{2s}^3 + 3b_{22}x_{2s}x_{1s}^2 - b_{22}k_{12}(x_{1s}^2 + x_{2s}^2) + (a_{21} + b_{22}k_1)x_{1s} + \\ +(a_{22} + b_{22}k_2)x_{2s} + a_{23}x_{3s} + \ldots + a_{2n}x_{ns} = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -b_{n,n}x_{ns}^3 + 3b_{n,n}x_{ns}x_{n-1,s}^2 - b_{n,n}k_{n.-1,\text{n}}(x_{n-1,s}^2 + x_{n,s}^2) + a_{n,1}x_{1s} + \\ +a_{n,2}x_{2s} + \ldots + (a_{n,n-1} + b_{n,n}k_{n-1})x_{n-1,s} + (a_{n,n} + b_{n,n}k_n)x_{ns} = 0 \end{cases} \quad (4)$$

Stationary states of the system (4) are defined as:

$$x_{1s} = 0, x_{2s} = 0, \ldots, x_{ns} = 0 \quad (5)$$

Other stationary states will be determined by solution of the equations:

$$-b_{ii}k_{i,i+1}x_{is} + a_{ii} + b_{ii}k_i = 0, \;\; x_{js} = 0 \text{ for } i \neq j, i = \overline{1,n} \quad (6)$$

The solution of equation (6) will be written as follows:

$$x_{is} = \frac{a_{ii}}{b_{ii}k_{i,i+1}} + \frac{k_i}{k_{i,i+1}} \;\; x_{js} = 0 \text{ for } i \neq j, i = \overline{1,n} \quad (7)$$

Stability of the stationary states

Research the robust stability of the stationary state (7) based on the method of Lyapunov's function. State equations (3) can be represented in the deviations with respect to the stationary state (7). For this we may use formal description allowing representation of the equation (3) in the deviations with respect to the stationary state. The state equation (3) with respect to the stationary state will be written as:

$$\begin{cases} \dot{x}_1 = -b_{11}x_2^3 + 3b_{11}x_2x_1^2 - b_{11}k_{12}(x_1^2 + x_2^2) + 6\frac{a_{11} + b_{11}k_1}{k_{12}}x_1x_2 - \\ -(a_{11} + b_{11}k_1)x_1 + (a_{12} + b_{11}k_2)x_2 + a_{13}x_3 + \ldots + a_{1n}x_n \\ \dot{x}_2 = -b_{22}x_2^3 + 3b_{22}x_2x_1^2 - b_{22}k_{12}(x_1^2 + x_2^2) - (a_{21} + b_{22}k_1)x_1 + \\ +(a_{22} + b_{22}k_2)x_2 + a_{23}x_3 + \ldots + a_{2n}x_n \\ \dot{x}_3 = -b_{33}x_4^3 + 3b_{33}x_4x_3^2 - b_{33}k_{34}(x_3^2 + x_4^2) + 6\frac{a_{33} + b_{33}k_3}{k_{34}}x_3x_4 + \\ +a_{31}x_1 + a_{32}x_2 - (a_{33} + b_{33}k_3)x_3 + (a_{34} + b_{33}k_4)x_4 + a_{35}x_5 + \ldots + a_{3n}x_n \\ \dot{x}_4 = -b_{44}x_4^3 + 3b_{44}x_4x_3^2 - b_{44}k_{34}(x_3^2 + x_4^2) + a_{41}x_1 + a_{42}x_2 - \\ -(a_{43} + b_{44}k_3)x_3 + (a_{44} + b_{44}k_4)x_4 + a_{45}x_5 \ldots + a_{4n}x_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{x}_{n-1} = -b_{n-1,n-1}x_n^3 + 3b_{n-1,n-1}x_nx_{n-1}^2 - b_{n-1,n-1}k_{n.-1,\text{n}}(x_{n-1}^2 + x_n^2) + \\ +6\frac{a_{n-1,n-1} + b_{n-1,n-1}k_n}{k_{n-1,n}}x_{n-1}x_n + a_{n-1,1}x_1 + a_{n-1,2}x_2 + \ldots - \\ -(a_{n-1,n-1} + b_{n-1,n-1}k_{n-1})x_{n-1} + (a_{n-1,n} + b_{n-1,n-1}k_n)x_n \\ \dot{x}_n = -b_{n,n}x_n^3 + 3b_{n,n}x_nx_{n-1}^2 - b_{n,n}k_{n.-1,\text{n}}(x_{n-1}^2 + x_n^2) + a_{n,1}x_1 + \\ +a_{n,2}x_2 + \ldots - (a_{n,n-1} + b_{n,n}k_{n-1})x_{n-1} + (a_{n,n} + b_{n,n}k_n)x_n \end{cases} \quad (8)$$

The components of the vector of antigradient from Lyapunov's vector function $V(x)=(V_1(x),V_2(x),\ldots)$ taken as:

$$\frac{\partial V_1(x)}{\partial x_1}=b_{11}k_{12}x_1^2-2b_{11}x_2x_1^2-3\frac{a_{11}+b_{11}k_1}{k_{12}}x_2x_1+\left(a_{11}+b_{11}k_1\right)x_1$$

$$\frac{\partial V_1(x)}{\partial x_2}=b_{11}x_2^3-b_{11}x_2x_1^2+b_{11}k_{12}x_2^2-3\frac{a_{11}+b_{11}k_1}{k_{12}}x_2x_1-\left(a_{12}+b_{11}k_2\right)x_2$$

$$\frac{\partial V_1(x)}{\partial x_3}=-a_{13}x_3,\frac{\partial V_1(x)}{\partial x_4}=-a_{14}x_4,\ldots,\frac{\partial V_1(x)}{\partial x_{n-1}}=-a_{1,n-1}x_{n-1},\frac{\partial V_1(x)}{\partial x_n}=-a_{1n}x_n.$$

$$\frac{\partial V_2(x)}{\partial x_1}=b_{22}k_{12}x_1^2-2b_{22}x_2x_1^2+\left(a_{21}+b_{22}k_1\right)x_1$$

$$\frac{\partial V_2(x)}{\partial x_2}=b_{22}x_2^3-b_{22}x_2x_1^2+b_{22}k_{12}x_2^2-\left(a_{22}+b_{22}k_2\right)x_2$$

$$\frac{\partial V_2(x)}{\partial x_3}=-a_{23}x_3,\frac{\partial V_2(x)}{\partial x_4}=-a_{24}x_4,\ldots,\frac{\partial V_2(x)}{\partial x_{n-1}}=-a_{2,n-1}x_{n-1},\frac{\partial V_2(x)}{\partial x_n}=-a_{2n}x_n.$$

……………………………………………

$$\frac{\partial V_{n-1}(x)}{\partial x_1}=-a_{n-1,1}x_1,\frac{\partial V_{n-1}(x)}{\partial x_2}=-a_{n-1,2}x_2,\ldots,$$

$$\frac{\partial V_{n-1}(x)}{\partial x_{n-1}}=b_{n-1,n-1}k_{n-1,n}x_{n-1}^2-2b_{n-1,n-1}x_nx_{n-1}^2-$$

$$-3\frac{a_{n-1,n-1}+b_{n-1,n-1}k_{n-1}}{k_{n-1,n}}x_nx_{n-1}+\left(a_{n-1,n-1}+b_{n-1,n-1}k_{n-1}\right)x_{n-1}$$

$$\frac{\partial V_{n-1}(x)}{\partial x_n}=b_{n-1,n-1}x_n^3-b_{n-1,n-1}x_nx_{n-1}^2+b_{n-1,n-1}k_{n-1,n}x_n^2-$$

$$-3\frac{a_{n-1,n-1}+b_{n-1,n-1}k_{n-1}}{k_{n-1,n}}x_nx_{n-1}-\left(a_{n-1,n}+b_{n-1,n-1}k_n\right)x_n$$

$$\frac{\partial V_n(x)}{\partial x_1}=-a_{n,1}x_1,\frac{\partial V_n(x)}{\partial x_2}=-a_{n,2}x_2,\ldots,$$

$$\frac{\partial V_n(x)}{\partial x_{n-1}}=b_{nn}k_{n-1,n}x_{n-1}^2-2b_{nn}x_nx_{n-1}^2+\left(a_{n,n-1}+b_{n,n}k_{n-1}\right)x_{n-1}$$

$$\frac{\partial V_n(x)}{\partial x_n}=b_{nn}x_n^3-b_{nn}x_nx_{n-1}^2+b_{nn}k_{n-1,n}x_n^2-\left(a_{n,n}+b_{nn}k_n\right)x_n$$

Total time derivative from components of Lyapunov's vector-function is equal to:

$$\begin{aligned}
&\frac{dV_1(x)}{dt} = -\left[b_{11}k_{12}x_1^2 - 2b_{11}x_2x_1^2 - 3\frac{a_{11}+b_{11}k_1}{k_{12}}x_2x_1 + \left(a_{11}+b_{11}k_1\right)x_1\right]^2 - \\
&-\left[b_{11}x_2^3 - b_{11}x_2x_1^2 + b_{11}k_{12}x_2^2 - 3\frac{a_{11}+b_{11}k_1}{k_{12}}x_2x_1 - \left(a_{12}+b_{11}k_2\right)x_2\right]^2 - \\
&-a_{13}^2x_3^2 - \ldots - a_{1n}^2x_n^2 \\
&\frac{dV_2(x)}{dt} = -\left[b_{22}k_{12}x_1^2 - 2b_{22}x_2x_1^2 + \left(a_{21}+b_{22}k_1\right)x_1\right]^2 - \\
&-\left[b_{22}x_2^3 - b_{22}x_2x_1^2 + b_{22}k_{12}x_2^2 - \left(a_{22}+b_{22}k_2\right)x_2\right]^2 - a_{23}^2x_3^2 - \ldots - a_{2n}^2x_n^2 \qquad (9)\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\frac{dV_{n-1}(x)}{dt} = -a_{n-1,1}^2x_1^2 - a_{n-1,2}^2x_2^2 - \ldots - \left[b_{n-1,n-1}k_{n-1,n}x_{n-1}^2 - 2b_{n-1,n-1}x_nx_{n-1}^2 - \right.\\
&\left.-3\frac{a_{n-1,n-1}+b_{n-1,n-1}k_{n-1}}{k_{n-1,n}}x_nx_{n-1} + \left(a_{n-1,n-1}+b_{n-1,n-1}k_{n-1}\right)x_{n-1}\right]^2 - \\
&-\left[b_{n-1,n-1}k_{n-1,n}x_n^2 - b_{n-1,n-1}x_nx_{n-1}^2 - 3\frac{a_{n-1,n-1}+b_{n-1,n-1}k_{n-1}}{k_{n-1,n}}x_nx_{n-1} - \right.\\
&\left.-\left(a_{n-1,n}+b_{n-1,n-1}k_n\right)x_n + b_{n-1,n-1}x_n^3\right]^2 \\
&\frac{dV_n(x)}{dt} = -a_{n,1}^2x_1^2 - a_{n,2}^2x_2^2 - \ldots - \left[b_{nn}k_{n-1,n}x_{n-1}^2 - 2b_{nn}x_nx_{n-1}^2 + \left(a_{n,n-1}+b_{n,n}k_{n-1}\right)x_{n-1}\right]^2 - \\
&-\left[b_{nn}x_n^3 - b_{nn}x_nx_{n-1}^2 + b_{nn}k_{n-1,n}x_n^2 - \left(a_{n,n}+b_{nn}k_n\right)x_n\right]^2
\end{aligned}$$

By components of the vector gradient of Lyapunov's vector-function there may be built the components of Lyapunov's vector-function in the following form

$$\begin{aligned}
&V_1(x) = \frac{1}{3}b_{11}k_{12}x_1^3 - \frac{3}{2}\frac{a_{11}+b_{11}k_1}{k_{12}}x_2x_1^2 + \frac{1}{2}\left(a_{11}+b_{11}k_1\right)x_1^2 - \\
&-\frac{2}{3}b_{11}x_2x_1^3 + \frac{1}{4}b_{11}x_2^4 - \frac{1}{2}b_{11}x_2^2x_1^2 + \frac{1}{3}b_{11}k_{12}x_2^3 - \frac{3}{2}\frac{a_{11}+b_{11}k_1}{k_{12}}x_1x_2^2 - \\
&-\frac{1}{2}\left(a_{12}+b_{11}k_2\right)x_2^2 - \frac{1}{2}a_{13}x_3^2 - \ldots - \frac{1}{2}a_{1n}x_n^2 \\
&V_2(x) = \frac{1}{3}b_{22}k_{12}x_1^3 + \frac{1}{2}\left(a_{21}+b_{22}k_1\right)x_1^2 + \frac{1}{4}b_{22}x_2^4 - \frac{2}{3}b_{22}x_2x_1^3 - \\
&-\frac{1}{2}b_{22}x_2^2x_1^2 + \frac{1}{3}b_{22}k_{12}x_2^3 - \frac{1}{2}\left(a_{22}+b_{22}k_2\right)x_2^2 - \frac{1}{2}a_{23}x_3^2 - \ldots - \frac{1}{2}a_{2n}x_n^2 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

$$V_{n-1}(x) = -\frac{1}{2}a_{n-1,1}x_1^2 - \ldots - \frac{1}{2}a_{n-1,n-2}x_{n-2}^2 + \frac{1}{3}b_{n-1,n-1}k_{n-1,n}x_{n-1}^3 -$$

$$-\frac{2}{3}b_{n-1,n-1}x_n x_{n-1}^3 - \frac{3}{2}\frac{a_{n-1,n-1} + b_{n-1,n-1}k_{n-1}}{k_{n-1,n}}x_n x_{n-1}^2 - \frac{1}{2}b_{n-1,n-1}x_n^2 x_{n-1}^2 +$$

$$+\frac{1}{2}\left(a_{n-1,n-1} + b_{n-1,n-1}k_{n-1}\right)x_{n-1}^2 - \frac{3}{2}\frac{a_{n-1,n-1} + b_{n-1,n-1}k_{n-1}}{k_{n-1,n}}x_{n-1}x_n^2 +$$

$$+\frac{1}{3}b_{n-1,n-1}k_{n-1,n}x_n^3 + \frac{1}{4}b_{n-1,n-1}x_n^4 - \frac{1}{2}\left(a_{n-1,n} + b_{n-1,n-1}k_n\right)x_n^2$$

$$V_n(x) = -\frac{1}{2}a_{n1}x_1^2 - \ldots - \frac{1}{2}a_{n,n-2}x_{n-2}^2 + \frac{1}{3}b_{nn}k_{n-1,n}x_{n-1}^3 - \frac{2}{3}b_{nn}x_n x_{n-1}^3 -$$

$$+\frac{1}{2}\left(a_{n,n-1} + b_{nn}k_{n-1}\right)x_{n-1}^2 + \frac{1}{4}b_{nn}x_n^4 - \frac{1}{2}b_{nn}x_n^2 x_{n-1}^2 + \frac{1}{3}b_{nn}k_{n-1,n}x_n^3 - \frac{1}{2}\left(a_{nn} + b_{nn}k_n\right)x_n^2$$

Lyapunov’s function in scalar form can be presented in the sum form:

$$V(x) = \frac{1}{3}\left(b_{11} + b_{22}\right)k_{12}x_1^3 - \frac{2}{3}\left(b_{11} + b_{22}\right)x_2 x_1^3 +$$

$$+\frac{1}{4}\left(b_{11} + b_{22}\right)x_2^4 - \frac{1}{2}\left(b_{11} + b_{22}\right)x_2^2 x_1^2 + \frac{1}{3}\left(b_{11} + b_{22}\right)k_{12}x_2^3 -$$

$$-\frac{3}{2}\frac{a_{11} + b_{11}k_1}{k_{12}}x_2 x_1\left(x_2 + x_1\right) +$$

$$+\frac{1}{2}\left[\left(b_{11} + b_{22}\right)k_1 + a_{11} + a_{21} - a_{31} - \ldots - a_{n1}\right]x_1^2 -$$

$$-\frac{1}{2}\left[\left(b_{11} + b_{22}\right)k_2 + a_{12} + a_{22} - a_{32} - \ldots - a_{n2}\right]x_2^2 +$$

$$+\ldots + \frac{1}{3}\left(b_{n-1,n-1} + b_{nn}\right)k_{n-1,n}x_{n-1}^3 - \frac{2}{3}\left(b_{n-1,n-1} + b_{nn}\right)x_n x_{n-1}^3 +$$

$$+\frac{1}{4}\left(b_{n-1,n-1} + b_{nn}\right)x_n^4 - \frac{1}{2}\left(b_{n-1,n-1} + b_{nn}\right)x_{n-1}^2 x_n^2 +$$

$$+\frac{1}{3}\left(b_{n-1,n-1} + b_{nn}\right)k_{n-1,n}x_n^3 +$$

$$+\frac{1}{2}\left[\left(b_{n-1,n-1} + b_{nn}\right)k_{n-1} - a_{1,n-1} - \ldots + a_{n-1,n-1} + a_{n,n-1}\right]x_{n-1}^2 -$$

$$-\frac{3}{2}\frac{a_{n-1,n-1} + b_{n-1,n-1}k_{n-1}}{k_{n-1,n}}x_n x_{n-1}\left(x_{n-1} + x_n\right) - \qquad (10)$$

$$-\frac{1}{2}\left[\left(b_{n-1,n-1} + b_{nn}\right)k_n + a_{1,n} + a_{2,n} + \ldots + a_{n,n}\right]x_n^2$$

The criterion of positive or negative definite function (10) is not obvious, therefore there may be used lemma of Morse from the catastrophe theory [4, 5]. Lyapunov's function (10) can locally be represented in quadratic form. The matrix of quadratic form can be calculated as the Hessian matrix from Lyapunov's vector function (9):

$$\left|\left.\frac{\partial^2 V(x)}{\partial x_1 \partial x_1}\right|_{(8)}\right| = \left(2\frac{a_{11}+b_{11}k_1}{k_1 b_{11}}+1\right)(b_{11}+b_{22})k_1+\left[a_{11}+a_{21}-\left(a_{31}+\ldots+a_{n,1}\right)\right], \quad \left|\left.\frac{\partial^2 V(x)}{\partial x_1 \partial x_j}\right|_{(8)}\right| = 0, \text{ for } j \neq 1$$

$$\left|\left.\frac{\partial^2 V(x)}{\partial x_2 \partial x_2}\right|_{(8)}\right| = \left(2\frac{a_{21}+b_{22}k_1}{k_2 b_{22}}+3\frac{(a_{21}+b_{22}k_1)^2}{b_{22}^2 k_{12}^2 k_2}-1\right)(b_{11}+b_{22})k_2-\left[a_{12}+a_{22}+\ldots+a_{n,2}\right],$$

$$\left|\left.\frac{\partial^2 V(x)}{\partial x_2 \partial x_j}\right|_{(8)}\right| = 0, \text{ for } j \neq 2$$

……………………………………………

$$\left|\left.\frac{\partial^2 V(x)}{\partial x_{n-1} \partial x_{n-1}}\right|_{(8)}\right| = \left(2\frac{a_{n-1,n-1}+b_{n-1,n-1}k_{n-1}}{k_{n-1} b_{n-1,n-1}}+1\right)(b_{n-1,n-1}+b_{nn})k_{n-1}+ ,$$

$$+\left[a_{n-1,n-1}+a_{n,n-1}-\left(a_{1,n-1}+\ldots+a_{n-2,n-1}\right)\right]$$

$$\left|\left.\frac{\partial^2 V(x)}{\partial x_{n-1} \partial x_j}\right|_{(8)}\right| = 0, \text{ for } j \neq n-1$$

$$\left|\left.\frac{\partial^2 V(x)}{\partial x_n \partial x_n}\right|_{(8)}\right| = \left(2\frac{a_{n,n-1}+b_{nn}k_{n-1}}{k_n b_{nn}}+3\frac{(a_{n,n-1}+b_{nn}k_{n-1})^2}{b_{nn}^2 k_{n-1,n}^2 k_n}-1\right)(b_{n-1,n-1}+b_{nn})k_n- ,$$

$$-\left[a_{1n}+a_{2n}+\ldots+a_{nn}\right]$$

$$\left|\left.\frac{\partial^2 V(x)}{\partial x_n \partial x_j}\right|_{(8)}\right| = 0, \text{ for } j \neq n$$

Lyapunov's function (10) may be represented in the quadratic form locally near of the stationary state

$$V(x)=\left\{\left(2\frac{a_{11}+b_{11}k_1}{k_1b_{11}}+1\right)(b_{11}+b_{22})k_1+a_{11}+a_{21}-(a_{31}+\ldots+a_{n,1})\right\}x_1^2+$$
$$+\left\{\left(2\frac{a_{21}+b_{22}k_1}{k_2b_{22}}+3\frac{(a_{21}+b_{22}k_1)^2}{b_{22}^2k_{12}^2k_2}-1\right)(b_{11}+b_{22})k_2-(a_{12}+a_{22}+\ldots+a_{n,2})\right\}x_2^2+$$
$$+\ldots+$$
$$+\left\{\left(2\frac{a_{n-1,n-1}+b_{n-1,n-1}k_{n-1}}{k_{n-1}b_{n-1,n-1}}+1\right)(b_{n-1,n-1}+b_{nn})k_{n-1}+\right. \quad (11)$$
$$\left.+a_{n-1,n-1}+a_{n,n-1}-(a_{1,n-1}+\ldots+a_{n-2,n-1})\right\}x_{n-1}^2+$$
$$+\left\{\left(2\frac{a_{n,n-1}+b_{nn}k_{n-1}}{k_nb_{nn}}+3\frac{(a_{n,n-1}+b_{nn}k_{n-1})^2}{b_{nn}^2k_{n-1,n}^2k_n}-1\right)(b_{n-1,n-1}+b_{nn})k_n-\right.$$
$$\left.-(a_{1n}+a_{2n}+\ldots+a_{nn})\right\}x_n^2$$

The criterions of stability of the stationary state (7) will be defined by positive definiteness of quadratic form (11) by the system of inequalities:

$$\begin{cases}\left(2\frac{a_{11}+b_{11}k_1}{k_1b_{11}}+1\right)(b_{11}+b_{22})k_1+a_{11}+a_{21}-(a_{31}+\ldots+a_{n,1})>0\\ \left(2\frac{a_{21}+b_{22}k_1}{k_2b_{22}}+3\frac{(a_{21}+b_{22}k_1)^2}{b_{22}^2k_{12}^2k_2}-1\right)(b_{11}+b_{22})k_2-\\ -(a_{12}+a_{22}+\ldots+a_{n,2})>0\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ \left(2\frac{a_{n-1,n-1}+b_{n-1,n-1}k_{n-1}}{k_{n-1}b_{n-1,n-1}}+1\right)(b_{n-1,n-1}+b_{nn})k_{n-1}+a_{n-1,n-1}+\\ +a_{n,n-1}-(a_{1,n-1}+\ldots+a_{n-2,n-1})>0\\ \left(2\frac{a_{n,n-1}+b_{nn}k_{n-1}}{k_nb_{nn}}+3\frac{(a_{n,n-1}+b_{nn}k_{n-1})^2}{b_{nn}^2k_{n-1,n}^2k_n}-1\right)(b_{n-1,n-1}+b_{nn})k_n-\\ -(a_{1n}+a_{2n}+\ldots+a_{nn})>0\end{cases} \quad (12)$$

The control system had built in class of three-parameter structurally - stable mappings for stationary state (7), is stable when you varying undetermined parameters of the object in the region (12).

CONCLUSION

In this paper developed a new approach for construction control systems with high potential of robustness. Structurally-stable mappings of the catastrophe theory, in particular, elliptical umbilic are used as the control laws. The use of the mappings helps to increase the range of robustness.

This paper describes a new idea – the increase of the range of robustness of control systems. Research of robust stability of control systems based on the construction of Lyapunov function. Lyapunov's function is formed as vector-function, antigradient of which is given by velocity vector's components. The results of this research can be used in the analysis and synthesis of methods of solving practical tasks related to robustness.

In result, it will help to increase the robust stability when designing control systems. Therefore, the quality of management of technological processes can be improved, which is especially desired when operating in presence of uncertainties and incomplete data.

REFERENCES

[1] Beisenbi M.A., Abdrakhmanova L.G., 2013, “Research of dynamic properties of parameter structurally stable maps by Lyapunov function,” International Conference on Computer, Network and Communication Engineering (ICCNCE 2013), Published by Atlantis Press, pp. 201 – 203.

[2] Barbashin E.A., 1967, Vvedenie v teoriyu ustoichivosti, Nauka, Moscow, Chap. 1.

[3] Beisenbi M.A., Suleimenova S. T., Kissykova N. M., 2016, “Research of control systems with a high potential for robust stability by Lyapunov’s function,” Proc. 3rd International Conference on Soft Computing and Computational Mathematics (ICSCCM 2016), Malaysia, pp. 16 – 19.

[4] Gilmor R., 1984, Prikladnaya teoriya katastrof, Mir, Moscow, Chap. 2.

[5] Thompson J. M. T., 1982, Instabilities and catastrophes in science and engineering, A Wiley – Interscience Publication, UK, pp. 17 – 47.