

Design of Robust Controller for Hemi-Spherical Tank System Using Volumetric Observer

Muthumari S¹, Rakesh Kumar S¹,

¹*School of Electrical and Electronics Engineering, SASTRA University, Thanjavur, Tamil Nadu, India.*

¹*Orcid Id: 0000-0001-9936-5736, 0000-0002-9521-2342*

Srimathy R², Kanimozhi G²

²*School of Electrical Engineering, VIT University, Chennai, Tamil Nadu, India.*

²*Orcid Id: 0000-0002-3956-9080, 0000-0001-7993-986X*

Abstract

Robustness of the controller is vital in many process industries. The robust controller will not only compensate for disturbance but also for the parametric variation in the process itself. This paper proposes the design of such a robust propositional-integral (PI) controller for a non-linear level control process, like hemi-spherical tank system, using volumetric observer. In a level control system, the volume has a linear relation to the differences in inflow and outflow, whereas the rate change in height is non-linearly related to the flow difference. The robustness of the proposed controller was enhanced by using volume error instead of level error. The volume error is derived by using a volumetric observer, which observes the tank volume and a volume estimator, which estimate the desired tank volume. This enables a linear controller like PI controller to be more robust over the entire span of tank height.

The robustness of the proposed controller was compared with the conventional controller and its performance is validated by using the averaged performance indices like averaged-ISE, ITAE and IAE. Simulation results illustrates that the proposed controller has faster response and lesser oscillations.

Keywords: Robust control, Non-linear control, Volume observer, Hemispherical tank

INTRODUCTION

Hemi-spherical tank system [1-4] finds its wide applications in many of the process industries. These tanks will have a parabolic cross-section area, which makes them ideal for even distribution of heat [5-6] when it comes for heat transfer through convection. It can also be used in liquid-gas separation process, where the liquid stays and the gases evaporate. Hemi-spherical tanks [6] are used in food processing industries, chemical dying units, batch reactors etc. Thus, the problem of controlling of liquid level in this tank becomes critical for these industries to ensure the quality of the product. Variable

shape of these tanks with respect to its height makes level control a challenging problem and demands for complex controllers.

In process industries, liquid level has been an important parameter [7] which is to be controlled at a desired value. A high level above the desired value may upset reaction equilibrium, cause damage to equipment, or result in spillage of valuable or hazardous material. A low level below the desired value may have bad consequences for the sequential operations [8]. So control of liquid level is an important and common task in process industries [9]

The majority of the control theory deals with the design of linear controllers for linear liquid tank systems. Ziegler-Nichols [10] has developed a well known PI controller design methods to provide a closed-loop response with a quarter-decay ratio. This technique has been used widely to tune the parameters of PI controller. Though, PI controllers are proved to be a perfect controller for simple and linear processes [11], it has limitations in handling nonlinear process and there are various techniques have been proposed to overcome these limitations [12].

In industries, most of the liquid level systems have inherent nonlinearity [13] due to change in shape of the tank as the height changes. In literature, adaptive control [14-17] and process linearization [1, 3, 18] are the widely used techniques to address the control problem of these nonlinear liquid level systems. In adaptive control techniques, the controller adapts its parameters [17] when the process parameters are varied due its nonlinearity. This techniques demand for an online parameter estimator to determine the changes in process parameters and an online controller tuner to vary the controller parameters. On the other hand, process linearization technique linearized the system [19-20] enabling linear controllers like PI controller [1] to control a nonlinear system.

The proposed work transforms non-linear control loop into a linear system over the entire span of level. This works employs the use of the structured nonlinearity of the liquid

tank system to design a robust control structure. Using this structured nonlinearity, a simple transformation has been proposed to translate the level into volume. The controller tries to minimize the volume error by regulating the inflow to the tank. As the inflow and volume have linear relationship, the entire closed loop structure becomes linear enabling a PI controller to be robust across all the operating regions.

The main contributions of this paper are as follows, (i) a detailed modeling of the hemispherical tank using mass-balance equations, (ii) assumption such as transport delay and level saturations are made in order to bring the model performance close to the real-time process, (iii) design of volumetric observer and estimator, which enables linearization of a hemispherical tank, (iv) simulation results to validate the performance of the proposed controller, and (v) performance indices like averaged-ISE, ITAE & IAE are also formulated, which will give the data in a more readable format for better understanding of controller performance.

METHODS

Modeling of Hemi-Spherical Tank

The basic mass-balance equation for any liquid tank system is defined as the rate of change of volume is proportional to the difference in flow rate in and out of the tank and is given by,

$$\frac{dV}{dt} = F_{in} - F_{out} \quad (1)$$

For a gravity discharge, the outflow depends on the present height of the tank, so (1) becomes,

$$\frac{dV}{dt} = F_{in} - K_v \sqrt{h} \quad (2)$$

For a hemi-spherical tank, the relation between instantaneous height and instantaneous radius is calculated by the triangle OAB in figure 1,

In ΔOAB ,

$$(R-h)^2 + r^2 = R^2 \quad (3)$$

On rearranging the equation (3),

$$r = \sqrt{2Rh - h^2} \quad (4)$$

The instantaneous volume of liquid in a hemi-spherical tank is given by,

$$V(t) = \frac{2}{3} \pi h(t) r^2(t) \quad (5)$$

On solving these equations and differentiating with respect to time the final time-domain model of a hemi-spherical tank with in-flow as input and height as output is given by,

$$\frac{dh}{dt} = \frac{F_{in} - K_v \sqrt{h}}{\frac{2}{3} \pi (4Rh - 3h^2)} \quad (6)$$

where, R and H are the maximum height and maximum radius of the hemispherical tank respectively. F_{in} is the inflow to the tank, K_v is the outflow valve co-efficient, V is the volume of the tank, h is the height of the tank at time (t) and r is the radius of the liquid in tank at time(t).

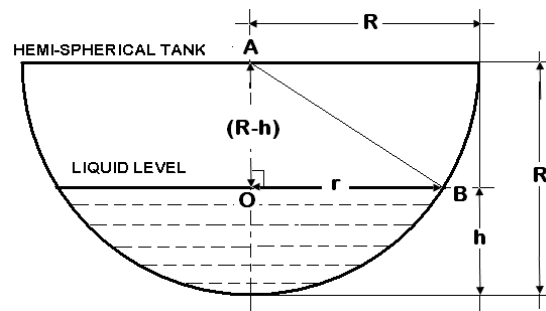


Figure 1: Schematic diagram for hemispherical tank

Lower Bound Height of the Tank

At liquid level zero, the rate change of volume becomes infinite. And at the lower level of the tank the out-flow pipeline fits as shown in figure 2. This will create a constant lower saturation limit of height.

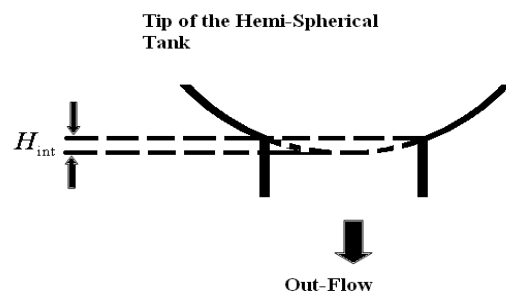


Figure 2: Bottom Tip of the Hemispherical Tank

Transport Delay of the Tank

The transportation delay time is defined as the time taken for the change in height to be effected for a corresponding change in inflow. For a change in inflow to create a change in height, the inflow has to reach the surface of the liquid in the tank and spread uniformly over the cross section area to fill up the space such that it can create a change in height for a change in inflow. The distance traveled by the liquid to reach the circumference of the tank for an instant of height (h) is shown in Figure 3 with arrow marks.

Assuming the flow as gravity flow, the dead time in seconds can be calculated as in equation (7),

$$T_d = \frac{H_i + (R - h(t)) + r(t)}{G \times t} \quad (7)$$

where, H_i is the total height between the reservoir and the top of the tank, r is the instantaneous radius for the corresponding level h and G is acceleration due to gravity.

Design of Robust Control Using Volumetric Observer

In the proposed design, a volume observer is used in the feedback of the closed loop system. An observer [15] is used to calculate the system's state variable using the system's measurable output. In the proposed work, the volumetric observer calculates the instantaneous volume of the non-linear tank using the measured instantaneous level of the liquid in the tank.

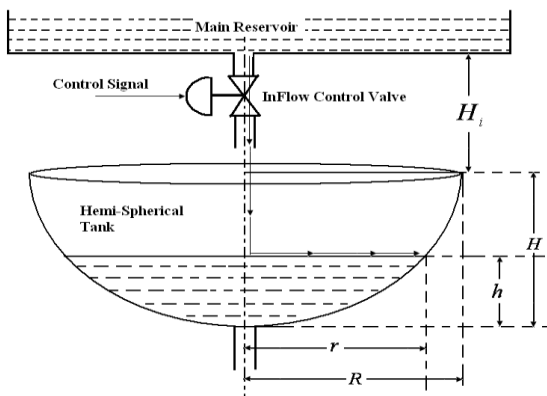


Figure 3: Transport delay in hemi-spherical tank

Since the proposed design uses instantaneous volume of the liquid in a tank as feedback signal. So, a volume estimator is also employed in the forward path of the control system. This estimator will estimate the volume required for the non-linear tank to achieve the desired height as shown in figure 4. For the hemi-spherical tank system, the volume observer and estimator are given by the equations (8) and (9) respectively.

$$V_{OS} = \frac{2}{3} \pi (2R h_{pv}^2 - h_{pv}^3) \quad (8)$$

$$V_{ES} = \frac{2}{3} \pi (2R h_{sp}^2 - h_{sp}^3) \quad (9)$$

where, V_{OS} is the observed volume output from the observer. h_{pv} is the present level of the liquid in the tank. V_{ES} is the estimated volume needed for a desired level. h_{sp} is the desired level which is to be maintained. Finally, a linear PI controller is used to calculate the required in-flow (F_{in}) using volumetric error ($V_{error} = V_{ES} - V_{OS}$).

Once the process is linearized, the conventional PI controller is employed to manipulate the inflow control valve. This regulates the inflow to the tank in order to achieve the desired liquid level. The parameters of the PI controllers are tuned by considering the linear model of the tank. The tank model is linearized by cascading the hemi-spherical tanks system with the volume observer. This relates the inflow to the tank with the observed volume, which is a linear relation. Thus, the introduction of the volume observer and estimator makes the closed loop structure to be linear and improves the robustness of the PI controller.

RESULTS AND DISCUSSION

The performance of the proposed robust PI controller is compared with the conventional PI controller. To maintain integrity, both the controllers are tuned using the same technique and their performances are validated using averaged performance indices like ISE, IAE and ITAE [13].

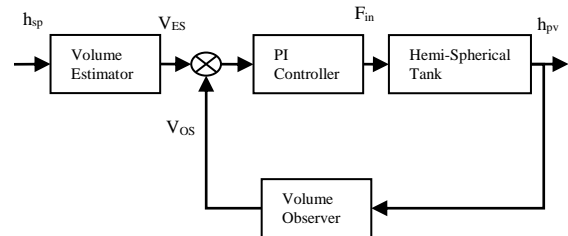


Figure 4: Block diagram of proposed robust control system

Averaged Performance Indices

Unlike conventional performance indices, the averaged performance indices provide an efficient and comparative data to validate the performance of the proposed controller. The averaged ISE value can be calculated as in equation (10) and (11) follows,

$$ISE_{avg} = \frac{(ISE_{rob} + ISE_{cov})}{2} \quad (9)$$

$$ISE_{per} = \frac{ISE_x}{ISE_{avg}} \Big|_{x=(rob,cov)} \quad (10)$$

The averaged ISE will give an insight of how the performance deviated from its average values as shown in Table 1. The averaged values will have base of 1 and if the value is less than 1, whose contribution to the average is minimum, which in turn reflects the minimum error value. On the other hand if the value is greater than 1 whose contribution will be maximum and will have large error. So the overall range will be from 0 to 2. Thus, the averaged performance metrics provides a more readable data than having these data in terms of powers of 10 or several digits as in conventional methods.

Table 1: Averaged Performance Indices

h (cm)	ISE		ITAE		IAE	
	R-PI	C-PI	R-PI	C-PI	R-PI	C-PI
20	1.13	0.87	0.28	1.72	0.86	1.14
40	0.85	1.15	0.09	1.91	0.58	1.42
60	0.64	1.36	0.05	1.95	0.43	1.57
80	0.63	1.37	0.08	1.92	0.48	1.52

Tracking Response of the controllers

The set point tracking performance of the proposed controller is validated against the conventional controller. The robust PI has almost the same peak overshoot over the entire operating range and offers lesser oscillatory response when compared to the conventional PI as shown in figure 5 and 6 respectively. The full extent of this advantage can be observed when there is a large step change which causes great increase in the settling time and the cyclic oscillation, when compared to the robust PI. Thus, the proposed robust PI offers better and stable performances.

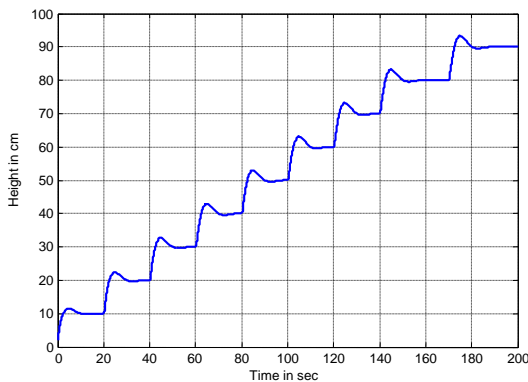


Figure 5: Set Point tracing response of proposed robust PI controller

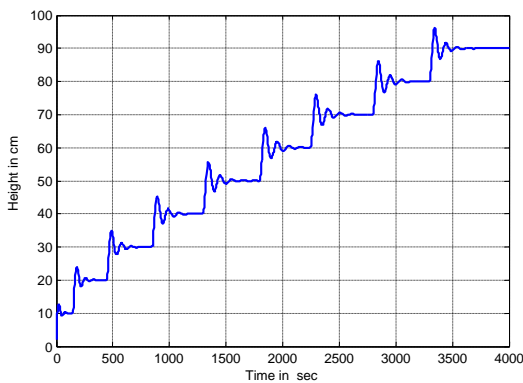


Figure 6: Set point tracing response of conventional PI controller

CONCLUSION

This paper proposes a design technique for imparting robustness in a control system, which is having inherent non-linearity. It employs the use of structured nonlinearity observed in the liquid tank systems for linearization of closed loop system. This technique is simple and promising to be implemented in real-time. It needs a simple computation system to compute an instantaneous volume for an instantaneous level, a linear PI controller and an interfacing system. Extension of the proposed controller to other nonlinear process is the future scope of this work.

REFERENCES

- [1] Dinesh Kumar, D., Meenakshipriya, B., and Sakthiya Ram, S., " Design of PSO based I-PD Controller and PID Controller for a Spherical Tank System," Indian journal of science and technology, vol. 9, no. 1, 2016.
- [2] Madhavasarma, P., and Sundaram, S., " Model Based Tuning of Controller for Non-Linear Hemispherical Tank Processes," Instrumentation Science & Technology, vol. 35, no. 6, 2007.
- [3] Anandanatarajan, R., and Mourougapragash, S., "Experimental evaluation of a controller using a variable transformation Smith predictor for a nonlinear process with dead time," ISA Transactions, vol. 47, no. 2, 2008.
- [4] Reshma, K. V., and Sumathi, S., " Control of Non Linear Spherical Tank Level Process," International Journal for Research in Applied Science and Engineering Technology, vol. 3, 2015.
- [5] Yumrutaş, R., and Ünsal, M., "A computational model of a heat pump system with a hemispherical surface tank as the ground heat source", Energy, vol. 25, no. 4, 2010.
- [6] Ji, F., Wang, X., Deng, G., Feng, J., and Liang, H., " Effect of the Shapes of Diversion Umbrellas on Temperature Field in a Large Spherical Tank During Heat Treatment Processes", ASME, Pressure Vessels and Piping Conference, vol. 2: Computer Technology and Bolted Joints, 2015.
- [7] Abinaya, N. S., and Rakesh Kumar, S., "Design of Float-Image Level Transmitter for Level Measurement Using Image Processing Technique," Sensors & Transducers, vol.145, no.10, 2012.
- [8] Ge, M., Chiu, M. S., and Wang, Q. G., " Robust PID controller design via LMI approach ," Journal of Process Control, vol.12, 2012.
- [9] Anandanatarajan, R., Chidambaram, M., and Jayasingh, T., "Design of controller using variable transformations

- for a nonlinear process with dead time," ISA Transactions, vol. 44, 2005.
- Innovative Computing, Information and Control, vol.5, 2009.
- [10] Zeigler, J. G., and Nichols, N.B., 1942," Optimum settings for automatic controllers," Trans.ASME, Vol. 64, pp.759 – 768.
- [11] Lee, I. B., and Sung, S. W., 1996," Limitations and counter measures of PID controllers," Industrial & Engineering Chemistry Research, 35, 2596–2610.
- [12] Anandanatarajan, R., Chidambaram, M., and Jayasingh, T., 2006," Limitations of a PI controller for a first order non-linear process with dead time," ISA Transactions, USA, 185–200.
- [13] Valarmathi, R., Theerthagiri, P. R., and Rakeshkumar, S., "Design and analysis of genetic algorithm based controllers for non linear liquid tank system," IEEE International Conference on Advances in Engineering, Science and Management (ICAESM), 2012.
- [14] Bhuvaneswari, N. S., Uma, G., and Rangaswamy, T. R., "Adaptive and optimal control of a non-linear process using intelligent controllers," Applied Soft Computing, vol. 9, no. 1, 2009.
- [15] Liu, Y. J., Tong, S., Li, D. J., and Gao, Y., " Fuzzy Adaptive Control With State Observer for a Class of Nonlinear Discrete-Time Systems With Input Constraint", IEEE Transactions on Fuzzy Systems, vol. 24, no. 5, 2016.
- [16] Sakthivel, G., Anandhi, T. S., and Natarajan, S. P., "Modeling and Real Time Implementation of Digital Pi Controller for a Non Linear Process," Journal of Innovative Research in Engineering and Sciences, vol. 2, no. 5, 2016.
- [17] Anand, S., Aswin, V., and Rakesh Kumar, S., " Simple tuned adaptive PI controller for conical tank process," IEEE International Conference on Recent Advancements in Electrical, Electronics and Control Engineering (ICONRAEeCE), 2012.
- [18] Kesavan,E., and Rakesh kumar,S., "PLC Based Adaptive PID Control of Non Linear Liquid Tank System using Online Estimation of Linear Parameters by Difference Equations," International Journal of Engineering and Technology (IJET), vol. 5, no. 2, 2013.
- [19] Christy, Y., Dinesh Kumar, D., and Scholar, P. G., " Modeling and Design of Controllers for Interacting Two Tank Hybrid System (ITTHS)," International Journal of Engineering and Innovative Technology (IJEIT), vol.3, no.7, 2014.
- [20] Wang, T., Tong, S., and Li, Y., "Robust adaptive fuzzy control for nonlinear system with dynamic uncertainties based on backstepping," International Journal of