

Super-Resolution Algorithms for Spectral Estimation in Cognitive Radio Systems

¹ Vu Van Yem and ² Nguyen Canh Minh

¹*School of Electronics and Telecommunications, Hanoi University of Science and Technology, Hanoi, Vietnam.*

²*Faculty of Electrical and Electronic Engineering, University of Transport and Communications, Hanoi, Vietnam.*

Abstract

Cognitive radio is new technology that uses spectrum more effectively. Spectrum sensing is one of the most challenging problems in cognitive radio systems. There are some sensing methods for indentifying the unused spectrum for opportunistic transmission. This paper develops one method based on multiple signal classification (MUSIC) algorithm for spectral estimation in cognitive radio system. Unlike traditional MUSIC algorithm that estimates direction of arrival of signal of interest, the modified MUSIC algorithm based on frequency difference of arrival (FDOA) estimates frequencies or carriers of users in cognitive radio network. The developed algorithm brings a more coherent spectrum and real power values than traditional MUSIC as well as fast fourier transform (FFT) approach. In this paper, we also discuss resolution issue and evaluate parameters having significant effects on it.

Keywords: Cognitive radio, Spectrum sensing, MUSIC, FDOA, Spectral estimation. Super-resolution algorithm.

INTRODUCTION

Cognitive radio proposed by Mitola III is a new technical concept in wireless communication technology that takes advantage of the available spectrum [1]. This technology is controlled by OODA loop which stands for Observation-Orientation-Decision-Action [1]-[3]. Concretely, At first, under affected by the outside world, cognitive radio observes surrounding environment (Observation), after that it will evaluate received knowledge (Orientation). Next, it will choose an alternative (Decision) to adjust its parameters such as transmit power, carrier frequency as well as modulation scheme in order to adapt itself to the environment (Action). Clearly, through the fundamental operation of OODA loop, we can realize that cognitive radio is prominent with its observation, adaptability and intelligence, which can exploit spectrum sources more effectively. To achieve these characteristics, one of the most challenging problems in cognitive radio systems is how to select a suitable spectrum sensing method, because it has to meet various requirements. Cognitive radio has to be aware of surrounding environment, especially interference temperatures. Spectrum sensing in cognitive radio system must

define the primary user's position, the type of signal, and must detect unused spectrums in order to exploit them. Based on spectrum knowledge, cognitive radio system will make decisions about transmitting and receiving signal without creating interferences to the primary users.

Some spectrum sensing methods have been proposed for cognitive radio systems, for example spectrum pooling using orthogonal frequency division multiplexing (OFDM) technology [4], using filter bank [9], based on multi-taper method (MTM) [5] and cyclic spectrum estimation [6][7]. However, all of these methods use fast fourier transform of the received signal, which is a conventional choice and it is not the best one to obtain a coherent spectrum. For the purpose of resolution, using high-resolution techniques instead of FFT one will have more coherent and accurate spectrum [8]. However, it is limited to simple demonstration of pseudo spectrum of cognitive radio node. In addition, the resolution issue, an important criterion to evaluate a spectral estimation method, has not been investigated yet. The MUSIC algorithm, a high resolution algorithm, was first presented by Ralph O. Schmidt for Direction Of Arrival estimation [9]. In this paper, we modify the conventional MUSIC algorithm for the purpose of spectral estimation in cognitive radio systems. This paper is organized as following. Section II presents the modified MUSIC algorithm for spectral estimation. After that, simulation results using MATLAB will be shown in Section III. In Section IV will analyze the resolution and the effects of parameters of the algorithm. Finally, a brief conclusion is given in Section V.

PROPOSED ALGORITHMS

Ideas of the algorithm

MUSIC algorithm is a kind of DOA estimation technique based on eigen-value decomposition, which is also called the subspace-based method [9]. Among high-resolution techniques, MUSIC emerges as the most promising one, because this technique can estimate multiple parameters, such as azimuth angle, elevation angle, polarization as well as propagation delay. As a subspace technique, MUSIC has advantages such as the independent distribution of frequencies,

fast global numerical convergence, no need for initial DOA information. Especially, MUSIC can minimize RMS more than other multidimensional subspace-fitting algorithms [10].

In cognitive radio systems, we focus on frequency problem mainly, and our goal is to estimate the power of signals at different frequencies, so the idea of FDOA-based MUSIC was sparked. FDOA-based MUSIC not only rejects the effect of antennas and noise, but also helps cognitive radio systems use spectrum more effectively. Especially, we can apply the modified MUSIC to one receiving antenna to make it operate like a normal array of antennas. The efficiency of antenna systems will be very high, and meet the requirement of cognitive radio systems. Furthermore, as far as we know, MUSIC is often used for estimating narrow-band signals, while FDOA is applied to localization of wide-band signal sources. Thus, the combination of FDOA and MUSIC technique can widen the application range of MUSIC. In this section, we improve both traditional MUSIC and root-MUSIC algorithm to choose the best one. We call the modified MUSIC algorithm a super-resolution algorithm. This algorithm still operates in according to the principal operations of MUSIC, but with little change in terms of system model as well as computational matrix.

Data model

Assuming that a system as illustrated in Fig.1 contains M - wideband far-field signal resources coming to one receiving antenna, and these signals divide into N frequency points. N and M values must satisfy the initial condition of MUSIC that is N>M.

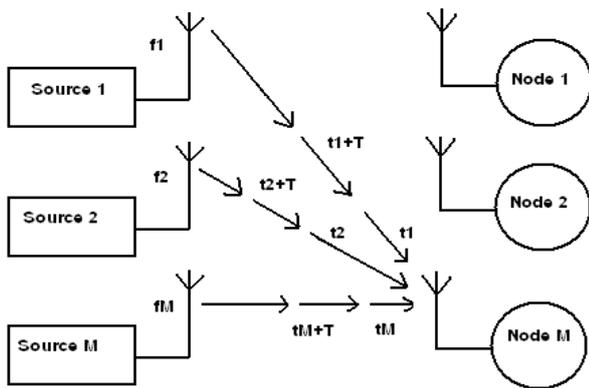


Figure 1: Model of a MIMO cognitive radio system.

The output i^{th} signal $x_i(t)$ from the antenna can be written as:

$$x_i(t) = \sum_{k=1}^M s_k(t) \exp(\Delta_k) + n_i(t) \quad (1)$$

With $s_k(t)$ is incident signal form k^{th} source and $n_i(t)$ is i^{th} white noise. Δ_k represents depending on algorithms

Transforming the equation (1) to matrix form, we have

$$X(t) = A.S(t) + N(t) \quad (2)$$

With $X(t) = [x_1(t)x_2(t).....x_i(t).....x_N(t)]^T$

$$S(t) = [s_1(t)s_2(t).....s_k(t).....s_M(t)]^T$$

$$N(t) = [n_1(t)n_2(t).....n_i(t).....n_N(t)]^T$$

$$A = [a(f_1)a(f_1).....a(f_k).....a(f_M)]$$

Based on root-MUSIC algorithm:

We develop a new algorithm based on root-MUSIC.

$$\Delta_k = -j2\pi\Delta \sin(2\pi ft_k) / \lambda$$

Herein, A is matrix of steered frequency vectors instead of steering vectors as usual. Each steered frequency vector has a form as the following:

$$a(f) = [1z^1z^2...z^k...z^{N-1}]$$

$z = e^{(-j2\pi\Delta \sin(2\pi ft_k) / \lambda)}$ with Δ is the inter-element separation and λ is the wavelength of the signal

Based on MUSIC algorithm

A new algorithm is improved from conventional MUSIC.

$$\Delta_k = -j2\pi ft_k$$

$$a(f) = [1e^{-j2\pi ft_1} e^{-j2\pi ft_2} ... e^{-j2\pi ft_3} ... e^{-j2\pi ft_{N-1}}]$$

In both above algorithms: $t_i = t_0 + (i-1)T$ with $i = 1 \div N$

At t_0 , the antenna begins to receive signals, and it takes T to scan all of incident signals. It means that if the antenna is receiving signal from 1st source, after a period of T, the antenna will receive signal from 1st source again; because this antenna used in a cognitive radio system, so it can be aware of the orders of receiving signals.

Autocorrelation functions:

$$R_{XX} = E\{X(t)X^H(t)\} = \frac{1}{T} \sum_{t=1}^T X(t)X^H(t) = A.R_S.A^H + \delta_o^2.I \quad (3)$$

$$R_S = E\{S(t)S^H(t)\}, \quad \delta_o^2.I = E\{N(t)N^H(t)\}$$

where S is the covariance matrix of the emitter signals and δ_o^2 is the noise power in each channel.

From (3), we have:

$$R_{XX} = \sum_{i=1}^N \lambda_i e_i e_i^H = E_S \Lambda_S E_S^H + \delta^2 E_N E_N^H \quad (4)$$

where $E_S = |e_1 \dots e_M|$: signal subspace

$E_N = |e_{M+1} \dots e_N|$ noise subspace.

Necessary conditions for eigenvectors:

- (1) $\mu_1 \geq \mu_2 \geq \dots \geq \mu_M > \mu_{M+1} = \mu_{M+2} = \dots = \mu_N = \delta_o^2$
- (2) $\{e_{M+1} \dots e_N\} \perp \{a(f_1) \dots a(f_M)\}$

We can generate the pseudo spectra based on MUSIC. So we construct an equation to estimate the power of different frequencies $P(f)$ and evaluate

$$P_{MUSIC}(f) = \frac{a(f)^H a(f)}{a(f)^H .E_N .E_N^H a(f)} \quad (5)$$

If we let take different values of f , we can plot what is called a pseudo spectrum, which has peaks where the zeros are. This is a good way to see how the spectra might look like, instead of using the nonparametric approach such as the periodogram.

SIMULATION RESULTS

In order to illustrate the performance of the procedure, we create a system including five signal sources and one receiving antenna. The frequencies of the carriers are assigned are 1.2; 1.4; 2.3; 2.6; and 3.7 GHz. The SNR of carriers are from 1 to 20dB randomly. In assumption, these signals are incoherent and linear independent to each other. In other words, these signals are uncorrelated, which is a necessary condition to make no effect on simulation results. The range of frequencies to estimate is from 1 to 5 GHz, and is divided into 15 frequency points. In this simulation, a data record of 5000 samples regarding to 5 carriers in white noise will be analyzed.

Fig.2 and Fig.3 presents the simulation results in MATLAB. As observed, the pseudo spectra are very coherent and sharp. These plots reveal that five given frequencies are being used, but among them there are white spaces for our cognitive radio system to transmit its signals. Comparing to Fig.2, the plot in Fig.3 has a lower resolution. It is understandable because root-MUSIC is special case of a uniform linear array (ULA), so it does not exhibit a loss-of-resolution threshold effect [9]. To detail, at low information-to-noise ratios (INR), root-MUSIC has a performance threshold much lower than one of MUSIC. Based on these results, the algorithm 1 becomes the best choice for cognitive radio systems.

This decision is more exact when we compare the simulation result of algorithm 1 to the result of FFT in Fig.3. Root-MUSIC can separate two signal sources with a smaller difference in regard to frequency than FFT. This statement can be proved clearly in the next section.

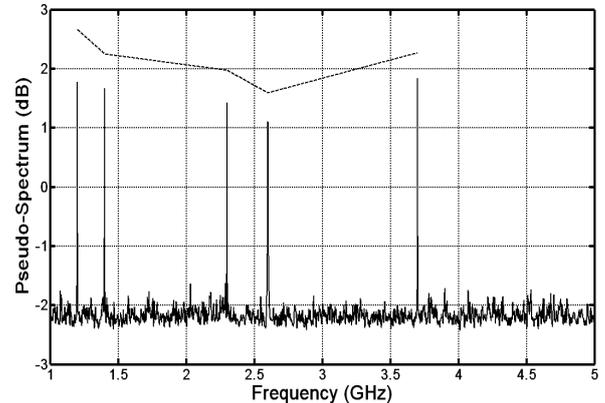


Figure 2: Simulation result for algorithm 1.

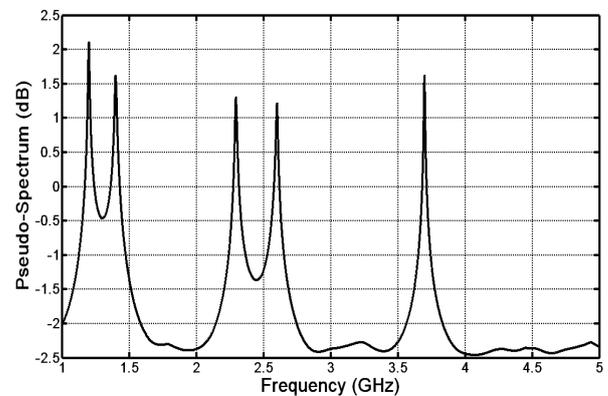


Figure 3: Simulation result for algorithm 2.

Normally, when using traditional MUSIC algorithm, we just demonstrate a pseudo spectrum of signals, not their real powers. However, with this improved MUSIC, we can estimate real powers of carriers, as you can see in the unconnected line in Fig.2.

ANALYSIS OF THE RESOLUTION

The effects of parameters on resolution

Resolution is the minimum frequency difference between two signal sources that this algorithm still discriminates them. It is the most important factor to evaluate one spectral estimation technique. These significant parameters needed to be concerned are SNRs (signal-to-noise ratio) of sources, the number of frequency points (N), the number of samples, and the period of T .

First of all, we consider the effect of SNRs. As mentioned in the previous section, at low INR or low SNR, the loss-of-resolution threshold effect will happen in root-MUSIC and reduce the resolution of the algorithm. Thus, the change of SNR is right proportional to the change of resolution. To be concrete, we will consider the relationship between SNR of a source and the resolution of MUSIC.

Modifying the equation to determine the resolution $\Delta\tau$ in [8] to be suitable for FDOA-based MUSIC algorithm, we have the following equation:

$$SNR \approx \frac{1}{T} \left\{ \frac{180(N-2)}{\pi^2 N} \left[1 + \sqrt{\frac{\pi^2 T (\Delta\tau \cdot \Delta f_{MUSIC})^2}{15(N-2)}} \right] (\Delta\tau \cdot \Delta f_{MUSIC})^{-4} \right\} \quad (6)$$

Where, N is the number of frequency tones, T is the number of snapshots per a tone, $\Delta\tau$ is time step, and Δf_{MUSIC} is MUSIC resolution of frequency which is the minimum difference of two signal sources. With T = 200, L = 15 we have a simulation result as shown in Fig.4. Although in theory, the more SNR is, the better resolution is; in fact, when SNR reaches 45dB, the resolution stays the same. We realized that while the values of SNRs change a lot, the resolution just changes little. In other words, SNR has little effect on the resolution. Through Figure 4, we also can draw a conclusion that when we estimate any frequency, the maximum possible error is 0.001GHz.

Secondly, one of parameters having the biggest effect on the resolution is the number of frequency points. As seen in (6), if SNR is constant, N is diverse proportional to Δf_{MUSIC} . A small increase of this value can ameliorate the spectrum demonstration significantly. It can be explained that when the value of N increase, the number of steered frequency vector in matrix A will increase too; and the value of $P_{MUSIC}(f)$ will be more exact. Herein, N is the number of frequency points, so it can get a much big value without a limit. However, it is manifest that the greater N is, the longer time MATLAB needs to run. Based on experiments, the value of 15 is good enough to get a small resolution.

Thirdly, admittedly, the number of samples contributes a big role in building a coherent spectrum. Like the previous parameter, when this value increases, the resolution will be decreased, but the time to run the simulation will be much longer. However, the number of samples cannot be as sensitive as N parameter; it needs a big increase to make a change in spectrum demonstration.

Finally, we will evaluate the scanning period of T. Like N parameter, the change of T results in matrix A and $P_{MUSIC}(f)$, so it has a big effect on the resolution.

Concisely, the effects of parameters will be presented in the following table.

| Parameters | SNR | N | Samples | T |
|------------|--------------|--------------------|------------|------------|
| | Small effect | Significant effect | Big effect | Big effect |

Simulation results

We implement a simulation program with a models of two signal sources in 1.2 and 1.201GHz, the number of frequency N = 15, both SNRs are 45dB, the number of samples are 2000, and T = 3s. We use both the proposed MUSIC algorithm and FFT to simulate in Matlab.

Fig.6 demonstrates the resolution of the chosen algorithm. In the plot, we can discriminate the signals of two frequencies in 1.2 and 1.201 GHz. It means that the achieved resolution is 0.001GHz or 1MHz. This result proves that our technique is super-resolution which can bring spectra more clear and exact than high resolution ones. Using FFT with the same system model, we got a simulation result as Fig.5. As we can see, the spectrum in Fig.5 is too blunt to recognize two signal sources. In other words, FFT has lower resolution than root-MUSIC.

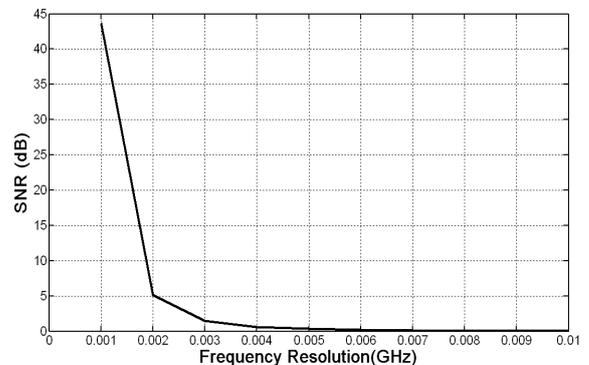


Figure 4: The relationship between SNR and frequency resolution.

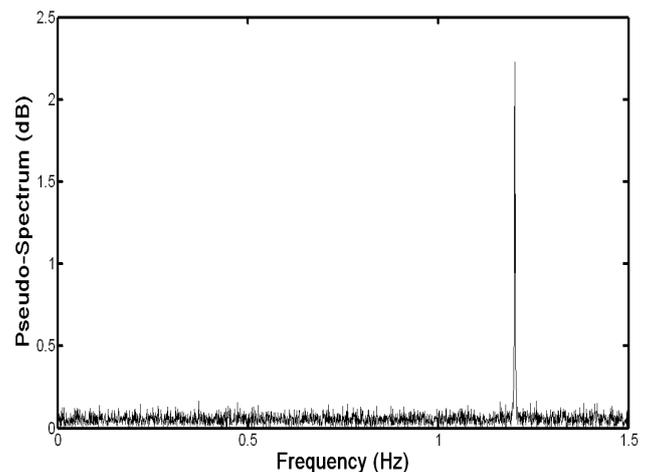


Figure 5: Resolution of FFT.

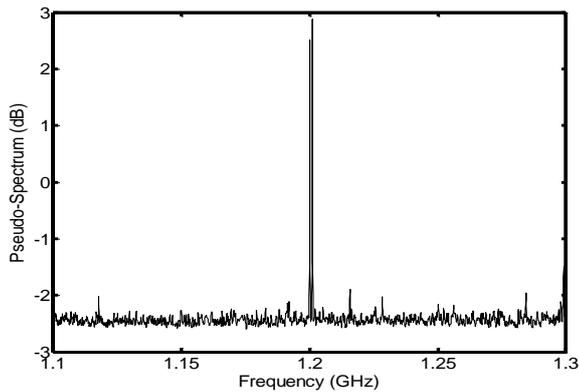


Figure 6: Resolution of the proposed algorithm.

CONCLUSION

In this paper, we develop a super-resolution algorithm based on root-MUSIC for cognitive radio systems. Firstly, the proposed algorithm rejects the drawbacks of conventional MUSIC. The spectrum is no longer reliable on the directions of antennas as well as incident signals, so the achieved result is more stable. More importantly, because it is FDoA-based MUSIC, it can apply to estimate wide-band sources; obviously, the range of applications will be widened. With the flexibility in choosing the number of frequency points, we can estimate a huge sources which still satisfy the primary condition ($N > M$). Especially, this method helps an antenna operate as an array of antennas, which not only reduces the necessary number of antennas, but also improves the efficiency of cognitive radio systems seriously. Secondly, with a small resolution, this approach can bring a more coherent and sharper spectrum than high-resolution algorithms. Most notably, it can provide real power levels, besides pseudo-spectrum as other spectrum estimations. With these prominent advantages, the super-resolution algorithm is worthy to become a candidate for spectral estimation for cognitive radio systems

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