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Comparative Study of Free Vibration of Cross-Ply Laminated Plates under First Order Shear Theory

K. Karthik[1], K.K. Viswanathan[1,2,*], Z.A. Aziz[1]

[1]UTM Centre for Industrial and Applied Mathematics (UTM-CIAM), Ibnu Sina Institute for Scientific & Industrial Research, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia.

[2] Kuwait College of Science and Technology, Doha District, Block 4, Safat 13133, Kuwait.

Y.V.S.S. Sanyasiraju[3]

[3]Department of Mathematics, Indian Institute of Technology Madras, Chennai 600036, India.

J.H. Lee[4]

[4]Dept. of Naval Architecture and Ocean Engineering, Inha University, 100 Inharo, Nam-gu, Incheon 22212, South Korea.

**Corresponding author*

Abstract

Free vibration of laminated cross-ply plates under first order shear deformation theory with clamped-clamped boundary condition using Radial Basis Function (RBF) and Spline Function are presented. The equations of motion derived using YNS theory and the solution is assumed in separable form to obtain a coupled differential equations. The differential equation are then approximated using RBF for first case and the displacement and rotational functions are approximated using Bickley-Spline function for the second case. The frequency parameter is analyzed with respect to odd and even number of layers, fiber orientations, side-to-thickness ratio and aspect ratio. Both RBF and Spline methods are compared and the results are presented in term of tables and graphs.

Keywords: Cross-ply; Free vibration; Radial basis function; Splines

INTRODUCTION

Composite materials are closely attached to civil, aerospace, automobile and aeronautic fields. The use of composite material in engineering field increased drastically due to its mechanical behaviour. Thus, many researcher show more interest to study the mechanical behaviour of these composite materials. Kirchhoff, (1850) [1] initiated the plate behaviours study while neglecting shear deformation. This ignorance leads to an inaccurate results for moderately thick plates and obtain higher frequency value than Mindlin plate theory [2]. Shear deformation theory was introduced by Stavsky [3] for isotropic plates and generalized to laminated anisotropic plates by Yang *et al* [4]. Many researchers have used various numerical approach to study the mechanical behaviour of plates using first order shear deformation theory (FSDT).

Radial basis function is related to scattered data approximation where it depends on Euclidian distance between collocation points. In this present work, RBF approximate ODE solution directly, which introduced by Kansa [5] for PDE equation. Kansa's Multiquadric (MQ) interpolation function, where a shape parameter has considered as a variable across the problem domain, and the shape parameter value obtained by an optimization approach. Kansa [6], and few other researcher used RBFs method to solve plate problems. Ferreira [7, 8] worked on Timoshenko beams and Mindlin plates using RBF and also formulate multiquadric RBF method for moderately thick laminated composite plates.

Liew *et al.*, Rodrigues *et al*., and Liu *et al*. [9-11] furthered their study on buckling analysis using radial basis function for laminated plates. Apart from them, Ferreira [12-15] and his group produced numerous research on plates using RBF with first order and higher order shear deformation theories since last few years, and Liu *at el*.[16] also investigated laminated composite plates using the radial point interpolation method. Meanwhile, Sanyasiraju [17], also had used RBF technique to solve some problems

Spline is another type of approximation where widely it used in numerical calculation. This method introduced by Schoenberg for a special study, then developed by Bickley [18] for two point boundary problems. Viswanathan and Navaneethakrishnan [19, 20], Viswanathan and Kim [21], and Viswanathan and Lee [22] showed interest in plate study, and used spline technique to solve various problems. Irie *et al.* [23], Irie and Yamada [24] also had done some studies on free vibration of rotating non-uniform discs and annular plate with variable thickness respectively.

The purpose of this study is to investigate the frequency parameter of cross-ply laminated plate under clamped-clamped (C-C) boundary condition using Radial Basis Function (RBF) and spline approximation. The formulation of this problem is based on Hamilton's principle, where the second order differential equations are obtained in term of mid-plane displacements and rotational functions. The solution is then assumed in separable form to get ordinary differential equations. Then the differential equations are approximated using Radial Basis Functions for the first case and the displacements and rotational functions are approximated using spline functions. In both the cases, the final equations became as generalized eigenvalue problem. To find the frequency parameter. The frequency parameter is analysed with respect to side-to-thickness ratio, aspect ratio, and number of layers considering different types of materials.

PROBLEM FORMULATION

Consider a rectangular plate with length a, width b and constant thickness h, which made up of even and odd number of layers are given in which the angle orientation fixed at 0˚ and 90˚ to analyze the problem. Based on YNS theory, the displacement components are consider as

$$u = u_0(x,y,t) + z\psi_x(x,y,t),$$
$$v = v_0(x,y,t) + z\psi_y(x,y,t),$$
$$w = w(x,y,t) \qquad \ldots(1)$$

where u, v, and w is displacement component in x, y and z directions respectively. u_0 and v_0 are the displacements at middle surface of the plate and ψ_x and ψ_y are shear rotation in middle surface of plate at any point and t is time. The Hamilton principle is given as

$$\int_0^T (\delta U + \delta V - \delta K)dt = 0 \qquad \ldots(2)$$

where δU, δV, and δK are strain energy, work done and kinetic energy respectively, and the governing equations are obtained from Hamilton's principle,

$$N_{x,x} + N_{xy,y} = I_0 \frac{\partial^2 u_0}{\partial t^2}$$

$$N_{xy,x} + N_{y,y} = I_0 \frac{\partial^2 v_0}{\partial t^2}$$

$$Q_{x,x} + Q_{y,y} = I_0 \frac{\partial^2 w}{\partial t^2}$$

$$M_{x,x} + M_{xy,y} - Q_x = I_1 \frac{\partial^2 \psi_x}{\partial t^2}$$

$$M_{xy,x} + M_{y,y} - Q_y = I_1 \frac{\partial^2 \psi_x}{\partial t^2} \qquad \ldots(3)$$

where N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x,,Q_y, I_0, and I_1 are defined in Appendix A. Substituting the stress-strain and strain-displacement relations, we get the following equation,

$$\left[A_{11}\frac{\partial^2}{\partial x^2} + A_{66}\frac{\partial^2}{\partial y^2} + I_0\omega^2\right]u + \left[(A_{12}+A_{66})\frac{\partial^2}{\partial x\partial y}\right]v +$$

$$\left[2B_{16}\frac{\partial^2}{\partial x\partial y}\right]\psi_x + \left[B_{16}\frac{\partial^2}{\partial x^2} + B_{26}\frac{\partial^2}{\partial y^2}\right]\psi_y = 0,$$

$$\left[(A_{12}+A_{66})\frac{\partial^2}{\partial x\partial y}\right]u + \left[A_{66}\frac{\partial^2}{\partial x^2} + A_{22}\frac{\partial^2}{\partial y^2} + I_0\omega^2\right]v +$$

$$\left[B_{16}\frac{\partial^2}{\partial x^2} + B_{26}\frac{\partial^2}{\partial y^2}\right]\psi_x + \left[2B_{26}\frac{\partial^2}{\partial x\partial y}\right]\psi_y = 0,$$

$$\left[2B_{16}\frac{\partial^2 u}{\partial x\partial y}\right] + \left[B_{16}\frac{\partial^2}{\partial x^2} + B_{26}\frac{\partial^2}{\partial y^2}\right]v + \left[(D_{11}+D_{66})\frac{\partial^2}{\partial x\partial y}\right]\psi_y$$
$$+\left[D_{11}\frac{\partial^2}{\partial x^2} + D_{66}\frac{\partial^2}{\partial y^2} - KA_{55} + I_1\omega^2\right]\psi_x + \left[-KA_{55}\frac{\partial}{\partial x}\right]w = 0,$$

$$\left[B_{16}\frac{\partial^2}{\partial x^2} + B_{26}\frac{\partial^2}{\partial y^2}\right]u + \left[2B_{26}\frac{\partial^2}{\partial x^2}\right]v + \left[(D_{12}+D_{66})\frac{\partial^2}{\partial x\partial y}\right]$$

$$\psi_x + \left[D_{66}\frac{\partial^2}{\partial x^2} + D_{22}\frac{\partial^2}{\partial y^2} - KA_{44} + I_1\omega^2\right]\psi_y + \left[-KA_{44}\frac{\partial}{\partial y}\right]$$
$$w = 0, \left[KA_{55}\frac{\partial}{\partial x}\right]\psi_x + \left[KA_{44}\frac{\partial}{\partial x}\right]\psi_y + \left[-KA_{55}\frac{\partial^2}{\partial x^2} - KA_{44}\right.$$

$$\left.\frac{\partial^2}{\partial y^2} - I_0\omega^2\right]w = 0 \qquad \ldots(4)$$

The displacement and rotation functions are assumed in separable form as

$$u(x,y) = U(x)\cos(n\pi y/b)e^{i\omega t},$$

$$v(x,y) = V(x)\sin(n\pi y/b)e^{i\omega t},$$

$$w(x,y) = W(x)\sin(n\pi y/b)e^{i\omega t},$$

$$\psi_x(x,y) = \Psi_X(x)\sin(n\pi y/b)e^{i\omega t},$$

$$\psi_y(x,y) = \Psi_Y(x)\cos(n\pi y/b)e^{i\omega t} \qquad \ldots(5)$$

with ω is the angular frequency and t is time. Non-dimensional parameters were introduced as

$\lambda = \omega a\sqrt{I_0/A_{11}}$, frequency parameter

$\phi = a/b$, aspect ratio

$X = x/a$, distance coordinate

$H = a/h$ side-to-thickness ratio …(6)

where A_{11} is standard extensional rigidity coefficient.

Substitute Eq. (5) and (6) into Eq. (4) to obtain the coupled differential equation in term of $U, V, W, \Psi_X,$ and Ψ_Y and given as follows,

$$\left[A_{11}\frac{d^2}{dX^2}+2A_{16}\frac{d^2}{dXdY}+A_{66}\frac{d^2}{dY^2}+I_0\omega^2\right]U+$$

$$\left[A_{16}\frac{d^2}{dX^2}+\left(A_{12}+A_{66}\right)\frac{d^2}{dXdY}+A_{26}\frac{d^2}{dY^2}\right]V+$$

$$\left[B_{11}\frac{d^2}{dX^2}+2B_{16}\frac{d^2}{dXdY}+B_{66}\frac{d^2}{dY^2}\right]\Psi_X+$$

$$\left[B_{16}\frac{d^2}{dX^2}+\left(B_{12}+B_{66}\right)\frac{d^2}{dXdY}+B_{26}\frac{d^2}{dY^2}\right]\Psi_Y=0$$

$$\left[A_{16}\frac{d^2}{dX^2}+\left(A_{12}+A_{66}\right)\frac{d^2}{dXdY}+A_{26}\frac{d^2}{dY^2}\right]U+$$

$$\left[A_{66}\frac{d^2}{dX^2}+2A_{26}\frac{d^2}{dXdY}+A_{22}\frac{d^2}{dY^2}+I_0\omega^2\right]V+$$

$$\left[B_{16}\frac{\partial^2}{\partial X^2}+\left(B_{12}+B_{66}\right)\frac{\partial^2}{\partial X\partial Y}+B_{26}\frac{d^2}{dY^2}\right]\Psi_X+$$

$$\left[B_{66}\frac{\partial^2}{\partial X^2}+2B_{26}\frac{\partial^2}{\partial X\partial Y}+B_{22}\frac{d^2}{dY^2}\right]\Psi_Y=0$$

$$\left[B_{11}\frac{d^2}{dX^2}+2B_{16}\frac{d^2}{dXdY}+B_{66}\frac{d^2}{dY^2}\right]U+$$

$$\left[B_{16}\frac{d^2}{dX^2}+\left(B_{12}+B_{66}\right)\frac{d^2}{dXdY}+B_{26}\frac{d^2}{dY^2}\right]V+$$

$$\left[D_{11}\frac{d^2}{dX^2}+2D_{16}\frac{d^2}{dXdY}+D_{66}\frac{d^2}{dY^2}-kA_{55}-kA_{45}\frac{\partial}{\partial Y}+I_1\omega^2\right]$$

$$\Psi_X+\left[D_{16}\frac{d^2}{dX^2}+\left(D_{12}+D_{66}\right)\frac{d^2}{dXdY}+D_{22}\frac{d^2}{dY^2}\right]\Psi_Y-$$

$$\left[kA_{45}\frac{d}{dY}+kA_{55}\frac{d}{dX}\right]W=0$$

$$\left[B_{16}\frac{d^2}{dX^2}+\left(B_{12}+B_{66}\right)\frac{d^2}{dXdY}+B_{26}\frac{d^2}{dY^2}\right]U+$$

$$\left[B_{66}\frac{d^2}{dX^2}+2B_{26}\frac{d^2}{dXdY}+B_{22}\frac{d^2}{dY^2}\right]V+$$

$$\left[D_{16}\frac{d^2}{dX^2}+\left(D_{12}+D_{66}\right)\frac{d^2}{dXdY}+D_{26}\frac{d^2}{dY^2}\right]\Psi_X+$$

$$\left[D_{66}\frac{d^2}{dX^2}+2D_{26}\frac{d^2}{dXdY}+D_{22}\frac{d^2}{dY^2}-kA_{44}-kA_{45}\frac{d}{dY}+I_1\omega^2\right]$$

$$\Psi_Y-\left[kA_{44}\frac{d}{dY}+kA_{45}\frac{d}{dX}\right]W=0$$

$$\left[kA_{55}+kA_{45}\right]\frac{d\Psi_X}{dX}+\left[kA_{44}+kA_{45}\right]\frac{d\Psi_Y}{dX}+$$

$$\left[kA_{55}\frac{d^2}{dX^2}+2kA_{45}\frac{d^2}{dXdY}+kA_{44}\frac{d^2}{dY^2}+I_0\omega^2\right]W=0 \quad \ldots(7)$$

where I_0 and I_1 is gives as

$$\left(I_0, I_1\right)=\sum_m\int_{z_{m-1}}^{z_m}\rho^{(m)}\left(1,z^2\right)dz$$

and $\rho^{(m)}$ is the material density of the m^{th} layer.

For anti-symmetric cross-ply lamination, the coefficients A_{16}, A_{26}, A_{45}, B_{16}, D_{16}, and D_{26} are identically zero, Viswanathan and Lee [19]. Thus the coupled differential equation, equation (7) can be written in matrix form as,

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ L_{21} & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\ L_{41} & L_{42} & L_{43} & L_{44} & L_{45} \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix}\begin{pmatrix} U \\ V \\ W \\ \Psi_x \\ \Psi_Y \end{pmatrix}=0 \qquad \ldots(8)$$

where L_{ij} (ij = 1, 2, 3, 4, 5)are the differential operators given in Appendix B.

SOLUTION PROCEDURE

The differential equation given by Eq. (8) consists of second order derivatives in term of $U(X)$, $V(X)$, $W(X)$, $\Psi_X(X)$, and $\Psi_Y(X)$. These differential equation are approximated using radial basis function and the displacement and rotational functions are approximated using spline functions.

Method of Radial Basis Function Approximation

Radial basis function is one of the mesh free method which depends on distance of points from centre. The distance from the centre represent as $g(||X - X_j, c||)$, where X_j is centre point, c is shape parameter, and $||X - X_j||$, is the Euclidian norm. Multiquadrics radial basis function is one of the approach to solve differential equations. The RBF interpolant is represented as

$$Lu(X) = s(X) = \sum_{j=1}^{N} a_j g_j \left(\left\| X - X_j \right\|, c \right) \qquad \text{…(9)}$$

The differential functions *U(X), V(X),* and *W(X)* and rotation functions Ψ_X and Ψ_Y were approximated by using RBF interpolant as

$$LU(X) = \sum_{j=1}^{N} a_j g_j \left(\left\| X - X_j \right\| \right),$$

$$LV(X) = \sum_{j=1}^{N} b_j g_j \left(\left\| X - X_j \right\| \right),$$

$$LW(X) = \sum_{j=1}^{N} c_j g_j \left(\left\| X - X_j \right\| \right),$$

$$L\Psi_X(X) = \sum_{j=1}^{N} d_j g_j \left(\left\| X - X_j \right\| \right),$$

$$L\Psi_Y(X) = \sum_{j=1}^{N} e_j g_j \left(\left\| X - X_j \right\| \right), \qquad \text{...(10)}$$

By substitute Eq. (10) into equilibrium equation, Radial basis function produce (5N+5) coefficients with 5N+5 homogeneous equation. Therefore these equation became as a generalized eigenvalue problem and resulting equation can be written as

$$L\mathbf{u} + \lambda\mathbf{u} = 0 \qquad \text{…(11)}$$

where L represent differential operator, **u** and λ represent eigenvectors and eigenvalues respectively.

Method of Spline Approximation

The displacement *U(X), V(X)* and *W(X)* and rotation functions Ψ_X and Ψ_Y are approximated by using cubic spline functions.

$$U(X) = a_0 + a_1 X + a_2 X^2 + \sum_{j=0}^{N-1} b_j (X - X_j)^3 H(X - X_j),$$

$$V(X) = c_0 + c_1 X + c_2 X^2 + \sum_{j=0}^{N-1} d_j (X - X_j)^3 H(X - X_j),$$

$$W(X) = e_0 + e_1 X + e_2 X^2 + \sum_{j=0}^{N-1} f_j (X - X_j)^3 H(X - X_j),$$

$$\Psi_X(X) = g_0 + g_1 X + g_2 X^2 + \sum_{j=0}^{N-1} p_j (X - X_j)^3 H(X - X_j),$$

$$\Psi_Y(X) = l_0 + l_1 X + l_2 X^2 + \sum_{j=0}^{N-1} q_j (X - X_j)^3 H(X - X_j), \qquad \text{…(12)}$$

where $H(X-X_j)$ is Heaviside function with N number of interval between [0,1]. The collocation points chosen from $X=X_s=s/N$, where s = 0,1,2,….,N, and the spline produce (5N+15) coefficients for 5N+5 homogeneous equation.

Here, clamped-clamped (C-C) boundary condition is considered, to get ten more equations and making a total of (5N+15) homogeneous equations, to obtain the generalized eigenvalue problem. The resulting equations can be written in form of

$$[M]\{q\} = \lambda^2 [P]\{q\} \qquad \text{…(13)}$$

Here [M] and [P] are square matrices and {q} is column matrix also known as eigenvector and λ is the eigenparameter.

NUMERICAL RESULTS AND DISCUSSION

In this study the frequency parameter analysed with respect to the aspect ratio (*a/b*) and length-to-thickness ratio (*a/h*) using different number of layers and materials. The plates are arranged in odd and even number of layers, with material combination of Kevlar-49/Epoxy (KGE), Graphite/Epoxy (AS_4/3501-6), (AGE) and E-glass/Epoxy (EGE) and the results are presented in both graphical and tabulated form.

Before proceeding the study, validation study has been carried out to validate the obtained results. Table 1 shows the validation of developed formulation against Khdeir [25]. Khdeir done research on symmetric cross-ply laminated composite plates and solve using Levy type solution. The properties of plate material used are as follows:

$E_1/E_2=40, G_{12}/E_2=G_{13}/E_2=0.6, G_{23}/E_2=0.5, \nu_{12}=0.25$

The fundamental frequency parameter, $\lambda=\omega a(\rho/A_{11})^{1/2}$, is analysed for cross-ply laminated plates under clamped-clamped boundary condition for *x*-axis where the *y*-axis is fixed at simply-supported boundary condition. The frequency parameter is analysed with respect to the length-to-thickness ratio (*a/h*) and aspect ratio (*a/b*), for odd and even number layered plates using two different methods, Radial Basis Function (RBF) and Spline method and the results are shown in tabulated and graphical form.

Table 1: The effect of the plate length-to-thickness ratio (*bh*) on the frequency parameter of a clamped-clamped three layered laminated square plate with $\theta = 0^\circ/90^\circ/0^\circ$

b/h	Khdeir	Present
2	6.756	6.702
5	12.333	12.411
10	20.315	20.239
15	26.183	25.907

Table 2: The effect of plate length-to-thickness ratio (*a/h*) on

the frequency parameter of a clamped-clamped three layered rectangle plate with $\theta = 0^\circ/90^\circ/0^\circ$ and a/b=1.5 with material arrangement AGE-KGE-AGE.

a/h	method	
	RBF	Spline
10	0.8904	0.879
20	0.8155	0.8032
30	0.7841	0.7711
40	0.7699	0.7565
50	0.7626	0.749
60	0.7584	0.7447

Table 3: The effect of plate length-to-thickness ratio (*a/h*) on the frequency parameter of a clamped-clamped three layered rectangle plate with $\theta = 0^\circ/90^\circ/0^\circ$ and a/b=1.5 with material arrangement KGE-AGE-KGE

a/h	method	
	RBF	Spline
10	1.0038	0.9894
20	0.9327	0.9171
30	0.9052	0.8889
40	0.8931	0.8765
50	0.887	0.8702
60	0.8835	0.8666

Table 4: The effect of plate length-to-thickness ratio (*a/h*) on the frequency parameter of a clamped-clamped three layered rectangle plate with $\theta = 0^\circ/90^\circ/0^\circ$ and a/b=1.5 with material arrangement KGE-EGE-KGE.

a/h	method	
	RBF	Spline
10	0.9978	0.9843
20	0.9272	0.9128
30	0.8997	0.8848
40	0.8877	0.8725
50	0.8816	0.8663
60	0.8782	0.8627

Table 2, 3, and 4 shows the frequency parameters, λ, with respect to the *a/h* for three layered plates with different materials, arranged in the form of AGE-KGE-AGE, KGE-AGE-KGE and KGE-EGE-KGE. From the Tables which says that, the values of the fundamental frequency parameter decreases with increasing in the length-to-thickness ratio. The value obtained using RBF method are higher than its corresponding values obtained by Spline. The maximum differences percentage for AGE-KGE-AGE layered plate with its corresponding values is 1.8064% and the minimum differences is 1.2803%. The highest differences percentage of second and third type of arrangements are 1.9128% and 1.7649% and smallest difference are 1.4345% and 1.3529% with mean difference of 0.0161 and 0.0148 respectively.

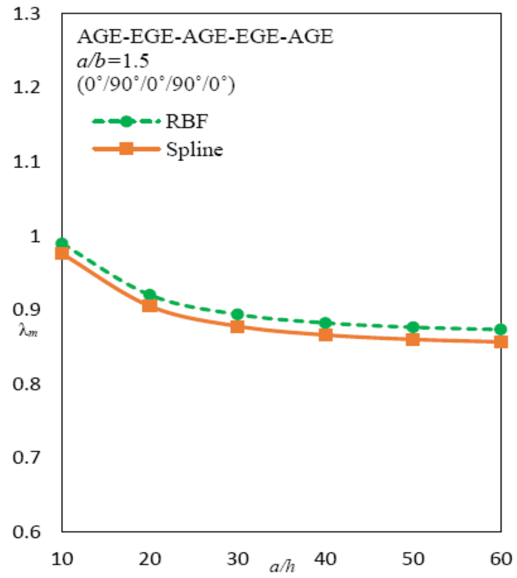

Figure 1: Comparison of two methods for effect of plate length-to-thickness ratio (*a/h*) on the frequency parameter for five layered rectangle plate using material AGE-EGE under C-C boundary conditions.

Among these three type of arrangement of layers, AGE-KGE-AGE arrangements gives lowest fundamental frequency.

Fig 1 and 2 shows the results for frequency parameter with respect to the length-to-thickness ratio for five layered plate ($0^\circ/90^\circ/0^\circ/90^\circ/0^\circ$) with different material arrangements analyzed using Spline and RBF methods. In Fig 1 and 2 the material are arranged as AGE-EGE-AGE-EGE-AGE and KGE-EGE-KGE-EGE-KGE respectively. The frequency parameter decreases as the side-to-thickness ratio increases, the gradient of the graph is steeper from *a/h* 10 to 30, and then the gradient of the line almost 0. For Fig. 1 the results obtained using RBF and Spline varies from 0.0138 to 0.0173 with mean difference 0.0160. Similarly for Fig. 2 the difference varies from 0.0137 to 0.0174 and the mean difference is 0.0163. The layers arrangement using material AGE-KGE combination shows the lowest frequency readings, where the highest frequency values is between the range of $0.95 < \lambda < 0.85$ and the lowest value is between 0.8 and 0.75

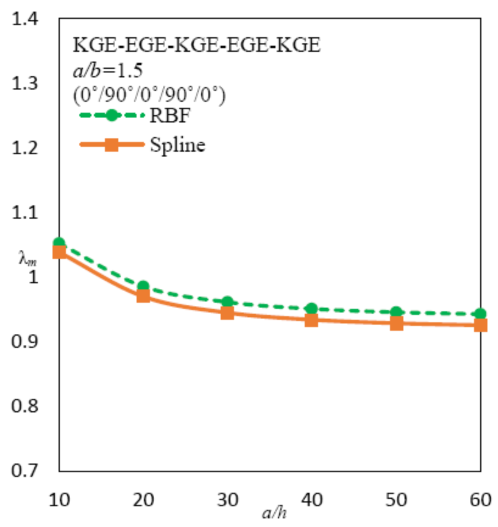

Figure 2: Comparison of two methods for effect of plate length-to-thickness ratio (*a/h*) on the frequency parameter for five layered rectangle plate using material KGE-EGE under C-C boundary conditions.

Table 5 and 6 shows the frequency parameter obtained for seven layered plates with material arrangements as AGE-EGE-AGE-EGE-AGE-EGE-AGE and KGE-EGE-KGE-EGE-KGE-EGE-KGE respectively and *a/b* fixed as 1.5. From the Table 5 and 6, the value of frequency parameter decreases as the value of *a/h* increases. The value calculated using RBF is slightly high compare to the values obtained using Spline method. The maximum difference for AGE-EGE, and KGE-EGE are 0.017 and 0.0182 respectively, and the minimum difference are 0.0136 and 0.0143 with mean difference 0.0160 and 0.0171 respectively. The combination of AGE-EGE materials produce lower frequency compared to KGE-EGE materials.

Table 5: The effect of plate length-to-thickness ratio (*a/h*) on the frequency parameter of a clamped-clamped seven layered rectangle plate with θ = 0˚/90˚/0˚/90˚/0˚/90˚/0˚ and *a/b*=1.5 with material arrangement AGE-EGE-AGE-EGE-AGE-EGE-AGE.

	Method	
a/b	RBF	Spline
10	1.011	0.9974
20	0.9387	0.9233
30	0.913	0.8967
40	0.9021	0.8854
50	0.8966	0.8798
60	0.8936	0.8766

Table 6: The effect of plate length-to-thickness ratio (*a/h*) on the frequency parameter of a clamped-clamped seven layered rectangle plate with θ = 0˚/90˚/0˚/90˚/0˚/90˚/0˚ and *a/b*=1.5 with material arrangement KGE-EGE-KGE-EGE-KGE-EGE-KGE.

	Method	
a/b	RBF	Spline
10	1.0823	1.068
20	1.0136	0.9971
30	0.99	0.9726
40	0.9802	0.9623
50	0.9753	0.9572
60	0.9725	0.9543

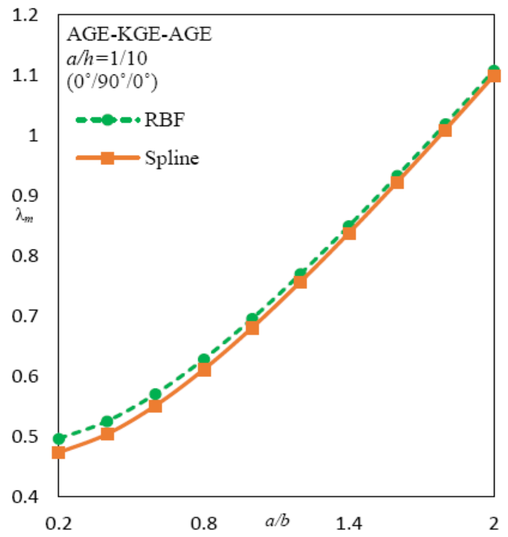

Figure 3: Comparison of two methods for effect of plate aspect ratio (*a/b*) on the frequency parameter of a clamped-clamped three layered rectangle plate with material AGE-KGE.

Fig. 3-5 shows the comparison of frequency parameter with respect to the aspect ratio for three layered plates with different material combinations. In general the frequency parameter increases as *a/b* increase. The frequency increase slowly in the range of $0.2 \leq \lambda \leq 0.8$ and increase rapidly for $\lambda > 0.8$. All three graphs shows that the value obtained using RBF is slightly higher compare to the Spline values. The differences in percentage of RBF values and its corresponding values obtained using Spline varies from 4.6812% to 0.7841%, 9.9955% to 0.7432% and 6.1527% to 0.7740% for Fig 4, 5, and 6 respectively. Among the three type of material combinations, AGE-KGE amalgamation gives lower frequency values.

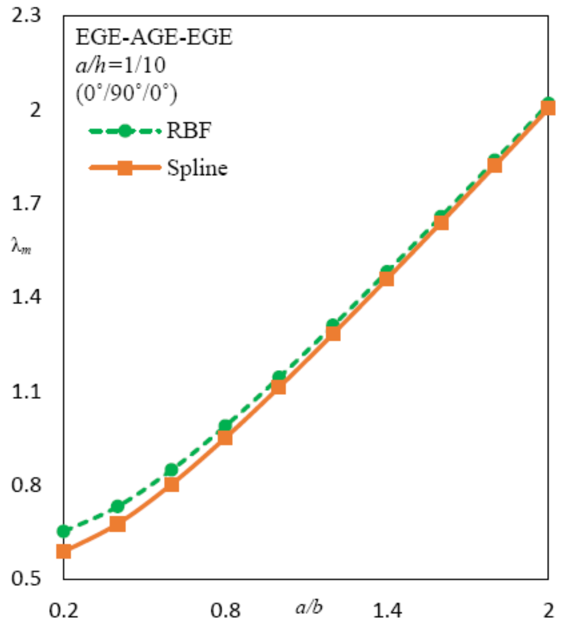

Figure 4: Comparison of two methods for effect of plate aspect ratio (*a/b*) on the frequency parameter of a clamped-clamped three layered rectangle plate with material EGE-AGE.

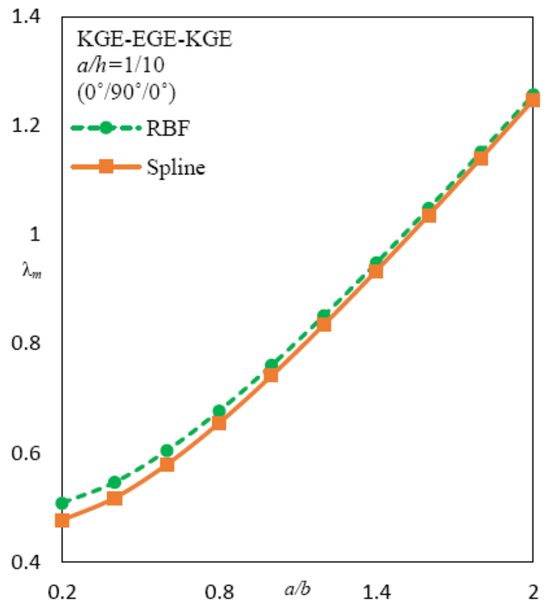

Figure 5: Comparison of two methods for effect of plate aspect ratio (*a/b*) on the frequency parameter of a clamped-clamped three layered rectangle plate with material KGE-EGE.

Table 7 - 9 depicts the effect of aspect ratio on the frequency for five, seven and nine layered plates respectively using different combination of materials by fixing *a/h*=1/10 under clamped-clamped boundary condition. The results are analysed using the material combination as KGE-AGE, EGE-KGE and AGE-KGE are shown in Table 7, 8 and 9 respectively. The frequency parameter value increases as increase in *a/b* value and the value obtained using RBF is slightly higher compared to its corresponding Spline results. The difference between its corresponding values directly proportional to *a/h* and as the number of layers increases the frequencies obtained increases. The jump of frequency values is high when the number of layers increases from seven to nine.

Table 7: The effect of plate aspect ratio (*a/b*) on the frequency parameter of a clamped-clamped five layered rectangle plate with $\theta = 0^\circ/90^\circ/0^\circ/90^\circ/0^\circ$ and materials KGE-AGE-KGE-AGE-KGE.

	Method	
a/b	RBF	Spline
0.2	0.5075	0.4705
0.4	0.5519	0.5191
0.6	0.619	0.5909
0.8	0.7021	0.6788
1	0.7963	0.7771
1.2	0.8982	0.8824
1.4	1.0053	0.9924
1.6	1.1161	1.1057
1.8	1.2298	1.2214
2.0	1.3454	1.3388

Table 8: The effect of plate aspect ratio (*a/b*) on the frequency parameter of a clamped-clamped seven layered rectangle plate with $\theta = 0^\circ/90^\circ/0^\circ/90^\circ/0^\circ/90^\circ/0^\circ$ and materials EGE-AGE-EGE-AGE-EGE-AGE-EGE

	Method	
a/b	RBF	Spline
0.2	0.6338	0.5572
0.4	0.7258	0.6625
0.6	0.8575	0.8071
0.8	1.0135	0.9735
1	1.1843	1.1523
1.2	1.3643	1.338
1.4	1.5502	1.5292
1.6	1.7403	1.7229
1.8	1.9331	1.9188
2.0	2.128	2.1161

The frequency parameter is analysed for four layered plates with respect to the side-to-thickness ratio shown in Table 10. The material arrangement for four layered plates is KGE-EGE-

KGE-EGE with ply angle 0˚/90˚/0˚/ 90˚. For six and eight layered plates material AGE-KGE and AGE-EGE were used with the arrangement AGE-KGE-AGE-KGE-AGE-KGE (0˚/90˚/0˚/90˚/0˚/90˚) and AGE-EGE-AGE-EGE-AGE-EGE-AGE-EGE (0˚/90˚/0˚/ 90˚/0˚/90˚/ 0˚/90˚) shown in Table 11 and Table 12 respectively. The vibration decreases as the side-to-thickness increases from 10 to 60 and the value obtain using RBF is higher compared to its corresponding Spline value for all three types of layers. The maximum difference between frequencies of four layers plates is 0.3173 and the minimum difference is 0.1994, with mean value of 0.2912. For six and eight layered plate the maximum difference between its two methods corresponding values is 0.2097 and 0.2265, and the minimum difference is 0.1096 and 0.1403 respectively

Table 9: The effect of plate aspect ratio (*a/b*) on the frequency parameter of a clamped-clamped nine layered rectangle plate with θ = 0˚/90˚/0˚/90˚/0˚/90˚/0˚/90˚/0˚ and materials AGE-KGE-AGE-KGE-AGE-KGE-AGE-KGE-AGE.

	Method	
a/b	RBF	Spline
0.2	0.4931	0.4677
0.4	0.5259	0.5029
0.6	0.5763	0.5564
0.8	0.6403	0.6235
1	0.7142	0.7002
1.2	0.7953	0.7838
1.4	0.8815	0.8722
1.6	0.9715	0.964
1.8	1.0644	1.0585
2	1.1594	1.1549

Table 10: The effect of plate length-to-thickness ratio (*a/h*) on the frequency parameter of a clamped-clamped four layered rectangle plate with θ = 0˚/90˚/0˚/90˚.

	Method	
a/b	RBF	Spline
10	1.0142	0.8148
20	1.0101	0.7243
30	1.0058	0.696
40	1.0015	0.6846
50	0.9973	0.679
60	0.9932	0.6759

Table 11: The effect of plate length-to-thickness ratio (*a/h*) on the frequency parameter of a clamped-clamped six layered rectangle plate with θ = 0˚/90˚/0˚/90˚/0˚/90˚.

	Method	
a/b	RBF	Spline
10	1.0328	0.9232
20	1.0312	0.8555
30	1.0297	0.8324
40	1.0281	0.8227
50	1.0265	0.8179
60	1.025	0.8152

Table 11: The effect of plate length-to-thickness ratio (*a/h*) on the frequency parameter of a clamped-clamped eight layered rectangle plate with θ = 0˚/90˚/0˚/90˚/0˚/90˚/0˚/90˚.

	Method	
a/b	RBF	Spline
10	1.1779	1.0376
20	1.1738	0.9689
30	1.1697	0.9472
40	1.1658	0.9384
50	1.1619	0.9341
60	1.1582	0.9317

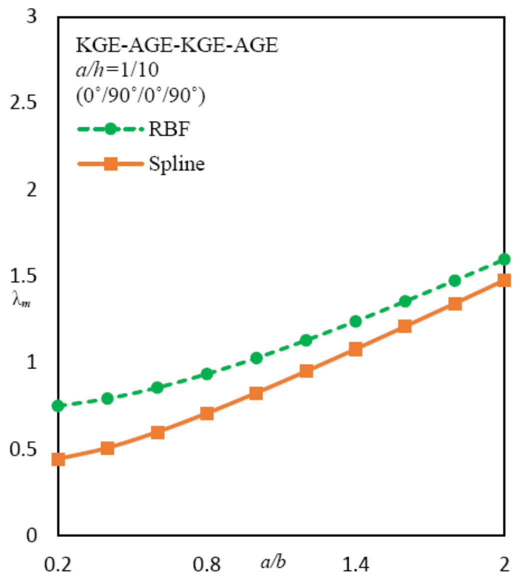

Figure 6: Comparison of two methods for effect of plate aspect ratio (*a/b*) on the frequency parameter of a four layered clamped-clamped rectangle plate.

Fig. 6-8 shows the vibration values calculated respect to aspect ratio varies from 0.2 to 2.0, for four, six and eight layered plates with different material arrangement. Figure 7 represent the frequency parameter calculated for four layered plates using material KGE and AGE with arrangement KGE-AGE-KGE-AGE, and Figure 8 for six layered plates with materials AGE and EGE with its arrangement AGE-EGE-AGE-EGE-AGE-EGE. The Figure 9 shows vibration calculated for eight layers plate using material KGE and EGE. Based on all three figures, it can be concluded that, the frequency of plate increases as the aspect ratio increases and the value calculated using RBF technique is higher compared to its corresponding value obtained by Spline technique. The difference between these two corresponding decreases as the *a/b* value increase. When the odd number of layers used to analyse the frequency parameter, the variation of RBF method values and its corresponding Spline method values greater compared to the even number of layered plates.

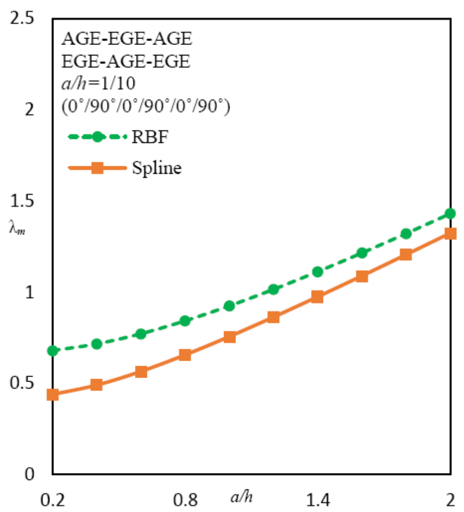

Figure 7: Comparison of two methods for effect of plate aspect ratio (*a/b*) on the frequency parameter of a six layered clamped-clamped rectangle plate.

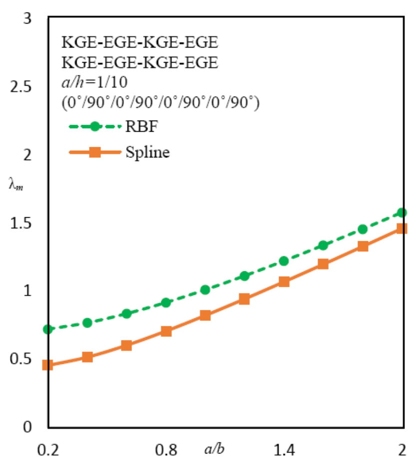

Figure 8: Comparison of two methods for effect of plate aspect ratio (*a/b*) on the frequency parameter of eight layered clamped-clamped rectangle plate.

CONCLUSIONS

The frequency parameter for laminated cross-ply plates including first order shear deformation theory under clamped-clamped boundary condition for odd and even number of layers are analysed. The displacement and rotational functions are approximated by two different approximations namely Radial Basis function and Spline function. The results are analysed with respect to the side-to-thickness ratio, aspect ratio and number of layers using two methods. The result's pattern for odd and even number of layered plates are discussed and the results obtained by both the methods are significant. The difference between Radial Basis Function method's result and Spline method's results for odd number of layered plates is smaller compared to difference for even number of layered plate.

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Appendix A

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & kA_{45} & kA_{55} \\ 0 & 0 & 0 & 0 & 0 & 0 & kA_{44} & kA_{45} \end{pmatrix}$$

$$\times \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \psi_{x,x} \\ \psi_{y,y} \\ \psi_{x,y} + \psi_{y,x} \\ \psi_x + w_{,x} \\ \psi_y + w_{,y} \end{pmatrix}$$

Where the extensional rigidities are represented by A_{ij}, the bending-stretching coupling rigidities, B_{ij}, bending rigidities, and Dij. A_{ij}, B_{ij}, and D_{ij}, given as follows;

$$A_{ij} = \sum_k \bar{Q}_{ij} (z_k - z_{k-1})$$

$$B_{ij} = 1/2 \sum_k \bar{Q}_{ij} (z_k^2 - z_{k-1}^2) \qquad \text{for } i, j = 1, 2, 6...$$

$$D_{ij} = 1/3 \sum_k \bar{Q}_{ij} (z_k^3 - z_{k-1}^3)$$

$$A_{ij} = k \sum_k \bar{Q}_{ij} (z_k - z_{k-1}) \qquad \text{for } i, j = 4, 5...$$

K is shear correction factor and ε_x, ε_y, γ_{xy}, γ_{xz} and γ_{yz} are given as

$$\varepsilon_x = \frac{\partial u_0}{\partial y}, \varepsilon_y = \frac{\partial v_0}{\partial y}, \gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x},$$

$$\gamma_{xz} = \psi_x + \frac{\partial w}{\partial x}, \text{ and } \gamma_{yz} = \psi_y + \frac{\partial w}{\partial y}.$$

Appendix B

$$L_{11} = \frac{d^2}{dX^2} - \beta^2 S_{10} + \lambda^2,\ L_{12} = -\beta\left(S_2 + S_{10}\right)\frac{d}{dX},$$

$$L_{13} = L_{31} = S_4 \frac{d^2}{dX^2} - \beta^2 S_{11}, L_{14} = -L_{41} = -\beta\left(S_5 + S_{11}\right)\frac{d}{dX},$$

$$L_{21} = \beta\left(S_2 + S_{10}\right)\frac{d}{dX},\ L_{22} = S_{10}\frac{d^2}{dX^2} - \beta^2 S_3 + \lambda^2,$$

$$L_{23} = -L_{32} = \beta\left(S_5 + S_{11}\right)\frac{d}{dX}, L_{24} = -L_{42} = S_{11}\frac{d^2}{dX^2} - \beta^2 S_6,$$

$$L_{33} = S_7 \frac{d^2}{dX^2} - \beta^2 S_{12} - KS_{14} + \frac{I_1}{I_0 a^2}\lambda^2,$$

$$L_{34} = -L_{43} = -\beta\left(S_8 + S_{12}\right)\frac{d}{dX}, L_{35} = -L_{53} = -KS_{14}\frac{d}{dX},$$

$$L_{44} = S_{12}\frac{d^2}{dX^2} - \beta^2 S_9 - KS_{13} + \frac{I_1}{I_0 a^2}\lambda^2,\ L_{45} = L_{54} = -K\beta S_{13},$$

$$L_{55} = KS_{14}\frac{d^2}{dX^2} - K\beta^2 S_{13} + \lambda^2,$$

$L_{15} = L_{25} = L_{51} = L_{52} = 0$ and $\beta = n\phi$

where the S_i *(i=2, 3, 4,….14)* quantities are defined as

$$S_2 = \frac{A_{12}}{A_{11}},\ S_3 = \frac{A_{22}}{A_{11}},\ S_4 = \frac{B_{11}}{aA_{11}},\ S_6 = \frac{B_{22}}{aA_{11}},\ S_7 = \frac{D_{11}}{a^2 A_{11}},$$

$$S_8 = \frac{D_{11}}{a^2 A_{11}},\ S_4 = \frac{D_{12}}{a^2 A_{11}}, S_9 = \frac{D_{22}}{a^2 A_{11}}, S_{10} = \frac{A_{66}}{A_{11}}, S_{11} = \frac{B_{66}}{aA_{11}},$$

$$S_{12} = \frac{D_{66}}{a^2 A_{11}},\ S_{13} = \frac{A_{44}}{A_{11}},\ S_{14} = \frac{A_{55}}{A_{11}}$$