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A New Class of Sets in Nano topological Spaces with an Application in Medical Diagnosis

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Abstract

The paper intends to introduce Ngrα-closed set and discuss some of its properties. Also, we identified the risk factors for the cause of heart attack through the concept of Nano topology.

Keywords: Nano topological space, Nr_α-open, Ngrα-closed, $T_{Ngr\alpha}$-space and Ngrα $T_{1/2}$-space, core, lower approximation and upper approximation.

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Introduction :

L.Thivagar et al [1] introduced the concept of nano topological spaces which was defined in terms of approximations and boundary region of a subset of a universe U using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. In this paper we introduce and study Nr_α-open,Ngrα-closed sets.Also$T_{Ngr\alpha}$-space and Ngrα $T_{1/2}$-space are introduced.The researchers conducted a case study to identify the key factors for the cause of heart failure. Heart failure is a condition in which the heart cannot pump enough blood to meet the bodys' needs. A 'heart attack' occurs when the flow of oxygen-rich blood to a section of heart muscle is blocked suddenly and when the heart does not receive the required oxygen. If blood flow is not restored quickly, the section of heart muscle begins to die. It is possible to eliminate many of these risk factors in order to reduce the chance of having a first or subsequent heart attack. High blood pressure is caused due to obesity, smoking, high cholesterol or diabetes that increases the risk of heart failure. If siblings, parents or grandparents would have had early heart attacks we may be at increased risk. Human beings are forced to undergo stress in ways that can increase the risk of heart attack.The paper highlights the risk factors for cause of heart attack using nano topology.

Preliminaries:

Definition 2.1[5]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indescernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The Pair (U, R) is said to be the approximation space. Let X⊆U

(i).The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.

That is $L_R(X) = \bigcup_{X\in U}\{R(x): R(x) \subseteq X\}$, where R(X) denotes the equivalence class determined by X∈U.

(ii).The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.

That is $U_R(X) = \bigcup_{X\in U}\{R(x): R(x) \cap X \neq \varphi\}$

(iii).The boundary of the region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.That is $B_R(X) = U_R(X) – L_R(X)$.

Properties 2.2[5]

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.

(ii) $L_R(\varphi) = U_R(\varphi) = \varphi$ and $L_R(U) = U_R(U) = U$

(iii) $U_R(X\cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X\cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X\cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

(ix) $U_RU_R(X) = L_RU_R(X) = U_R(X)$

(x) $L_RL_R(X) = U_RL_R(X) = L_R(X)$

Definition 2.3[1]

Let U be the Universe and R be an equivalence relation on U. Then for X⊆U, $\tau_R(X)$ = {U, φ, $L_R(X)$, $U_R(X)$, $B_R(X)$} is called the nano topology on U. By property 2.2,$\tau_R(X)$ satisfies the following axioms:

(i) U and φ $\in \tau_R(X)$.

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii)The intersection of the elements of any finite subcollection of $\tau_R(X)$is in $\tau_R(X)$

We call (U,$\tau_R(X)$) is a nano topological space.The elements of $\tau_R(X)$ are called a nano open sets and the complement of a nano open sets is called nano closed sets.

Throughout this paper (U,$\tau_R(X)$) is a nano topological space with respect to X where X⊆U, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R.

Definition2.4 [1]

Let(U,$\tau_R(X)$) be a nano topological space, the set β= {U, $L_R(X)$, $B_R(X)$} is called a bases for the nano topology $\tau_R(X)$ on U with respect to X.

Definition 2.5[1]

If (U,$\tau_R(X)$) is a nano topological space with respect to X, where X⊆U and if A⊆U, then (i)The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by Nint(A).

(ii)The nano closure of the set A is defined as the intersection of all nano closed subsets containing A and is denoted by Ncl(A).

Definition 2.6

Let (U,$\tau_R(X)$) be a nano topological space and A⊆U. Then A is said to be

(i) nano regular open [1] if A = Nint(Ncl(A))

(ii) nano α-open [1]if A ⊆ Nint(Ncl(Nint(A)))

(iii) nano preopen [1] if A ⊆ Nint(Ncl(A))

(iii) nano regular generalized closed set [6] if Nrcl(A) ⊆ V whenever A ⊆ V and V is Nano regular open in (U,$\tau_R(X)$).

Definition 2.7 [1]

Let (U,$\tau_R(X)$) be a nano topological space and A⊆U. Then A is said to be nano α-closed(respectively, nano regular closed) if its complement is nano α-open (nano regular open).

Ngrα-closed Sets :

Definition 3.1

A subset A of a nano topological space (U, $\tau_R(X)$) is called Nr_α-open if there is a nano regular open set V such that V⊂A⊂Nαcl(V).

Definition 3.2

A subset A of a nano topological space (U, $\tau_R(X)$) is called Ngrα-closed if Ncl(Nint(A))⊂V whenever A⊂V and V is Nr_α-open in (U, $\tau_R(X)$).

Theorem 3.3

(i) Every nano-closed set is Ngrα-closed.

(ii) Every nano regular-closed set is Ngrα-closed.

(iii) Every nano-α closed set is Ngrα-closed.

Remark 3.4

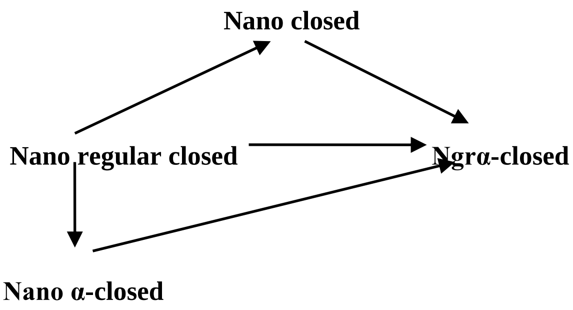
Converse of the above theorem need not be true as seen from the following example.

Example 3.5

Let U = {a, b, c, d}, X = {a, b}, $\tau_R(X)$ = {U, φ,{a}, {b, d},{a, b, d}}.The nano closed sets are { U, φ, {c}, {a, c},{b, c, d}}.The nano regular closed sets are { U, φ,{a, c},{b, c, d}}.The nano α-closed sets are { U, φ, {c},{a, c},{b, c, d}}.The Ngrα-closed sets are{ U, φ, {b},{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d}, {a, b, c}, {a, c, d},{b ,c ,d}}.

Remark 3.6

From the above discussion, we have the following implications (Figure 1).



Theorem 3.7

The union of two Ngrα-closed sets is also a Ngrα-closed set.

Proof

Let A and B be two Ngrα-closed sets in (U,$\tau_R(X)$). Let Ncl(Nint(A))⊆V, Ncl(Nint(B))⊆V, A ⊆V, B ⊆V and V be Nr_α-open. Then we have, A∪B ⊆V. Now Ncl(Nint(A∪B)) = Ncl(Nint(A))∪ Ncl(Nint(B))⊆V. Therefore A∪B is a Ngrα-closed set in U.

Remark 3.8

The intersection of two Ngrα-closed sets is not Ngrα closed set as shown from the following example.

Example 3.9

Let U = {a ,b, c, d} , X ={a, b} , $\tau_R(X)$ = {U, φ, {a},{b, d},{a, b, d}}. Ngrα-closed are {U, φ,{b},{c}, {d}, {a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c}, {a, b, d},{a, c, d},{b, c, d}}. Here {a, c}∩{a, b, d} = {a} is not Ngrα-closed .

Theorem 3.10

If A is a Ngrα-close subset of (U,$\tau_R(X)$), then Ncl(Nint(A)) – A contains no non-empty Nr_α-open.

Proof

Suppose that A is Ngrα-closed in U. We prove the result by contradiction. Let F be a Nr_α-open such that F⊂Ncl(Nint(A)) –A and F = φ . Then A⊆U–F. Since A is Ngrα-closed set, F is Nr_α-open and U–F is nano is also Nr_α-open, we have Ncl(Nint(A))⊂U-F. So F⊂U–Ncl(Nint(A)). Therefore F⊂(Ncl(Nint(A))∩(U–Ncl(Nint(A))) = φ, this is a contradiction. Hence Ncl(Nint(A)) – A does not contain any non-empty Nr_α-open set.

Theorem 3.11

If a subset A of (U, $\tau_R(X)$) is Ngrα-closed, then Ncl(Nint(A)) – A does not contain any nano regular open set in U.

Proof

Proof follows from the fact that every nano regular open set is Nr_α-open.

Theorem 3.12

Let A be a Ngrα-closed subset of (U,$\tau_R(X)$). Then A is nano regular closed if and only if Ncl(Nint(A)) – A is a Nr_α-open.

Proof

Suppose A is a nano regular-closed. Then Ncl(Nint(A)) = A and Ncl(Nint(A)) – A =φ, which is Nr_α-open. Conversely, let Ncl(Nint(A))–A be a Nr_α-open in U. Since A is Ngrα-closed, by theorem 3.10 , Ncl(Nint(A)) – A does not contain any non-empty Nr_α-openset. Then Ncl(Nint(A)) – A =φ, hence A is nano regular closed.

Theorem 3.13

If A is Ngrα closed subset of (U, $\tau_R(X)$) such that A⊂B⊂Ncl(Nint(A)), then B is Ngrα-closed set in (U, $\tau_R(X)$).

Proof

Let A be a Ngrα-closed subset of (U, $\tau_R(X)$) such that A⊂B⊂Ncl(Nint(A)). Let V be Nr_α-open set such that B⊂V, then A⊂V. Since A is Ngrα-closed, Ncl(Nint(A)⊂V. Now we have Ncl(Nint(B))⊂ Ncl(Nint(Ncl(Nint(A)))) = Ncl(Nint(A))⊂V. This implies that B is Ngrα-closed.

Theorem 3.14

Let A be aNgrα-closed. Then A is Nr_α-closed if and only if Ncl(Nint(A) – A is a Nr_α-open.

Proof

Suppose A is a nano regular-closed. Then cl(Nint(A)) = A and Ncl(Nint(A)) – A = φ, which is a Nr_α-open in U. Conversely, let Ncl(Nint(A)) – A be a Nr_α-open set in U. Since A is a Ngrα-closed, by theorem 3.10, Ncl(int(A)) – A does not contain any non-empty Nr_α-open in U. Then Ncl(Nint(A)) – A = φ, hence A is nano regular closed.

Theorem 3.15

If A is Ngrα-closed and nano regular open, then A is nano rg-closed, nano pre closed and nanoclopen.

Proof

Since every nano regular open set is nano open and Nr_α-open. Let A⊂A be Ngrα-closed and nanoregular open , Ncl(Nint(A))⊂ A and Nrcl(A) ⊆Ncl(A) = Ncl(Nint(A))⊂A. Therefore A is nanorg-closed.

Definition 3.16

A space (U, $\tau_R(X)$) is called $T_{Ngr\alpha}$ –space if every Ngrα closed set is nano closed.

Definition 3.17

A space (U, $\tau_R(X)$) is called Ngrα- $T_{1/2}$space if every Ngrα-closed is nano α-closed.

Theorem 3.18

For a Nano topological space (U, $\tau_R(X)$) the following are equivalent

(i) U is Ngrα-$T_{1/2}$ space.

(ii) Every singleton of U is either Nr_α-closed (or) nano α-open.

Proof

(i)⇒(ii)

Let x∈U and assume that {x} is not Nr_α-closed. Then clearly U–{x} is also not Nr_α-open and U–{x} is Ngrα-closed. By hypothesis, U–{x} is nano α-closed and thus {x} is nano α-open.

(ii)⟹(i)

Let A⊂U be Ngrα-closed. Let x∈ Ncl(Nint(A)). To show x∈A.

Case(i) : If a set {x} is nano Nr_α-closed and x∉A, then A⊂U–{x}. Since A is Ngrα-closed and U–{x} is Nr_α-open, Ncl(Nint(A))⊂U–{x} and hence x∉Ncl(Nint(A)), this is a contradiction. Therefore, x∈A.

Case (ii)

The set {x} is nano α-open. Since x∈Ncl(Nint(A)), then {x}∩A≠φ implies x∈A. This shows that A is Nano α-closed.

Theorem 3.19

(i)NαO(U, $\tau_R(X)$)⊂NGRαO(U, $\tau_R(X)$)

(ii)A space is Ngrα-$T_{1/2}$ space if and only if NαO(U, $\tau_R(X)$) = NGRαO(U, $\tau_R(X)$).

Proof

(i)Obvious.

(ii)Let U be a nano $T_{1/2}$ space and A∈NGRαO(U, $\tau_R(X)$),Thus, NGRαO(U, $\tau_R(X)$)= NαO(U, $\tau_R(X)$). Conversely,let NαO(U, $\tau_R(X)$) = NGRαO(U, $\tau_R(X)$). A is Ngrα-closed implies U – A is Ngrα-open. Hence A is nano α-closed

NGRA-CONTINUOUS AND NGRA-IRRESOLUTE

Definition 4.1

Let (U,τ_R(X)) and (V, τ_R(Y)) be two nano topological spaces. Then a mapping f: (U,τ_R(X))→ (V, τ_R(Y)) is Ngrα-continuous if f^{-1}(F) is Ngrα-closed in U if for every nano-closed set F of V.

Definition 4.2

Let (U,τ_R(X)) and (V, τ_R(Y)) be two nano topological spaces. Then a mapping f: (U,τ_R(X))→ (V, τ_R(Y)) is Ngrα-irresolute if f^{-1}(F) is Ngrα-closed in U if for every Ngrα-closed set F of V.

Theorem 4.3

Let f: (U,τ_R(X))→ (V, τ_R(Y)) and g: (V, τ_R(Y))→ (W, τ_R(Z)) be any two functions.Then

(i) g∘f is Ngrα-continuous if g is nano-continuous and f is Ngrα-continuous.

(ii) g∘f is Ngrα-irresolute if g is Ngrα-irresolute and f is Ngrα-irrresolute.

(iii)g∘f is Ngrα-continuous if g is Ngrα-continuous and f is Ngrα-irresolute.

(iv)g∘f is nano α-continuous if g is nano α- irresolute and f is Ngrα-continuous and V is a Ngrα-$T_{1/2}$ space.

Proof

(i)Let F be nano closed in W. Then g^{-1}(F) is nano closed in V, Since g is nano continuous. By hypothesis, $f^{-1}(g^{-1}(F))$ is Ngrα-closed in U. Hence g∘f is Ngrα-continuous.

(ii),(iii) and (iv). These proofs are obtained similarly as the proof of (i).

APPLICATIONS OF NANO TOPOLOGY

Example 5.1

Here we apply the nano topology to find the key factors of 'Heart Attack' using topological reduction of attributes in incomplete information system.

The following table gives information about patients those who are having High Blood Pressure, Alcohol and smoking, Stress and strain, Diabetics and Family history.

Table 1

Patients	**High Blood Pressure**	**Alcohol and smoking**	**Diabetics**	**Stress and strain**	**Family history**	**Result**
B_1	√	√	√	Nil	Nil	Yes
B_2	√	Nil	Nil	√	√	Yes
B_3	Nil	√	√	Nil	√	Yes
B_4	√	√	√	Nil	Nil	No
B_5	Nil	√	√	Nil	Nil	No
B_6	Nil	√	√	Nil	√	No
B_7	√	Nil	√	Nil	√	Yes
B_8	√	Nil	√	√	Nil	No

Here U = {B_1, B_2 ,B_3 ,B_4 ,B_5, B_6 ,B_7 , B_8 }, the set of patients and A = { High Blood Pressure, Alcohol and smoking, Diabetics, Stress and strain and Family history, Heart attack} which is divided into two classes, C= {BP,AS,DB,SS,FH} and D = { Heart Attack}. The family of equivalence classes, U/C corresponding to C is given by U/R(C) = {{B_1 ,B_4}, {B_2} , {B_3 ,B_6} , {B_5} , {B_7} {B_8} }.

Case 1 :(Patients with Heart Attack) Let X = {B_1, B_2,B_3,B_7}, the set of patient with Heart Attack . Then L_C(X) = {B_2, B_7}, U_C(X) = {B_1, B_2, B_3, B_4, B_6, B_7} and B_R(X) = {B_1, B_3, B_4, B_6}. Therefore, τ_C(X) = {U, φ, {B_2, B_7}, {B_1, B_2, B_3, B_4, B_6, B_7}, {B_1, B_3, B_4, B_6}}. The basis of τ_C(X) is given by β_C(X) = {U, {B_2, B_7},{B_1, B_3, B_4, B_6}}.

Step 1: When the attribute 'High Blood Pressure' is removed from C, U/R(C-BP) = {{B_1, B_4, B_5}, {B_2}, {B_3,B_6}, {B_7], {B_8}} and hence the lower and upper approximations of X corresponding to C-{BP} are given by $L_{C\text{-}\{BP\}}${X} = {{B_2, B_7}, $U_{C\text{-}\{BP\}}$(X) = {B_1, B_2, B_3, B_4,B_5, B_6, B_7} and the corresponding boundary region is $B_{C\text{-}\{BP\}}$(X) = {B_1, B_3, B_4, B_6}.Therefore the corresponding nano topology and its basis are given by$\tau_{C-\{BP\}}$(X) = { U,φ, {B_2, B_7} , {B_1, B_2, B_3, B_4,B_5, B_6, B_7}, {B_1, B_3, B_4, B_6}} and $\beta_{C\text{-}\{BP\}}$(X) = {U, {B_2, B_7} , {B_1, B_3, B_4, B_6}}≠β_C(X).

Step 2: When the attribute 'Alcohol and smoking' is removed from C, U/R(C-AS) = {{B_1, B_4}, {B_2}, {B_3,B_6}, {B_5}, {B_7], {B_8}} and hence $\tau_{C-\{AS\}}$(X) = { U,φ, {B_2, B_7} , {B_1, B_2, B_3, B_4, B_6, B_7}, {B_1, B_3, B_4, B_6}} and $\beta_{C\text{-}\{AS\}}$(X) = {U, {B_2, B_7} , {B_1, B_3, B_4, B_6}} = β_C(X).

Step 3: When the attribute 'Diabetics' is removed from C, U/R(C-DB) = {{B_1, B_4}, {B_2}, {B_3,B_6}, {B_5}, {B_7], {B_8}} and hence $\tau_{C-\{DB\}}$(X) = { U, φ, {B_2, B_7} , {B_1, B_2, B_3, B_4, B_6, B_7}, {B_1, B_3, B_4, B_6}} and $\beta_{C\text{-}\{AS\}}$(X) = {U,{B_2, B_7} , {B_1, B_3, B_4, B_6}} = β_C(X).

Step 4: When the attribute 'Stress and Strain' is removed from C, U/R(C-SS) = {{B_1, B_4}, {B_2}, {B_3,B_6}, {B_5}, {B_7], {B_8}} and hence $\tau_{C-\{SS\}}$(X) = { U,φ, {B_2, B_7} , {B_1, B_2, B_3, B_4, B_6, B_7}, {B_1, B_3, B_4, B_6}} and $\beta_{C\text{-}\{SS\}}$(X) = {U, {B_2, B_7} , {B_1, B_3, B_4, B_6}} = β_C(X).

Step 5: When the attribute 'Remove family History' is removed from C, U/R(C-FH) = {{B_1, B_4}, {B_2}, {B_3,B_5, B_6}, {B_7}, {B_8}} and hence $\tau_{C-\{FH\}}$(X) = { $\mathcal{U}$,φ, {B_2, B_7} , {B_1, B_2, B_3, B_4, B_5, B_6, B_7}, {B_1, B_3, B_4, B_5, B_6}} and $\beta_{C\text{-}\{FH\}}$(X) = {U, {B_2, B_7} , {B_1, B_3, B_4, B_5, B_6}} ≠ β_C(X). Therefore, CORE = {High Blood Pressure, Family history}.

Case 2 (Patients not with Heart Attack)

Let X = {B_4, B_5, B_6 ,B_8}, the set of patients without Heart Attack. Therefore, τ_C(X) = {U, φ, {B_5, B_8}, {B_1, B_3, B_4, B_5, B_6, B_8}, {B_1, B_3, B_4, B_6}}. The basis of τ_C(X) is given by β_C(X) = {U, {B_5, B_8},{B_1, B_3, B_4, B_6}}.

Step 1: When the attribute 'High Blood Pressure' is removed from C, U/R(C-BP) = {{B_1, B_4, B_5}, {B_2}, {B_3,B_6}, {B_7], {B_8}}. Therefore,$\tau_{C-\{BP\}}$(X) = { U, φ, {B_8} , {B_1, B_3, B_4,B_5, B_6, B_8}, {B_1, B_3, B_4, B_5 ,B_6}} and $\beta_{C\text{-}\{BP\}}$(X) = {U, {B_8} , {B_1, B_3, B_4, B_5,B_6}} ≠ β_C(X).

Step 2: When the attribute 'Alcohol and smoking' is removed from C, U/R(C-AS) = {{B_1, B_4}, {B_2}, {B_3,B_6}, {B_5}, {B_7], {B_8}} and hence $\tau_{C-\{AS\}}$(X) = { $\mathcal{U}$,φ, {B_5, B_8} , {B_1, B_3, B_4, B_5, B_6, B_8}, {B_1, B_3, B_4, B_6}} and $\beta_{C\text{-}\{AS\}}$(X) = {U, {B_5, B_8} , {B_1, B_3, B_4, B_6}} = β_C(X).

Step 3: When the attribute 'Diabetics' is removed from C, U/R(C-DB) = {{B_1, B_4}, {B_2}, {B_3,B_6}, {B_5}, {B_7], {B_8}} and hence $\tau_{C-\{DB\}}$(X) = { U, φ, {B_5, B_8} , {B_1, B_3, B_4, B_5, B_6, B_8}, {B_1, B_3, B_4, B_6}} and $\beta_{C\text{-}\{DB\}}$(X) = {U, {B_5, B_8} , {B_1, B_3, B_4, B_6}} = β_C(X).

Step 4: When the attribute 'Stress and Strain' is removed from C, U/R(C-SS) = {{B_1, B_4}, {B_2}, {B_3,B_6}, {B_5}, {B_7], {B_8}} and hence $\tau_{C-\{SS\}}$(X) = { U, φ, {B_5, B_8} , {B_1, B_3, B_4, B_5, B_6, B_8}, {B_1, B_3, B_4, B_6}} and $\beta_{C\text{-}\{SS\}}$(X) = {U, {B_5, B_8} , {B_1, B_3, B_4, B_6}} = β_C(X).

Step 5: When the attribute 'Remove family History' is removed from C, U/R(C-FH) = {{B_1, B_4}, {B_2}, {B_3,B_5, B_6}, {B_7}, {B_8}} and hence $\tau_{C-\{FH\}}$(X) = { $\mathcal{U}$,φ, {B_8} , {B_1, B_3, B_4, B_5, B_6, B_7,B_8}, {B_1, B_3, B_4, B_5, B_6,B_7}} and $\beta_{C\text{-}\{FH\}}$(X) = {U, {B_8} , {B_1, B_3, B_4, B_5, B_6, B_7}} ≠ β_C(X). Therefore, CORE = {High Blood Pressure, Family history}.

Observation: From the CORE, we observed that 'High Blood Pressure' and 'Family History' are the key factors for heart attack. Proper medical care and change in the behavioral pattern can prevent the risk.

REFERENCES

[1] LellisThivagar M and Carmel Richard, On Nano Forms of weakly open sets, International. j. Math. and stat.Inv.,Vol.1,No.1 (2013),31-37.

[2] Pawlak Z., Rough sets – Theoretical aspects of reasoning aspects of reasoning about data, Kluwer Academic Publishers, London, (1991).

[3] Pawlak Z and Skowron A., Rudiments of Rough sets, Information Sciences,177,No.1,(2007),3-27.

[4] Pawlak Z.,Rough set theory and its applications, Journal of telecommunications and Information Technology, 3,(2002),7-10.

[5] Reilly I.L and Vamanamurthy, On α-sets in topological spaces, Tamkang J. Math., 16(1985), 7-11.

[6] Sulochana Devi P and Bhuvaneswari K, On Nano Regular Generalized and Nano Generalized Regular Closed sets in Nano Topological Spaces, IJETT(13), 8(2014),386-390.