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Data Fusion with Model Error Estimators and Stability Analysis

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Abstract

An approach for sensor data fusion that utilizes model error estimators is presented. These estimators are based on the method of invariant embedding and H-infinity concept; and both continuous time and discrete time nonlinear systems are evaluated. Also, an observer for nonlinear continuous time dynamic system is proposed that utilizes the gain and the associated matrix Riccati type differential equation from the combined invariant embedding (IE) and H-infinity (HI) theories. Then, Lyapunov energy (LE) functional is used for deriving the condition for the local asymptotic stability for the observer's error dynamics. The performances of the model error estimators-based data fusion scheme and of the continuous observer are evaluated by simulation carried out in MATLAB. The presented results validate the theoretical asymptotic behaviour of the nonlinear observer, as well as the convergence properties of the data fusion scheme, since both are based on IE/HI theory. This type of validation study is a novel feature of this contribution.

Keywords: Sensor data fusion, invariant embedding, H-infinity concept, model error, nonlinear observer, Lyapunov functional based asymptotic result.

INTRODUCTION

In many real life situations one needs accurate identification of nonlinear terms and parameters in the model of a dynamic system, for which principle of model error is employed. The methods based on this approach gives not only the estimates the states of the system (from its measurements), but also the model discrepancy (or so called model error, ME) as a time history. Once, the discrepancy time history is available, one can fit another model to it and estimate its parameters using a regression-LS method. Then, combination of the previously used, in all probability a deficient model, in the state estimation procedure and the new additional model obtained by LS method, would yield the accurate model of the underlying nonlinear dynamic system, which has in fact generated the data [1, 2]. The approaches evaluated, in this paper are based on the method of invariant embedding (IE) and H-infinity (HI) based modification of the IE method. These model error estimators are used in state vector fusion process *as if*, it combines the processed information from two individual 'fictitious' sensor channels. Actually two deficient models are used in the first place, and the ME estimators are used which give corrected states from the measurements. The concept of data fusion is employed to enhance the predictive capability of the estimated state trajectory. The presented approach is novel in utilization of IE and IE-HI based model error estimators in data fusion process. Also, in many cases engineers use an observer to reconstruct the states of a dynamic system using the system's input/output signals. Such observers/estimators are called deterministic, for they handle only measurement noise and not the process/state noise [3].In this paper, a nonlinear observer for continuous time system using the gain from the theory of IE/HI is presented; and it also uses the matrix Riccati type differential (RTD) equation which is needed in the computation of the gain. The asymptotic stability result for the observer is derived using the Lyapunov energy functional.

The performances of the model error estimators and the nonlinear observer are demonstrated by implementing these algorithms in MATAB. The simulation results support the asymptotic result of the IE & HI based observer error dynamics, as well as validate the data fusion scheme.

INVARIANT EMBEDDING METHOD

First, the idea is to determine the model error from the available noisy measurements for a given nonlinear dynamic system; it is presumed that the empirical data are obtained from an actual nonlinear system [4]. Then, one fits only a primarily known model that might be deficient, because the true model is not available. The mathematical description of the nonlinear system is given as

$$\dot{x} = f(x(t), u(t), t) + d(t) \tag{1}$$

In (1), d(.) is the 'un-modelled' deterministic disturbance, which is assumed to be piecewise continuous; and this is not the process noise term as found in a model used in Kalman filter (KF). In control theory, term d(.) represents a control force or input, u(t), which is determined using an optimization technique by minimizing the cost function

$$J = \sum_{k=1}^{N} [z(k) - h(\hat{x}(k), k)]^T R^{-1} [z(k) - h(\hat{x}(k), k)] + \int_{t_0}^{t_f} d^T(t) Q d(t) dt \quad (2)$$

In (2), it is assumed that $E\{v(k)\} = 0$; $E\{v(k)v^T(k)\} = R(k)$ which is assumed to be known; 'h' is the measurement model; and z(.) are the measurements. The weighting matrix Q plays a role of a tuning device for the estimator and represents a 'deterministic covariance' or more logically called Gramian matrix of the input disturbance, d. In (2), R(.) is 'covariance' matrix of the measurement noise, or Gramian matrix of the measurement disturbance. The merit of the present approach is that it obtains state estimates in the presence of un-modelled effects, d, as well as accurate estimates of these effects, d. No statistical assumptions are required. The cost function criteria used for estimation are based on the principle of least squares, LS. In the process, the postulated model itself is improved, since this estimate of the un-modelled effects can be further modelled and the new sub-model can be obtained: accurate model (of the original system) = deficient model (postulated model) + model fitted to the discrepancy (i.e. un-modelled effects). Interestingly, the problem of determination of the model error is via minimization of the cost functional (2), and gives rise to the so called two point boundary value problem (TPBVP) [5]. The method of invariant embedding obtains the recursive solution to the TPBV problem. In many cases, it is useful to analyze a general solution of which the original problem posed is one particular case, and the method of IE belongs to this class: the particular solution that is being sought is embedded in the general class and after the general solution is obtained, the particular solution can be obtained by using the special conditions (that were kept invariant) in final analysis. The resultant equations from the TPBVP are, detail avoided,

$$\dot{x} = \phi(x(t), \lambda(t), t) \quad (3)$$

$$\dot{\lambda} = \psi(x(t), \lambda(t), t) \quad (4)$$

It is seen that the dependencies for ϕ *and* ψ on $x(t)$ *and* $\lambda(t)$ arise from the form of (1), and (2): a general TPBVP with associated boundary conditions as $\lambda(0) = a$ and $\lambda(t_f) = b$. Though the terminal condition $\lambda(t_f) = b$ (on the Lagrange multiplier or the co-state) and time are fixed, these are treated as free variables making the problem more general, which in any way includes the specific problem. The final resulting algorithms are given next.

CONTINUOUS TIME ALGORITHM

Let a nonlinear dynamic system be represented by

$$\dot{x} = f(x(t), t) + d(t) \quad (5)$$

$$z(t) = Hx(t) + v(t) \quad (6)$$

Then, the model error estimator based on IE method is given as

$$\dot{\hat{x}} = f(x(t), t) + 2S(t)H^T R^{-1}(z(t) - Hx(t)) \quad (7)$$

$$S(t) = S(t)f_{\hat{x}}^T + f_{\hat{x}}S(t) - 2S(t)H^T R^{-1} HS(t) + \frac{1}{2}Q^{-1} \quad (8)$$

Explicitly the model error is given as

$$\hat{d}(t) = 2S(t)H^T R^{-1}(z(t) - Hx(t) \quad (9)$$

Equation (8) is called matrix Riccati type differential (RTD) equation, and can be solved by using the transition matrix method [3].

DISCRETE TIME ALGORITHM

Let the nonlinear system be given as

$$X(k+1) = g(X(k), k) \quad (10)$$

$$Z(k) = h(X(k), k) \quad (11)$$

In (10), 'g' is the vector valued function and in (11), Z is the vector of observables. Equations (10) and (11) are rewritten to express the model error

$$X(k+1) = f(x(k), k) + d(k) \quad (12)$$

$$Z(k) = h(x(k), k) + v(k) \quad (13)$$

In (12), '*f*' is the nominal model that is a deficient model. The discrete time IE recursive algorithm is given as

$$\hat{x}(k+1) = f_{\hat{x}}(\hat{x}(k), k) + 2S(k+1)H^T(k+1)R^{-1}[z(k+1) - h(\hat{x}(k+1), k+1)] \quad (14)$$

$$S(k+1) = [I + 2P(k+1)H^T(k+1)R^{-1}H(k+1)]^{-1}P(k+1) \quad (15)$$

$$P(k+1) = f_{\hat{x}}(\hat{x}(k), k)S(k)f_{\hat{x}}^T(\hat{x}(k), k) + \frac{1}{2}Q^{-1} \quad (16)$$

$$\hat{d}(k) = 2S(k)H^T(k)R^{-1}[(z(k) - h(\hat{x}(k), k)] \quad (17)$$

THE MODEL ERROR ESTIMATORS BASED ON H-INFINITY FRAMEWORK

The H∞(HI) norm provides a measure of a worst-case system's transfer function gain, given as [6, 7]:

$$\|G\|_\infty = sup_w |G(j\omega)| \quad (18)$$

In (18), $|G(j\omega)|$ is a factor by which the amplitude of a sinusoidal input with frequency ω is magnified by the system, and is seen that the HI norm is simply a measure of the largest factor or gain. The HI norms of error system matrix [7] are given as:

for continuous system:

$$\|\tilde{z}\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{z}(j\omega)^H \tilde{z}(j\omega)d\omega = \int_{-\infty}^{\infty} \tilde{z}(t)^H \tilde{z}(t)dt \tag{19}$$

for discrete system:

$$\|\tilde{z}\|_2^2 = \sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{z}(n)^H \tilde{z}(n) = \frac{1}{2\pi j}\oint\int_{-\infty}^{\infty} \tilde{z}(e^{j\omega T})^H \tilde{z}(e^{j\omega T})\frac{de^{j\omega T}}{e^{j\omega T}} \tag{20}$$

The error is given as $\tilde{z} = z - \hat{z}$, and $d = \begin{bmatrix} W \\ n \end{bmatrix}$ as input disturbance. For an HI norm criterion, the transfer function, from the input disturbances to the estimate error, is $G_{\hat{z}d}$, has a system gain with an upper bound

$$\|G_{\hat{z}d}\|_\infty^2 < \gamma^2 \tag{21}$$

The performance bound criterion is also written as

$$\underset{w}{sup}\frac{\|z-\hat{z}\|_2^2}{\|{}^W_n\|_2^2} < \gamma^2 \tag{22}$$

The model error estimators are given next, detail avoided.

CONTINUOUS TIME ALGORITHM

Let the mathematical description of dynamic system be given as

$$\dot{x} = f(x(t), u(t), t) + d(t) \tag{23}$$

$$z(t) = Hx(t) + v(t) \tag{24}$$

The model error estimator equations are given as

$$\dot{\hat{x}}(t) = 2H^T R^{-1} S(t)\big(z(t) - Hx(t)\big) + f(x(t), t) \tag{25}$$

$$\dot{S}(t) = S(t)f_{\hat{x}}^T - 2\, S(t)H^T R^{-1} HS(t) - \frac{1}{2}\gamma^{-2}Q^{-1} + f_{\hat{x}}S(t) \tag{26}$$

$$\hat{d}(t) = 2S(t)H^T R^{-1}\big(z(t) - Hx(t)\big) \tag{27}$$

Equation (26) is matrix RTD equation.

DISCRETE TIME ALGORITHM

Consider a nonlinear system given by

$$x(k+1) = f(x(k), k) + d(k) \tag{28}$$

$$z(k) = h(x(k), k) + v(k) \tag{29}$$

The model error estimator equations are given as

$$\hat{x}(k+1) = f_{\hat{x}}(\hat{x}(k), k) + 2S(k+1)H^T(k+1)R^{-1}[z(k+1) - h(\hat{x}(k+1), k+1] \tag{30}$$

$$S(k+1) = [I + 2P(k+1)H^T(k+1)R^{-1}H(k+1)]^{-1}P(k+1) \tag{31}$$

$$P(k+1) = f_{\hat{x}}(\hat{x}(k), k)S(k)f_{\hat{x}}^T(\hat{x}(k), k) - \frac{1}{2}\gamma^{-2}Q^{-1} \tag{32}$$

$$\hat{d}(k) = 2S(k)H^T(k)R^{-1}[z(k) - h(\hat{x}(k), k)] \tag{33}$$

These model error estimators based on the IE and IE/HI concepts are later evaluated for discrete and continuous time nonlinear system in a state vector data fusion scheme. Next, a nonlinear observer is presented that is based on the combined concepts of IE and HI.

IE/HI BASED NONLINEAR OBSERVER

The nonlinear system considered is

$$\dot{x} = f(x(t), t) + d(t)$$

$$y(t) = Hx(t) + v(t) \tag{34}$$

In (34), d(.) is a deterministic signal or an unknown deterministic disturbance, and y(.) is a measurement vector; v(.) can be regarded as a random noise with zero mean and covariance matrix as R; or as an unknown disturbance with known intensity R. A nonlinear observer for the system of (34) is given as

$$\dot{\hat{x}}(t) = f(\hat{x}(t), t) + \hat{d}(t) + L(t)(y(t) - \hat{y}(t))$$

$$\hat{y}(t) = H\hat{x}(t) \tag{35}$$

The observer gain is chosen from (25) as

$$L(t) = 2S(t)H^T R^{-1} \tag{36}$$

For (36), S(t) is obtained as the solution of the matrix RTD equation (26),

$$\dot{S}(t) = S(t)A^T(t) + A(t)S(t) - 2SH^T R^{-1} HS(t) - \frac{1}{2}Q^{-1}\gamma^{-2}$$

$$;with\ S(0) = S_0; S(T) = a\ steady\ state\ value \tag{37}$$

The Jacobian matrix A(t) needed in (37) is obtained as

$$A(t) = \frac{\delta f(.)}{\delta \hat{x}(t)} \quad (38)$$

The equations (36), and (37) are from the combined theories of invariant embedding and H infinity for continuous time nonlinear dynamic system/estimator. In the domain of deterministic IE/HI estimators and observers, the matrices Q and R, are called weighting matrices or the Gramians. In that case, the corresponding variables are termed as the 'generalized random' variables. The residuals and the state errors from the observer are given as

$$r(t) = y(t) - \hat{y}(t); e(t) = x(t) - \hat{x}(t) \quad (39)$$

The state error dynamics are obtained by subtracting (35) from (34):

$$e(t) = A(t)e(t) + d(t) - \hat{d}(t) + \psi(t) - L(t)He(t) - L(t)v(t) \quad (40)$$

In (40), the nonlinear function is

$$\psi(.) = -A(t)e(t) + f(x,t) - f(\hat{x},t);$$

$$\psi(.) = \psi(x(t), \hat{x}(t)) \quad (41)$$

THE ASYMPTOTIC RESULT FOR STATE ERROR DYNAMICS

Certain important conditions need to be considered for establishing the local asymptotic behaviour of the error dynamics of (40):

a) The solution of the RTD equation (37) should be considered as bounded [8, 9]

$$s_l I \leq S(t) \leq s_u I \quad (42)$$

Here, $s_l, s_u > 0$are positive constants; and are the lower and upper bounds on S(t) respectively; the matrix S(t) is supposed to be positive definite and symmetric; I is the identity matrix. This condition is natural, because we need a bounded solution to the observer error dynamics.

b) The nonlinearity in (40) of the error dynamics is bounded [8]

$$\|\psi(.)\| \leq \rho_1 \|x(t) - \hat{x}(t)\|^2 \quad (43)$$

This is again a natural requirement for the same reason as for the condition a).

c) The deterministic error is bounded

$$\|d(t) - \hat{d}(t)\| = \|e_d(.)\| \leq \rho_2 \|x(t) - \hat{x}(t)\|^2 \quad (44)$$

In basic observer theory, such a deterministic error, or model deficiency is generally not considered. Now, if all the three conditions a) to c) are satisfied, and the time derivative of the LE functional is negative definite, then, the observer error dynamics in (40) are locally asymptotically stable; of course some more feasible conditions are needed that will evolved when the stability result is derived. Now, consider the normalized LE functional

$$V(t) = e^T Y(t) e(t) \quad (45)$$

In (45), $Y(t)$ is the normalizing matrix regarded as an information Gramian, for, $Y(t) = S^{-1}(t)$. Since, for the deterministic observers and IE/HI estimators, one deals with unknown, but deterministic disturbance processes; S(t) is considered as the state-error Gramian matrix; and hence, the variables x(.), y(.), and e(t) are called the generalized 'random' variables. The LE functional (45) is positive definite as seen from the following condition; the inequality a); (42):

$$\frac{1}{s_u}\|e(t)\|^2 \leq e^T(t)Y(t)e(t) \leq \frac{1}{s_l}\|e(t)\|^2 \quad (46)$$

Then, in order to establish the asymptotic convergence result for the considered nonlinear IE/HI based observer, the time derivative of the LE functional, (45); {after the use of the constraints of the error dynamics (40), of gain (36) and of the Gramian S(t) (37), in the time derivative formula}, should be negative definite for all time t. The error dynamics are written (with the knowledge that the quantities A, L, H, x, v are dependent on time t) as:

$$e(t) = A(t)e(t) + e_d(t) + \psi(t) - LHe(t) - Lv \quad (47)$$

Then, the time derivative of (13) is obtained as

$$V(t) = e^T(t)Y(t)e(t) + e^T(t)Y(t)[A(t)e(t) + e_d(t) + \psi(t) - LHe(t) - Lv] + [e^T(t)A^T(t) + e_d^T(t) + \psi^T(.) - e^T(t)H^T L^T - v^T L^T]Y(t)e(t) \quad (48)$$

Next, substitute

i) for $Y(t) = -Y(t)S(t)Y(t)$, gain (36), and the RTD equation (37) in (48);

ii)$\|H^T H\| \leq h^2; \|R^{-1}\| \leq r$, (here, h, and r, are positive constants);

iii)$\frac{1}{q_u} I \leq Q^{-1}\gamma^{-2} \leq \frac{1}{q_l} I; \|e(t)\|^2 \leq \varepsilon^2$;with $k = \varepsilon$;

iv) $\|v(t)\| \leq \rho_3 \|e(t)\|^2 \quad (49)$

and bounds (43), and (44) in (48) {with the constants in all these

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bounds assumed known and/or pre-specified quantities}, and then after cancelling out several common terms and simplifying (without any approximations) one gets, in terms the norms of the individual terms:

$$V(t) = \frac{1}{2s_u^2 q_u}\|e(t)\|^2 - \frac{2h^2}{r}\|e(t)\|^2 + \frac{2\rho_2}{s_u}\varepsilon\|e(t)\|^2 + \frac{2\rho_1}{s_u}\varepsilon\|e(t)\|^2 - \frac{4h\rho_3}{r}\varepsilon\|e(t)\|^2 \tag{50}$$

Just rearranging (50), one obtains

$$V(t) = -\left[\frac{4h\rho_3}{r}\varepsilon - \frac{1}{2s_u^2 q_u}\right]\|e(t)\|^2 - \left[\frac{2h^2}{r} - \frac{2(\rho_1\rho_2)}{s_u}\varepsilon\right]\|e(t)\|^2 \tag{51}$$

In (51), $\rho_1\rho_2 = \rho_1 + \rho_2$.

$$V(t) = -\left[\frac{4h\rho_3}{r}\varepsilon - \frac{1}{2s_u^2 q_u}\right]\|e(t)\|^2 - \left[\frac{2h^2}{r} - \frac{2(\rho_1\rho_2)}{s_u}\varepsilon\right]\|e(t)\|^2 \tag{52}$$

$$V(t) = -\left\{k = \varepsilon > \frac{r}{8s_u^2 q_u h\rho_3}; k = \varepsilon < \frac{h^2 s_u}{r\rho_{12}}\right\}\|e(t)\|^2 \tag{53}$$

EVALUATION OF THE IE/HI BASED OBSERVER

The IE/HI based observer for nonlinear dynamic system is evaluated first, using numerical simulation carried out in MATLAB. The data are generated for a period of 20 sec. with a sampling interval of 0.01 sec. The nonlinear dynamic system [10] used for the purpose is given by:

$$x_1(t) = -0.1x_1(t) + 0.3x_1(t)x_2(t)$$

$$x_2(t) = 10x_2(t) - 0.7x_1(t)x_2(t) - 0.1x_2^2(t) \tag{54}$$

$$y(t) = x_2(t)$$

The model (54) is the Lotka-Volterra (L-V) dynamic system that is often used as an interactive model for nonlinear systems. It has become very popular for study of the mathematical system theory in relation to population biology systems [10]. For the system of (54), the IE/HI based nonlinear observer is given as

$$\hat{x}_1(t) = -0.1\hat{x}_1(t) + 0.3\hat{x}_1(t)\hat{x}_2(t) + L_1\big(y(t) - \hat{x}_2(t)\big)$$

$$\hat{x}_2(t) = 10\hat{x}_2(t) - 0.7\hat{x}_1(t)\hat{x}_2(t) - 0.1\hat{x}_2^2(t)L_2\big(y(t) - \hat{x}_2(t)\big) \tag{55}$$

The observer gain is taken as in (36), and the initial conditions of the states used for the simulation and the observer are, respectively: $x(0) = [15 \quad 2]'$; and $\hat{x}(0) = [10 \quad 5]'$. The RTD equation (37) is solved by using the transition matrix method [3]. First, the Gramian matrix S(t) is transformed, and then the RTDE is partitioned and re-formulated as:

$$a = S(t)b \tag{56}$$

$$b = -A^T b + \left(\frac{2}{r}H^T H\right)a \tag{57}$$

$$a = -\frac{1}{2}(\gamma^{-2}Q^{-1})b + Aa \tag{58}$$

The equations (57) and (58) are like any state space differential equations, and are solved by the transition matrix method. Then, using (56), the solution S(t) of the RTDE is obtained. For the generation of the simulated data, the dynamic equations (54), and (55) are solved by Euler integration method with a small step size. The required Jacobian matrix, A(t) to be used in RTDE is represented in the discrete form as:

$$A(k) = \begin{bmatrix} -0.1 + 0.3x_2(k-1) & 0.3x_1(k-1) \\ -0.7x_2(k-1) & 10 - 0.7x_1(k-1) - 0.2x_2(k-1) \end{bmatrix} \tag{59}$$

In (59), A(.), being time-dependent is evaluated at every time step. The performance of the IE/HI-based nonlinear observer is illustrated in Figure 1for two states x_1 and x_2. Figure 2 shows the convergence of the eigenvalues of the matrix S(t) (left plot), and the true and predicted measurements (right plot). The time history match, Figure 1, is found to be very good. The performance of the IE/HI-based observer has been found to be very satisfactory, as also supported by the % fit errors obtained by using the formula: PFE=100*norm (errors)/norm (true signal), Table 1. Figure 2clearlyshows that the IE/HI-based nonlinear observer is asymptotically stable, because the eigenvalues of the Gramian matrix asymptotically converge. The theoretical result established is corroborated, at least qualitatively, by the actual behaviour of the IE/HI-based observer's error dynamics. It should be straightforward to extend the present results to multisensory data fusion needs [1,11] that can be realised using the IE/HI-based estimators.

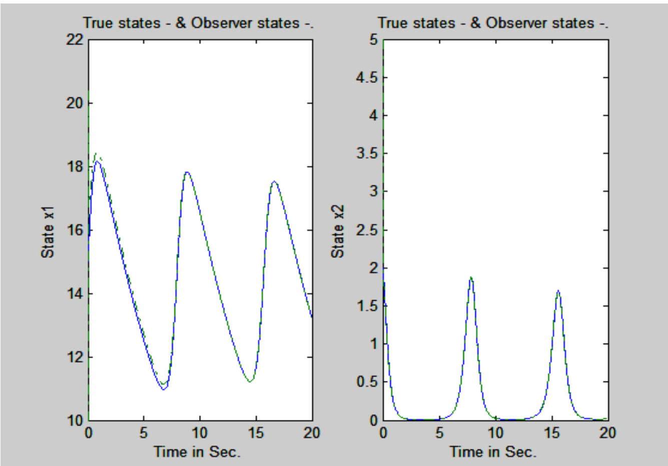

Figure 1: Time history (the true (-) and observer

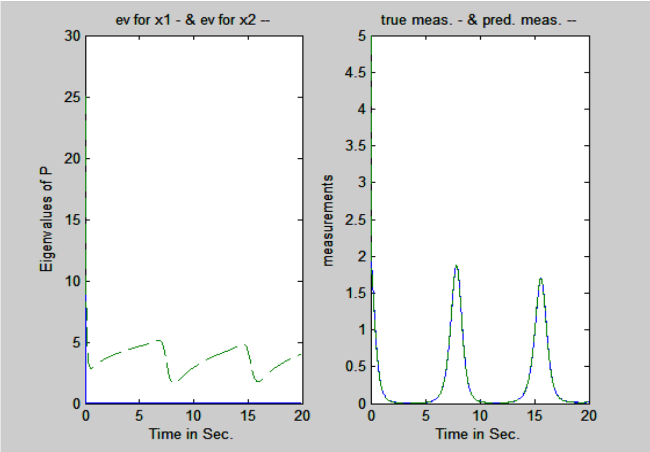

Figure 2: Eigenvalues of S(t) for states: x_1 (-), & x_2 (--); states (-.)) for the IE/HI based nonlinear observer. {left graph}; True (-), predicted meas. (--) {right graph}.

Table 1: PFEs for the IE/HI-based observer

PFEs: for r=0.001; Q=[0.9 0;0 0.85]	$\gamma^2 = 0.04$	$\gamma^2 = 0.09$	$\gamma^2 = 0.25$	$\gamma^2 = 0.49$
State x_1	2.2517	0.1891	1.1738	1.1065
State x_2	1.6233	1.3955	1.4803	1.5329
Measurements	1.6016	1.3816	1.4764	1.5328

EVALUATION OF THE DATA FUSION SCHEME

The integration of data and knowledge from several sensor and/or sources is known as data fusion. It provides a way to integrate measured information/states originating from different sources to produce more precise and comprehensive/final information/knowledge or model about an entity or event of interest. It is an emerging and evolving technology with applications to: a) automated target recognition, b) battlefield surveillance, c) guidance and control of autonomous vehicles, d) as monitoring of complex machinery, e) medical diagnosis, and f) robotics. A data fusion scheme that would use IE and IE-HI based model error estimators is shown in Figure 3. The data from a system are received from/via some information channels S_1 and S_2; then, a model is postulated (that is primarily not known accurately) and hence,is a deficient model (DFM) of the system: DFM1, DFM2 (and DFM3). The IE and IE-HI based ME estimators are used two estimate the states of the system as well as the model errors. The estimated states are fused using the state vector fusion formulae:

$$Fused\ State: x_f = x_1 + S_1 * inv(S_1 + S_2) * (x_2 - x_1) \quad (60)$$

$$Covariance\ (Gramian) = S_f = S_1 - S_1 * inv(S_1 + S_2) * (S_1)' \quad (61)$$

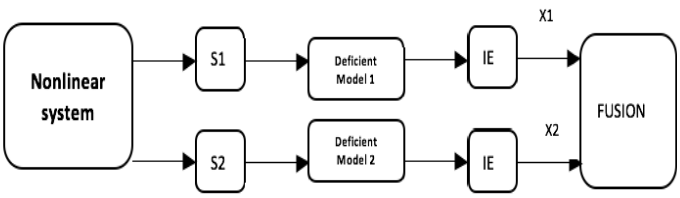

Figure 3: Data fusion scheme for the joint IE/HI model error estimators.S1, S2→datachannels; X1,X2: two estimates of the same state variable x1}

The following nonlinear discrete system is simulated:

$$X_1(k+1) = 0.8X_1(k) + 0.223X_2(k) + 2.5\cos(0.3k) + 0.8\sin(0.2k) - 0.05X_1^3(k) \quad (62)$$

$$X_2(k+1) = 0.5X_2(k) + 0.1\cos(0.4k) \quad (63)$$

The model error is determined by eliminating the terms from (62) in turn: a) X_1^3, b) X_1, X_1^3, and c) X_1, X_2, X_1^3. To this model error, a regression model is fitted: $d(k) = a_1X_1(k) + a_2X_1^2(k) + a_3X_1^3(k) + a_4X_2(k)$ to estimate the parameters of the discrete nonlinear system. 100 samples of data are generated using (62) and (63). It is to be noted that although the term containing X_1^2 is not present in the true model of the system, it is included to check the performance of the algorithm. Table2 shows parameter estimates based on the IE and IE-HI model error estimators. The estimates compare very well with the true values of the parameters. In all the cases, the parameter that is removed from the model is estimated; and in all the cases, the term a_2 is estimated with a value, which is practically zero since, it is anyway not present in the model. Tables 3-5 show the percentage measurement residuals' fit errors, state errors and HI norm which are reasonably low. Figures 4-7 depict performance graphs for the discrete time case for the data fusion, from which it can be seen that the data fusion scheme that uses (at a time any) two DFMs, gives very satisfactory state vector fusion results. The norm of the 'covariance' Gramian{S(t), studied in the context of data fusion scheme using IE/HI estimators} is shown in a typical graph in Figure 6, which again corroborates the asymptotic result derived for the observer error dynamics, since the observer is based on the IE/HI based gain and state error Gramian S(t); of course the discrete time results would be equivalent and accurate for small sampling interval.

Table 2: Nonlinear parameter estimation results – Discrete-time system

Case	*Parameter*	a_1 X_1	a_2 X_1^2	a_3 X_1^3	a_4 X_2	Terms removed
	True values	**0.8**	**0**	**-0.05**	**0.223**	-
DFM1	IE method	*(0.8)*	-1.03e-5	-0.0497	*(0.223)*	X_1^3, X_1^2
	IE+HI method $\gamma = 5$	*(0.8)*	-0.0000	-0.05	*(0.223)*	
DFM2	IE method	0.7961	-8.3e-6	-0.0498	*(0.223)*	X_1, X_1^3
	IE+HI method $\gamma = 7.5$	0.8	-0.00	-0.0499	*(0.223)*	
DFM3	IE method	0.8000	-3.07e-7	-0.05	0.2224	X_1, X_2, X_1^3
	IE+HI method $\gamma = 0.2$	0.8	-0.00	-0.05	0.223	

(.) true values used in the model, DFM: Deficient model

Table 3: Percentage Residual Fit Errors

Method	Model	Measurement
IE	DFM1	0.5391
	DFM2	0.8495
IE/HI	DFM1	0.4291
$\gamma = 0.5$	DFM2	0.7395

Table 4: Percentage State Errors

Method	Def Model	State X_1	State X_2
IE	DFM1	0.0257	0.9230
	DFM2	0.0271	1.0851
	Fusion	0.0180	0.6679
IE/HI	DFM1	0.0234	0.8391
$\gamma = 0.5$	DFM2	0.0247	0.9864
	Fusion	0.0164	0.6072

Table 5: H-Infinity Norm

Case	Method	norm
DFM1 & DFM2	IE	0.0770
	IE+HI $\gamma = 0.5$	0.0765
DFM2 & DFM3	IE	0.0783
	IE+HI $\gamma = 7.5$	0.0721
DFM3 & DFM1	IE	0.0743
	IE+HI $\gamma = 0.2$	0.0698

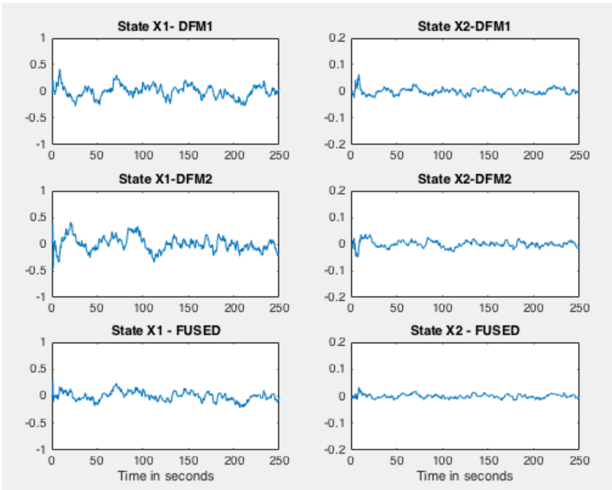

Figure 4: Estimated/fused states IE method

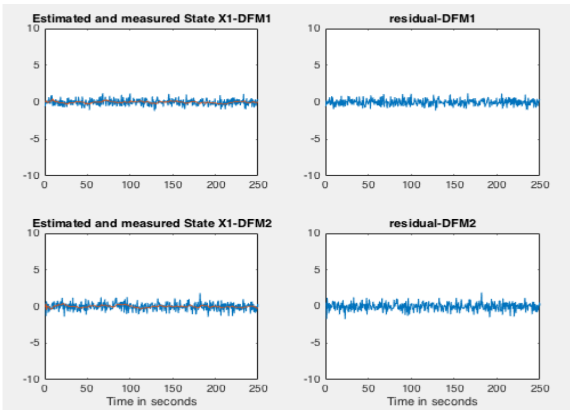

Figure 5: Estimated/measured states & Residuals

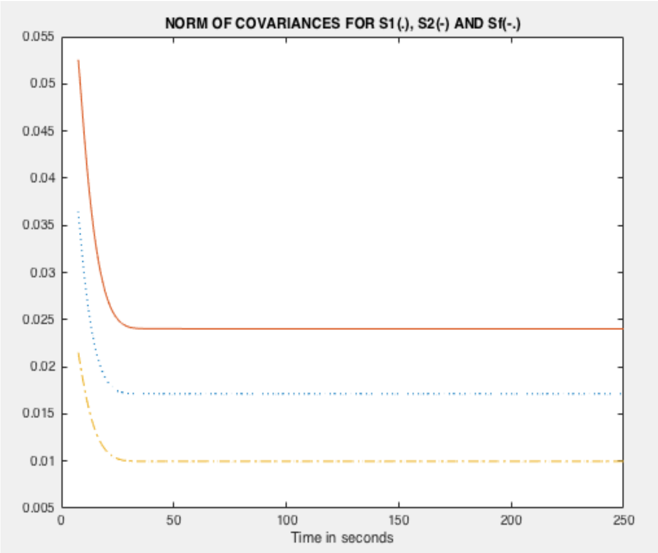

Figure 6: Norm of state error Gramians/IE method

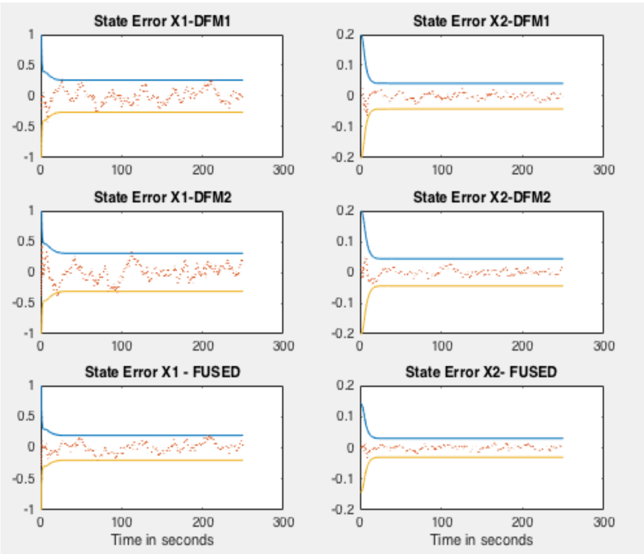

Figure 7: State errors with their bounds

Next, the following nonlinear continuous time system is considered

$$\dot{X}_1(t) = 2.5\cos(t) - 0.68X_1(t) - X_2(t) - 0.0195X_2^3(t) \quad (64)$$

$$\dot{X}_2(t) = X_1(t) \quad (65)$$

The model error is determined by eliminating the terms from (64) in turn: (i)$X_2^3(t)$,(ii) X_1, X_2, X_2^3, and utilizing the deficient models in the corresponding estimation algorithm; then, a regression model is fitted to the discrepancy $d(t) = a_1X_1(t) + a_2X_2(t) + a_3X_2^3(t)$ to estimate the parameters of the CTS. Values Q=diag(0.001, 30) and R=18 are used for achieving convergence. The parameters are estimated from the model discrepancies using LS method. Table 5 shows the estimates of the coefficients. The estimates compare well with the true values of the parameters. Tables 6-8 show the percentage fit errors, state errors and HI norm for continuous time case, which are reasonably low. Figures 8-11 depict performance graphs for the continuous time case for the data fusion, also from which it can be seen that the data fusion scheme that uses (at a time any) two DFMs, gives very satisfactory state vector fusion results. The norm of the 'covariance' Gramian (S(t), studied in the context of data fusion scheme using IE/HI estimators) is shown in typical graph in Figure 10, which again corroborates the asymptotic result derived for the observer error dynamics, since the observer is based on the IE/HI based gain and state error Gramian S(t).

Table 6: Nonlinear parameter estimation results – Continuous time system

Case	*Parameter*	a_1 X_1	a_2 X_2	a_3 X_2^3	*Terms removed*
	True Value	**0.68**	**1**	**0.0195**	-
DFM1	IE method	*(0.68)*	*(1)*	0.0187	X_2^3
	IE+HI method γ=22.902	*(0.68)*	*(1)*	0.0196	
DFM2	IE method	0.5576	0.9647	0.0198	X_1, X_2, X_2^3
	IE+HI method γ=0.3524	0.68	1.08	0.0197	

Table 7: Percentage Residual Fit Errors

Method	Model	Measurement
IE	DFM1	0.4290
	DFM2	0.7391
IE/HI	DFM1	0.3990
γ=0.3524	DFM2	0.7091

Table 8: Percentage State Errors

Method	Model	State X_1	State X_2
IE	DFM1	0.0214	0.7692
	DFM2	0.0226	0.9042
	Fusion	0.0150	0.5566
IE/HI	DFM1	0.0198	0.7100
γ=0.3524	DFM2	0.0209	0.8346
	Fusion	0.0138	0.5137

Table 9: H-Infinity Norm

Case	Method	norm
DFM1 & DFM2	IE	0.0870
	IE+HI γ=0.3524	0.0846

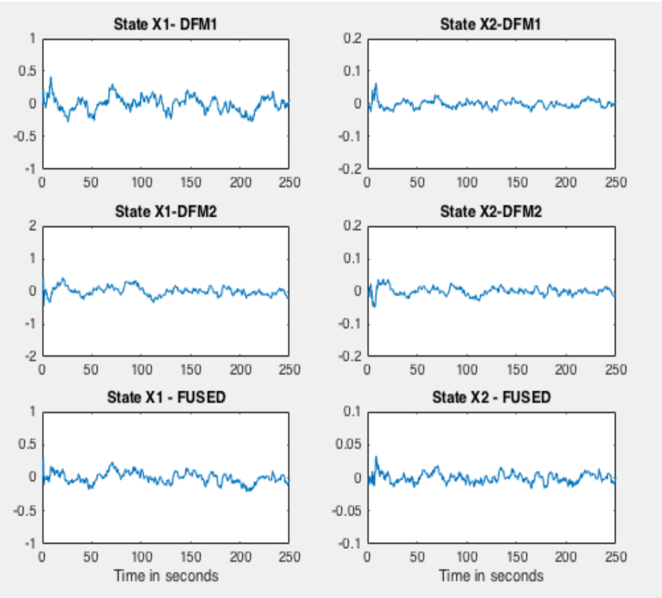

Figure 8: Estimated/fused states IE-HI method

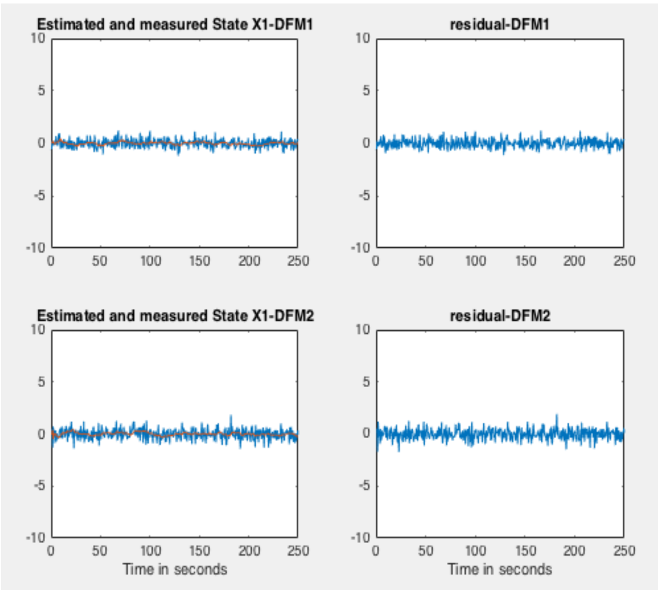

Figure 9: Estimated/measured states & Residuals

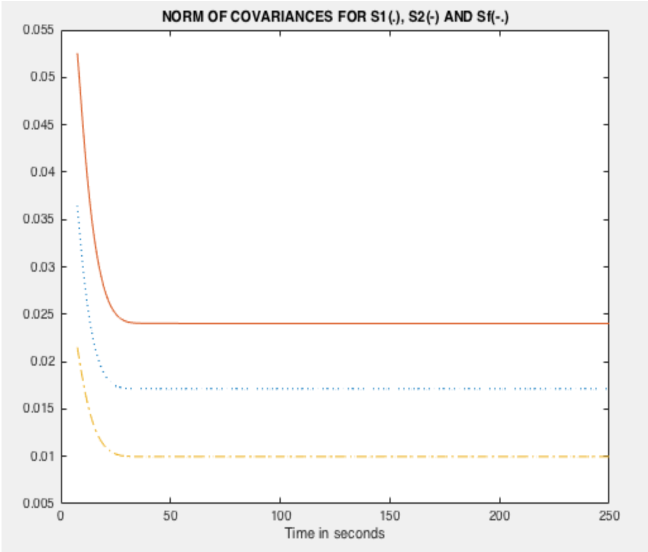

Figure 10: Norm of SE Gramians/IE-HI method

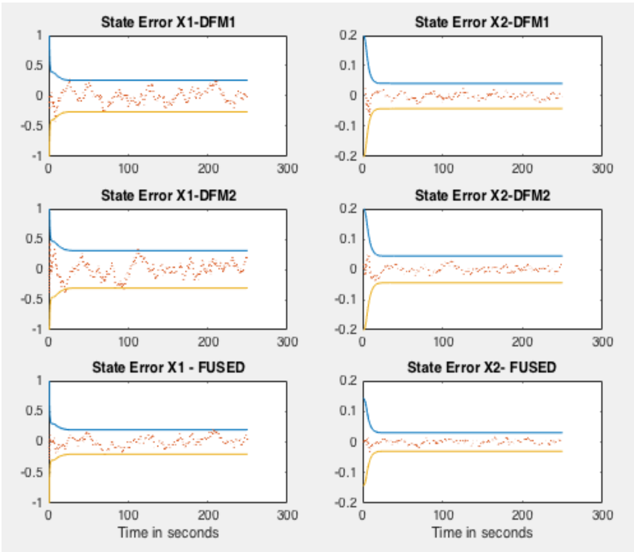

Figure 11: State errors with their bounds .

CONCLUSION

A data fusion scheme that utilizes the IE and IE/HI based model error estimators for two deficient models (at a time, in turn) is presented in a state vector fusion mode, as if these two information-results are coming from two independent (fictitious sensors) channels. The various metrics evaluated for continuous and discrete time systems using MATLAB based simulations and the performance graphs validate the efficacy of the scheme. An asymptotic stability result based on Lyapunov energy functional has been established for a nonlinear observer in the joint framework of invariant embedding and H-infinity theories. The performance of the nonlinear observer has been validated using the L-V two species nonlinear model simulated in MATLAB. The performance has been found to be very satisfactory and the evaluation results also support the asymptotic result. The latter result is supported by the behaviour of the eigenvalues of the Gramian matrix, S(t). The analytical and simulations results presented in this paper validate the satisfactory convergence and performance of the IE and IE/HI based model error estimators, either for observer or the data fusion process.

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