Computational Results of Hybrid Learning in Adaptive Neuro Fuzzy Inference System for Optimal Prediction

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Abstract

Hybrid learning provides an optimal predicting the result of the cognitive skill level under the process of the forward pass and backward pass. Many researchers proposed different techniques such as Naive Bayes classification, decision tree, C4.5 algorithms, and Mamdani approach and it generates the result for predicting by small data sets. In this research, Adaptive Neuro Fuzzy Inference System can be implemented and error tolerance can be learned by hybrid learning method which can be used to reduce an error in calculating the least square method for analyzing the error occurs from the first layer to the fourth layer and adjusting the old parameter value for reducing the error using the gradient descent method. From this analysis, an optimal result can be computed for predicting the cognitive skill level in crisp output.

Keywords: Hybrid learning; Sugeno Inference system; Triangular membership function; Least Square method; Gradient descent method; Prediction error; Epochs; Step size

INTRODUCTION

ANFIS utilizes the hybrid-learning rule and manages complex decision-making or diagnosis systems. ANFIS has been proven to be an effective tool for tuning the membership functions of fuzzy inference systems [1]. A data learning technique that uses a fuzzy inference system model to transform a given input into a target output. This prediction involves membership functions, fuzzy logic operators, and if-then rules. This expert knowledge is usually of the form if-then rules.

IF - a set of antecedent conditions is satisfied  
THEN - a set of consequences can be inferred.

In the forward pass of the hybrid learning algorithm, functional signals go forward up to four layer and network weights allowed permanent. On other hand in the backward pass, weights are adjusted according to rule for error correction [2] [12]. Error signal is the actual response of the network and assigned the rule base are adjusted using with a back propagation algorithm [3] to construct the crisp value of students skill level.

SCOPE OF RESEARCH

In this research, Sugeno fuzzy inference model can be used for the fuzzy rules and a system of rules to make decisions through the fuzzy logical operator and generates a single truth value that determines the outcome of the rules [10] [4]. A triangular membership function can be used for associated with terms that appear in the antecedents or consequents of rules [6]. Defuzzification produced the final output of a fuzzy system to give a crisp value [12] [13]. For decision-making purposes [5] [15], the output fuzzy sets must be defuzzifier to crisp value from three input of logical reasoning, numerical ability, perceptual speed.

MATERIAL AND METHOD

The knowledge acquisition module allows user to seek the inputs variables are fuzzified by triangular membership functions [11] for defined applied actual values from an input variable, to determine the degree of truth for each rule. An antecedents and consequents involve linguistic variables [9] which can be formed in fuzzy rule base by the characterized of if-then rules. The truth value of antecedent of each rule can identified and applied to the consequent part of each rule. This results of one fuzzy subset to be assigned to each output variable for each rule.

The steps involves to performing Fuzzy Inference operation in Fuzzy Inference System in ANFIS of Sugeno method,

1. Compare the input variables with the membership functions to obtain the membership values of each linguistic label [8]. (This process is called fuzzification).
2. Aggregate the membership values on the premise part to get firing strength of each rule.
3. Depending on the firing strength rule, generate the qualified consequent using fuzzy or crisp rule.
4. Aggregate the qualified consequent to produce a crisp output (this step is called defuzzification). Adaptive Neuro Fuzzy Inference System (ANFIS) constructs a fuzzy inference system (FIS) to learn the information about the data set whose membership function parameters are tuned (adjusted) using back propagation algorithm in hybrid learning method.

6. The system model of the proposed ANFIS-based system is shown in figure 1. The forward pass can propagate an input vector through the network layer up to four layers. In the backward pass, the error can be sent back through the network to back propagation. Here it can be proposed by Sugeno Fuzzy Inference model to develop a systematic approach for generating fuzzy rules from a given input [7] [8]. The output membership functions of linear can be produced by inference system based on Sugeno-type for analyzing the performance cognitive skill for students to analyze an ability level.

Figure 1: System model using ANFIS

In general, Sugeno-type systems can be used to model any inference system in which the output membership functions in linear.

Fuzzification is the process of mapping a crisp value of an input to membership degrees in different Fuzzy Linguistic variables [10].

1. A membership function can defined a point of input space is mapped to a degree of triangular membership between 0 and 1 and is denoted by \( \mu \). Here the process of membership function can be follow in level 1 of the methodology for analyzing the cognitive skill of students.

2. The Triangular membership function [6] is followed by the formula of Equation 1 as given below,

\[
\begin{align*}
\mu_A(x) &= \begin{cases} 
\frac{1}{a-b} (x-a), & a \leq x \leq b \\
\frac{1}{c-b} (c-x), & b \leq x \leq c \\
0, & \text{otherwise}
\end{cases}
\end{align*}
\]

(1)

\[
w_i = \mu A(x) + \mu B(y) + \mu C(z)
\]

Rule Base: In Rule Evaluation, Sugeno fuzzy inference model can be follow for the step of antecedent part of the If – Then rules are evaluated by using the triangular membership function and the output of fired rule can be evolved by Rule Aggregation process. In fuzzy logic controller collects the fuzzy rules forms the rule base for the fuzzy logic system.
Finally, the fuzzy inference system can generate a single truth value that determines the outcome of the rules by using an equation (2 and 3):

If $x$ is $A_1$ and $y$ is $B_1$ and $z$ is $C_1$ then $f_1 = c_{11}x + c_{12}y + c_{13}z + c_{10}$

If $x$ is $A_n$ and $y$ is $B_n$ and $z$ is $C_n$ then $f_n = c_{n1}x + c_{n2}y + c_{n3}z + c_{n0}$

Evaluating the rule premises results in

$$w_i = \mu_{A_i}(x)\mu_{B_i}(y)\mu_{C_i}(z), i = 1, 2, 3$$

(2)

Where $w_i$ represents the defuzzification of fuzzy output ability factor value of cognitive skill.

$$O_{3j} = \frac{w_i}{w_1 + w_2 + w_3}$$

(3)

The nodes in this layer are adaptive and perform the consequence of the rules of equation 4.,

$$O_{4j} = \frac{w_i f_i}{w_1 + w_2 + w_3} = \frac{w_i (p_i x + q_i y + r_i z + u_i)}{w_1 + w_2 + w_3}$$

$$= (0.03)^2 = 0.0009$$

(4)

The application of a control rule is also called firing. Aggregation is the methodology which is used in deciding what control action should be taken as a result of the firing of several rules, the decision rules are constructed for input parameter and control output values to find what actions should be taken as a result of firing several rules and finally the aggregation of minimum control outputs are taken into consideration to maximize the grade of output to resolve the uncertain linguistic input to produce crisp output.

From figure 4 and figure 5 represents the rule editor of triangular membership function of three input and one output can be consider as If then Rule of AND operator for functioning in fuzzy logic controller. If-then rules are framed according to the values defined in the membership function with the help of rule editor, which is implemented with the rule viewer.

Defuzzification is inverse process of Fuzzification, the inputs of linguistic variable are put into the measurement for performing to the Sugeno member function method and assigned the rule base classifier [12] [13]. A system model has several inputs and outputs, each variable having its own minimum and maximum values [8].

The output parameter of cognitive factors of student intellectual can be examined by five fuzzy sets very low, low, medium, high and very high. The parameters of $\left(p_i, q_i, r_i\right)$ are determined and referred as a consequent parameters. Where $P_i$ represents the Numerical ability value and $q_i$ represents the logical reasoning value and $r_i$ represents the perceptual speed value. Finally the crisp value of cognitive skill of student’s level can be determined by given the input variables by using the equation 5.

$$O_{5j} = \sum_j w_i f_i = \sum_i w_i f_i$$

(5)

In this research work, 125 different rules can be derived for analyzing an ability level of students. According to this method, with reference to the training data set of student’s ability values, the aggregation of control output as shown in figure 6 and figure 7.

For instance, an input value of numerical level is 11, logical reasoning is 8 and perceptual speed is 10, an fuzzy inference system can perform by lingustic input values which can be functioned in sugeno method based on triangular membership function and evaluate the membership value for analysing in rule base. After analysing the rule editor the weight of the scoring of students can be displayed in output is 0.81 which can be viewed in rule viewer as shown figure 5 respectively.
Surface Viewer is a three dimensional curve that represents the mapping from the ability level of students and the output parameter can be shown by figure 6 and figure 7 viewed by graph plotted as per the given data. The inputs of ability data can be sending through triangular membership function in different range, and then final output can be produced by fuzzy controller using MATLAB.

**Figure 6:** Surface Viewer mapping cognitive skill of student’s in1 and in3 of perceptual and Numerical ability level

From figure 6, represent the surface view of perceptual speed and numerical ability level can be predicted from one input value of student level which can be analysed by surface view, represented as high level.

From figure 7, represent the surface view of logical reasoning and perceptual speed can be predicted from one input value of student level which can be analysed by surface view, represented as high level.

**Figure 7:** Surface Viewer mapping cognitive skill of student’s in2 and in3 of logical reasoning and perceptual speed

**HYBRID LEARNING**

In hybrid learning algorithm, the forward pass signal can be forwarded up to layer 4 and then the consequent parameters are identified by the estimation of least squares [14]. A Least square estimator method was applied to repair the consequent parameter in the fourth layer. The consequent parameters are linear; least square estimator method can be applied to accelerate the convergence rate in hybrid learning process.

An actual output is propagated back to the first layer when an error can occurred during the comparison between outputs generated. At the same time, parameter premises in the first layer are updated using learning methods of gradient descent. Initialize all weights with small random numbers, typically between -1 and 1 [13]. The pseudo code of hybrid learning method as follows,

Initialize all weights with small random numbers, typically between -1 and 1

repeat

for every pattern in the training set

Present the pattern to the network

// Propagated the input forward through the network:
for each layer in the network
    for every node in the layer
        1. Calculate the weight sum of the inputs to every node
        2. Add the threshold to the sum
        3. Calculate the activation for the node
    end

// Propagate the errors backward through the network
for every node in the output layer
    calculate the error signal
end

for all hidden layers
    for every node in the layer
        1. Calculate the node's signal error
        2. Update each node's weight in the network
    end

// Calculate Global Error
Calculate the Error Function
end

while ((maximum number of iterations < than specified) AND (Error Function is > than specified))

**IMPLEMENTATION**

From table1, represents the generation of training data in fuzzy inference system. Here 125 combinations of rules can be used for firing rule analysis. In this research, 1000 instance of training data can be used for testing based on hybrid learning in 100 epochs with 42 premise of non-linear parameter and 28 consequent of linear parameter.
Table 1: Generation of Fuzzy Inference System

<table>
<thead>
<tr>
<th>FIS Generation</th>
<th>125</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of rule combination</td>
<td>125</td>
</tr>
<tr>
<td>Input MF</td>
<td>Very Low</td>
</tr>
<tr>
<td>Output MF</td>
<td>Very Low</td>
</tr>
<tr>
<td>InRange</td>
<td>[1 15]</td>
</tr>
<tr>
<td>outrange</td>
<td>[0 1]</td>
</tr>
<tr>
<td>InMFTypes</td>
<td>Trimf</td>
</tr>
<tr>
<td>OutMFTypes</td>
<td>Linear</td>
</tr>
<tr>
<td>No. of Nodes</td>
<td>62</td>
</tr>
<tr>
<td>Non-linear Parameter</td>
<td>42 (Premise)</td>
</tr>
<tr>
<td>Linear Parameter</td>
<td>28 (Consequent)</td>
</tr>
<tr>
<td>Training data pairs</td>
<td>1000</td>
</tr>
<tr>
<td>Testing and checking</td>
<td>1000</td>
</tr>
<tr>
<td>No. of fuzzy rules</td>
<td>7</td>
</tr>
<tr>
<td>Epoch reached</td>
<td>100 epoch</td>
</tr>
<tr>
<td>% of error in epoch</td>
<td>1.36832</td>
</tr>
</tbody>
</table>

The ability level of student can be experimented in layer by layer implemented by Adaptive Neuro Fuzzy Inference System. In case 1, an input value of three ability values are 11, 8 and 10, which can be analyzed by five layers.

- **Layer 1**

  Collect the current scoring value of the student regarding ability by set of facts and represent using domain knowledge. The output of each node for Numerical, logical reasoning and perceptual speed values can be represented by follows,

  \[ w_i = \mu_{A_i}(x)\mu_{B_j}(y)\mu_{C_i}, i = 1, 2, 3 \]

  Where

  \[ O_{1i} = \mu_{A_1}(x) = \mu_{A_1} = 11; O_{1i} = \mu_{B_1}(y) = \mu_{B_1} = 8; \]

  \[ O_{1i} = \mu_{C_1}(z) = \mu_{C_1} = 10 \]

  Here \( A_i \) represents ability range with Numerical value,
  \( B_i \) represents ability range of Logical reasoning value
  \( C_i \) represents ability range of Perceptual Speed value

  Then calculate Membership function plots (MFP) value by the reference of (1) and \( \mu \) value is given below,

  \[ \mu_{A_1}(x) = \mu_{High}(x) = 0.8 \]

  \[ \mu_{B_1}(x) = \mu_{Medium}(x) = 1.4 \]

  \[ \mu_{C_1}(x) = \mu_{High}(x) = 0.5 \]

- **Layer 2**

  The Skill level can be analyzed by given range of \( \mu \) value using ‘OR’ operator as refer from formula (2)

  It is represented by \( O_{2i} = w_i = \mu_{A_i}(x) + \mu_{B_i}(y) + \mu_{C_i}(z) \)

  \[ W_i = 0.8+1.4+0.5= 2.7 \]

- **Layer 3**

  IF (condition) part of the rule matches a fact of numerical, logical and perceptual then the rule is fired and its THEN (action) part is executed.

  If   \( x \) is \( A_1 \) and \( y \) is \( B_1 \) and \( z \) is \( C_1 \)

  Then \( f_i = c_{11}x+c_{12}y+c_{13}z+c_{10} \)

  If Numerical is 11 and logical is 8 and perceptual speed is 10, then the weight of the rule can be analyzed in layer 3, where the weight of \( W_1, W_2, W_3 \) is 1 and \( O_{3i} \) can be calculated by (3)

  \[ O_{3i} = w_i f_i = \frac{2.7}{1+1+1} = 0.9 \]

- **Layer 4**

  Finally, the firing rules can be analyzed by the formula (4)

  \[ O_{4i} = w_i f_i = 0.9(2.7) = 2.43 \]

- **Layer 5**

  Defuzzification is the process of converting the final output of a fuzzy system to a crisp value can be examined as 0.17 which represents the risk is in Normal level.

  \[ O_{5j} = \frac{2.43}{3} = 0.81 \]

  For the training scenario, the error in prediction considering by the equation 6.
\[ E_k = \frac{1}{2} (T_{ok} - O_{ok})^2 \]  

\[ E_k = \frac{1}{2} (0.75 - 0.81)^2 \]

\[ = (-0.03)^2 = 0.0009 \]

A. Least Square Estimator

Each epoch is composed of forward pass and backward pass, the node outputs go forward until Layer 4 and consequent parameters are identified by least squares estimator; Best solution for \( q \) that minimizes \( ||Aq - z||^2 \) is the least-squares estimator \( b \) by the equation 7.

\[ b = (X^T X)^{-1} X^T y \]

B. Backward Propagation

In Backward pass, the premise parameters are updated by gradient descent an error signals propagate backward. Let the premise parameters be fixed. ANFIS output is given by linear combination of consequent parameters \( p, q \) and \( r \):

Step 1: Initialize the weights \( (W_1, W_2, W_3) \) with random values and calculate Error (Sum Square Error) by equation 8.

\[ SSE = \sum_{k=1}^{n} \left( d_k - x_{i,k} \right)^2 \]  

\[ \text{(8)} \]

Step 2: Calculate the gradient i.e. change in SSE when the weights \( (W_1, W_2, W_3) \) are changed by a very small value from their original randomly initialized value. It helps to move the values of \( W_1, W_2, W_3 \) in the direction in which Sum Square Error (SSE) is minimized.

Step 3: By using Gradient, to adjust the weights to reach the optimal values where SSE is minimized by using an equation 9.

\[ \Delta \alpha = -\eta \frac{\partial E}{\partial \alpha} \]

\[ \text{(9)} \]

Step 4: Use the new weights for prediction and to calculate the step size by equation 10, which can be changed in order to accelerate the convergence rate in adaptive networks.

\[ \eta = \frac{k}{\sqrt{\sum \alpha \left( \frac{\partial E}{\partial \alpha} \right)^2}} \]

\[ \text{(10)} \]

Step 5: Repeat steps 2 and 3 till further adjustments to weights doesn’t significantly reduce the Error.

The parameters associated with the membership functions will change through the learning process. An adjustment parameter can be computed by a gradient vector, which provides a measure of how well the fuzzy inference system is modeling the input/output data for a given set of parameters.

From the figure 8 represents the degree of membership function on input 1 which can be viewed by over fitted using least square method.

Figure 8: Degree of membership function

Figure 9: Step size on 100 Epochs

From figure 9, represents the step size can be calculated on each epoch when reach at 100. On each epoch from 1, Step size increases from 0.011000 to 0.0256 after epoch 25, the step size can decreases from 0.023110 to 0.007252 then designated epoch number reached, ANFIS training completed at epoch 100. From figure 10 represents an adjusting new weight of the parameter updated for prediction.
From figure 11, represent the epoch error of 1000 training data, it can be reduced on each epoch under adjusting the updatable parameter in Gradient descent method and finally an error come to reduce up to 1.368 on 100 epochs.

From figure 12, represents the time taken for analysing the all training data in seconds.

**CONCLUSION**

In this paper, it can be concluded that in Sugeno fuzzy inference system, hybrid learning method can gives an optimal prediction rather than back propagation method and other method. By using triangular membership function associated with terms that appear in the antecedents or consequents of rules leads to optimal fuzzy controller should be adapted in order to ensure optimal performance based on hybrid learning by combination of least square method and gradient descent method. In the forward pass of the hybrid learning algorithm, functional signals go forward up to layer 4 and the consequent parameters are identified by the least squares estimate. The back propagation is used to identify the nonlinear parameters (premise parameters). In this paper, it can be concentrated the forward pass and backward pass process held in inference engine. Finally an error can be reduced 0.03 on 100 epochs efficiently reach optimal prediction of analysing the cognitive skill of students in efficient.

**REFERENCES**


