

Reliability Analysis of Rodding Anode Plant in Aluminium Industry

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Abstract

The paper presents a reliability analysis of rodding anode plant in aluminum industry. Manufacturing process of raw aluminum passes through eight stations viz., butt shot blast station 1, butt & thimble removal press station 2 with standby arrangement, combined btp (butt & thimble press) station 3, stub straighten station 4, stub shot blast station 5, stub coating and drying station 6, casting station 7, and anode rod inspection station 8. Failure of any of the stations brings the system to a complete halt, except the butt & thimble removal press stations because of the parallel standby arrangement, and does not affect the system operation completely unless both fails. Six years maintenance data on component failures, repairs and associated costs are collected for the purpose of this analysis and various rates are estimated from the data. Measures of system effectiveness such as mean time to plant failure (MTPF), availability of the plant, busy period of repairman and expected number of repair have been obtained. Semi-Markov processes and regenerative point techniques are used in the analysis.

Keywords: Reliability, semi-Markov process, regenerative point technique, failure, repair

NOTATIONS:

O_i	Unit is operative at state i
S_2	One machine at station 2 in standby mode
$2r$	One machine at station 2 is under repair
D_i	Down at state i
F_{ir}, F_{iwr}	Station i is failed and is under repair, waiting for repair.
$f_i(t), F_i(t)$	$p.d.f$ and $c.d.f$ of failure rate of the station i
$g_i(t), G_i(t)$	$p.d.f$ and $c.d.f$ of repair rate of the station i
q_{ij}, Q_{ij}	Probability density function ($p.d.f$), cumulative distribution function ($c.d.f$) from a regenerative state i to a regenerative state j without visiting any other regenerative state $(0, t]$.
$q_{ij}^{(k)}, Q_{ij}^{(k)}$	Probability density function ($p.d.f$), cumulative distribution function ($c.d.f$) from a regenerative state i to a regenerative state j

	with visiting (k) state in $(0, t]$.
$p_{ij}, p_{ij}^{(k)}$	Probability of transition from a regenerative state i to a regenerative state j without visiting any other state in $(0, t]$, probability of transition from a regenerative state i to a regenerative state j with visiting (k) state in $(0, t]$.
$*/LT$	Symbol for a Laplace Transform
$**/LST$	Symbol of a Laplace-Steiltjes transform
$m_{ij}, m_{ij}^{(k)}$	The unconditional mean time taken to transit to any regenerative state from the epoch of entry into regenerative state j without visiting any failed states, visiting failed state (k) once.
μ_i	Mean sojourn time in the regenerative state i before transiting to any other state.
©	Laplace convolution.
Ⓢ	Steiltjes convolution.
$\Phi_i(t)$	Cumulative distribution function ($c.d.f$) of the first passage time from a regenerative state i to a failed state
$M_i(t)$	The probability that the system initially up in regenerative state i , is up at a time t without going to any regenerative state
$A_i(t)$	The probability of the unit entering into upstate at instant t , giving that the unit entered in regenerative state i at $t = 0$
$B_i(t)$	Prpbability that the repairman is busy in inspection of instant t , given that the system entered regenerative state i at $t = 0$
$V_i(t)$	Expected number of visits of the repairman, given that the system entered regenerative state i at $t = 0$
$W_i(t)$	Probability that that the repairman is busy in regenerative state i at time t without passing any other regenerative state.

INTRODUCTION

Reliability analysis of industrial systems has been widely presented in the literature by many researchers due to its potential importance in industries. Complex systems are subject to failures because of many reasons which affect the

profitability of the manufacturing industry and hence reliability analysis plays an important role in understanding the system performance while dealing with real industrial problems under different operating conditions and assumptions. Gulshan et al. [1] wrote about system analysis with perfect repair under partial failure mode and priority for repair to completely failed unit, Attahiru & Zhao [2] considered repairable system with three-units, Tuteja et al. [3]-[5] worked for two-units system with regular repairman who is not always available, system with perfect repair at partial failure or complete failure mode, and the profit evaluation of a two-units cold standby system with tiredness and two types of repairmen. Rizwan et al. [6]-[13] analyzed cold and hot standby systems with single-unit and two-units under different failure and repair situations where the reliability indices of interest are obtained and the cost benefit analysis of the systems are carried out. Mathew et al. [14]-[20] extensively analyzed the continuous casting plant and studied the variations under different operating conditions of the plant. Detailed analysis was reported for desalination plant by Padmavathi et al. [21] with online repair under emergency shutdowns, Rizwan et al. [22] with repair/maintenance strategy on first come first served basis, Padmavathi et al. [23]-[27] continued on desalination plant with priority for repair over maintenance, comparative analysis between the plant models,

analysis under major and minor failures consideration, analysis by prioritizing repair over maintenance under major / minor failures, and comparative analysis between the plant models portraying two operating conditions of the plant as to which model is better than the other. The methodology was further extended for various industrial systems analyses by Gupta and Gupta [28] with post inspection concept, Ram et al. [29] waiting repair strategy, Malhotra and Taneja [30] both units operative on demand, Niwas et al. [31] obtained mean time to system failure and profit of a single unit system with inspection for feasibility of repair beyond warranty. Later, Rizwan et al. [32]-[34] focused on waste water treatment plant & anaerobic batch reactor and reliability indices of interest were obtained in order to assess the plant/reactor performance. Recently, Taj et al. [35] analyzed a single machine subsystem of a cable plant with six maintenance categories. Thus, the methodology for industrial system analysis under various failure and maintenance situations has been widely presented in the literature and proved to be a good tool for industrial system / plant analysis under different operating conditions.

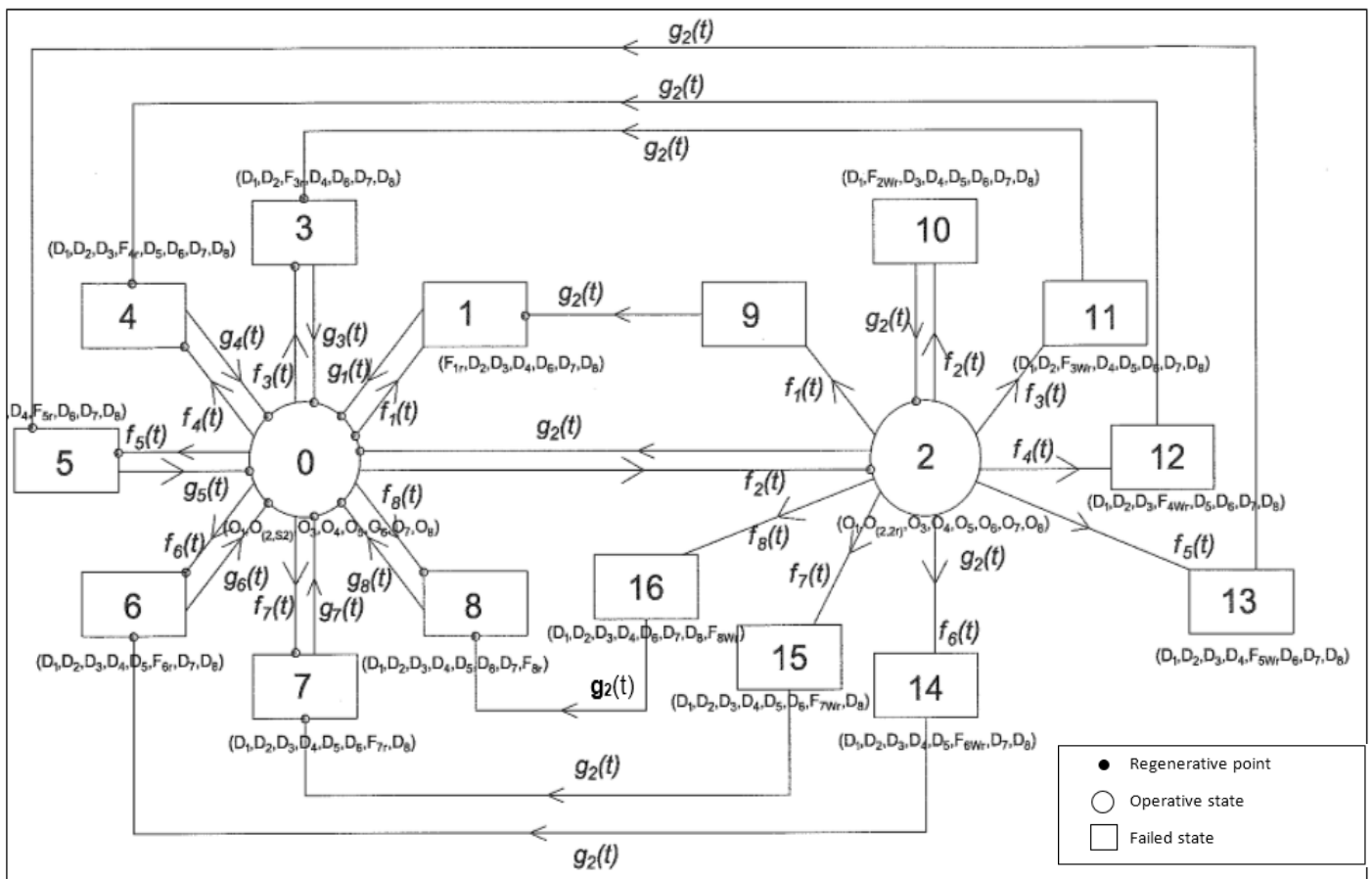


Figure 1: State transition diagram

Aluminum being widely used as a source input for many manufacturing industries is a good reason for this analysis from reliability perspective. One such aluminum manufacturing industrial plant operating in Oman has been considered for this purpose, and analysis is carried out in order to understand the operating behavior of the plant. The anode rod plant manufactures raw aluminum blocks. Six years maintenance data on component failures, repairs and various associated costs are collected for the purpose of this analysis. Failure rates of the components, repairs and various associated costs are estimated from the data. Plant has eight stations viz., butt shot blast station 1, butt & thimble removal press station 2 with standby arrangement, combined btp (butt & thimble press) station 3, stub straighten station 4, stub shot blast station 5, stub coating and drying station 6, casting station 7, and anode rod inspection station 8. The plant operates round the clock, and failure in any of the stations impacts the plant to a complete shutdown situation. However, butt & thimble removal press station 2, has standby arrangement, and do not affect the system operation completely unless both of them fail. The state transition diagram of the plant is shown in Fig. 1. Based on the various operating states of the plant, a detailed analysis is carried out using semi Markov process and regenerative point techniques. Outcome of the entire analysis is measured in terms of overall system effectiveness such as mean time to plant failure (MTPF), availability of the plant, expected busy period of the repairman, and expected number of repairs.

The data summary reflects the following related estimations:

Table I: Estimated values for the plant

S. No.	Station No.	Repair rate	Failure rate
1)	Station 1	$\alpha_1 = 0.11382393$	$\lambda_1 = 0.01768162$
2)	Station 2	$\alpha_2 = 0.18064516$	$\lambda_2 = 0.02045033$
3)	Station 3	$\alpha_3 = 0.21238938$	$\lambda_3 = 0.00050736$
4)	Station 4	$\alpha_4 = 0.21138211$	$\lambda_4 = 0.01429055$
5)	Station 5	$\alpha_5 = 0.18921776$	$\lambda_5 = 0.00322732$
6)	Station 6	$\alpha_6 = 0.23120$	$\lambda_6 = 0.003358$
7)	Station 7	$\alpha_7 = 0.24016145$	$\lambda_7 = 0.000341$
8)	Station 8	$\alpha_1 = 0.11382393$	$\lambda_1 = 0.01768162$

A. MODEL DESCRIPTIONS AND ASSUMPTIONS

- 1) Initially at state 0, the plant is in good and operational mode.
- 2) Only station 2 has the standby arrangement.
- 3) Failure at any station other than station 2, leads to a complete shutdown state.

- 4) All necessary maintenances are off-line which means necessary repairs or replacements need plant, to be in switch-off mode.
- 5) Maintenances are all random and need to be addressed on requirement by a single repairman.
- 6) The failure of any component at a particular station is taken as a single failure.
- 7) All failure times are assumed to have exponential distribution whereas other times are general.
- 8) After each repair the plant works as good as new and returns to new state.
- 9) Repairman comes as soon as a unit or component fails, and other failures need to wait until previous failures have been resolved.

B. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Possible transitions states of the plant are shown in Fig. 1. States 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16 are the failed states whereas 0 & 2 are the operative states. The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7 and 8 are regeneration points and thus are regenerative states. States 9, 10, 11, 12, 13, 14, 15, and 16 are non-regenerative states.

The transition probabilities are given by:

$$dQ_{01}(t) = \lambda_1 e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} dt \tag{1}$$

$$dQ_{02}(t) = \lambda_2 e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} dt \tag{2}$$

$$dQ_{03}(t) = \lambda_3 e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} dt \tag{3}$$

$$dQ_{04}(t) = \lambda_4 e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} dt \tag{4}$$

$$dQ_{05}(t) = \lambda_5 e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} dt \tag{5}$$

$$dQ_{06}(t) = \lambda_6 e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} dt \tag{6}$$

$$dQ_{07}(t) = \lambda_7 e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} dt \tag{7}$$

$$dQ_{08}(t) = \lambda_8 e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} dt \tag{8}$$

$$dQ_{10}(t) = g_1(t) dt \tag{9}$$

$$dQ_{20}(t) = g_2(t) e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} dt \tag{10}$$

$$dQ_{30}(t) = g_3(t) dt \tag{11}$$

$$dQ_{40}(t) = g_4(t) dt \tag{12}$$

$$dQ_{50}(t) = g_5(t) dt \tag{13}$$

$$dQ_{60}(t) = g_6(t)dt \quad (14)$$

$$dQ_{70}(t) = g_7(t)dt \quad (15)$$

$$dQ_{80}(t) = g_8(t)dt \quad (16)$$

$$dQ_{29}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \overline{G_2(t)} dt \quad (17)$$

$$dQ_{2,10}(t) = \lambda_2 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \overline{G_2(t)} dt \quad (18)$$

$$dQ_{2,11}(t) = \lambda_3 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \overline{G_2(t)} dt \quad (19)$$

$$dQ_{2,12}(t) = \lambda_4 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \overline{G_2(t)} dt \quad (20)$$

$$dQ_{2,13}(t) = \lambda_5 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \overline{G_2(t)} dt \quad (21)$$

$$dQ_{2,14}(t) = \lambda_6 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \overline{G_2(t)} dt \quad (22)$$

$$dQ_{2,15}(t) = \lambda_7 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \overline{G_2(t)} dt \quad (23)$$

$$dQ_{2,16}(t) = \lambda_8 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \overline{G_2(t)} dt \quad (24)$$

$$dQ_{21}^{(9)}(t) = (\lambda_1 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \odot 1) g_2(t) dt \quad (25)$$

$$dQ_{22}^{(10)}(t) = (\lambda_2 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \odot 1) g_2(t) dt \quad (26)$$

$$dQ_{23}^{(11)}(t) = (\lambda_3 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \odot 1) g_2(t) dt \quad (27)$$

$$dQ_{24}^{(12)}(t) = (\lambda_4 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \odot 1) g_2(t) dt \quad (28)$$

$$dQ_{25}^{(13)}(t) = (\lambda_5 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \odot 1) g_2(t) dt \quad (29)$$

$$dQ_{26}^{(14)}(t) = (\lambda_6 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \odot 1) g_2(t) dt \quad (30)$$

$$dQ_{27}^{(15)}(t) = (\lambda_7 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \odot 1) g_2(t) dt \quad (31)$$

$$dQ_{28}^{(16)}(t) = (\lambda_8 e^{-(\lambda_1+\lambda_2+\dots+\lambda_8)t} \odot 1) g_2(t) dt \quad (32)$$

The non-zero element p_{ij} can be obtained from (1) to (32), using

$$p_{ij} = \lim_{s \rightarrow 0} \int_0^\infty q_{ij}(t) dt \text{ or } p_{ij} = \lim_{s \rightarrow 0} q_{ij}^*(s) \quad (33)$$

$$p_{01} + p_{02} + p_{03} + p_{04} + p_{05} + p_{06} + p_{07} + p_{08} = 1 \quad (33)$$

$$p_{10} = p_{30} = p_{40} = p_{50} = p_{60} = p_{70} = p_{80} = 1 \quad (34)$$

$$p_{20} + p_{29} + p_{2,10} + p_{2,11} + p_{2,12} + p_{2,13} + p_{2,14} + p_{2,15} + p_{2,16} = 1 \quad (35)$$

$$p_{20} + p_{21}^{(9)} + p_{22}^{(10)} + p_{23}^{(11)} + p_{24}^{(12)} + p_{25}^{(13)} + p_{26}^{(14)} + p_{27}^{(15)} + p_{28}^{(16)} = 1 \quad (36)$$

The mean sojourn time is the expected time taken by the plant in a particular state before transition to any other state and it is also called as the mean survival time in the state,

$\mu_i = E(t) = P(T > t) = \int_0^\infty t dQ_{ij}(t) dt$ in state i is given by:

$$\mu_0 = \frac{1}{\lambda_1 + \lambda_2 + \dots + \lambda_8} \quad (37)$$

$$\mu_1 = \frac{\lambda_1}{(\lambda_1 + \lambda_2 + \dots + \lambda_8)^2} \quad (38)$$

$$\mu_2 = \frac{2\alpha_2 + \lambda_1 + \dots + \lambda_8}{(\alpha_2 + \lambda_1 + \lambda_2 + \dots + \lambda_8)^2} \quad (39)$$

$$\mu_3 = \frac{1}{\alpha_3} \quad (40)$$

$$\mu_4 = \frac{1}{\alpha_4} \quad (41)$$

$$\mu_5 = \frac{1}{\alpha_5} \quad (42)$$

$$\mu_6 = \frac{1}{\alpha_6} \quad (43)$$

$$\mu_7 = \frac{1}{\alpha_7} \quad (44)$$

$$\mu_8 = \frac{1}{\alpha_8} \quad (45)$$

The unconditional mean time $m_{ij} = \int_0^\infty t dQ_{ij}(t)$ or $\lim_{s \rightarrow 0} -\frac{d}{ds} (q_{ij}^*(s))$ taken by the plant to transit for any state j when it has taken from epoch of entrance into regenerative state i is mathematically stated as:

$$m_{01} + m_{02} + \dots + m_{08} = \mu_0 \quad (46)$$

$$m_{10} = \mu_1 \quad (47)$$

$$m_{30} = \mu_3 \quad (48)$$

$$m_{40} = \mu_4 \quad (49)$$

$$m_{50} = \mu_5 \quad (50)$$

$$m_{60} = \mu_6 \quad (51)$$

$$m_{70} = \mu_7 \quad (52)$$

$$m_{80} = u_8 \quad (53)$$

$$m_{20} + m_{29} + m_{2,10} + m_{2,11} + m_{2,12} + m_{2,13} + m_{2,14} + m_{2,15} + m_{2,16} + m_{21}^{(9)} + m_{22}^{(10)} + m_{23}^{(11)} + m_{24}^{(12)} + m_{25}^{(13)} + m_{26}^{(14)} + m_{27}^{(15)} + m_{28}^{(16)} = \mu_2 \quad (54)$$

THE MATHEMATICAL ANALYSIS

A. MEAN TIME TO PLANT FAILURE

Regarding the failed states as absorbing states and employing the arguments used for regenerative processes, we have the following recursive relations for ϕ_i :

$$\phi_0(t) = \sum_{j=0}^8 Q_{0j} + Q_{02}(t) \otimes \phi_2(t) \quad (55)$$

$$\phi_2(t) = Q_{20}(t) \otimes \phi_0(t) + \sum_{j=9}^{16} Q_{2,j} \quad (56)$$

Taking Laplace – Stieltje’s transform of (56) & (57) and solving them for $\phi_0^{**}(s)$, the mean time to plant failure when the plant starts from the state 0 is given by:

$$MTPF = \lim_{s \rightarrow 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N}{D} \quad (57)$$

Where,

$$N = \mu_0 + p_{02}(m_{20} + m_{29} + m_{2,10} + m_{2,11} + m_{2,12} + m_{2,13} + m_{2,14} + m_{2,15} + m_{2,16}); \text{ and } D = 1 - p_{02}p_{20}$$

B. AVAILABILITY ANALYSIS OF THE PLANT

Using the probabilistic arguments and defining $A_i(t)$ as the probability of the unit entering into upstate at instant t , giving that the unit entered in regenerative state i at $t = 0$, the following recursive relations are obtained $A_i(t)$:

$$A_0(t) = M_0(t) + q_{01}(t) \otimes A_1(t) + q_{02}(t) \otimes A_2(t) + q_{03}(t) \otimes A_3(t) + q_{04}(t) \otimes A_4(t) + q_{05}(t) \otimes A_5(t) + q_{06}(t) \otimes A_6(t) + q_{07}(t) \otimes A_7(t) + q_{08}(t) \otimes A_8(t) \quad (58)$$

$$A_1(t) = q_{10}(t) \otimes A_0(t) \quad (59)$$

$$A_2(t) = M_2(t) + q_{20}(t) \otimes A_0(t) + q_{21}^{(9)}(t) \otimes A_1(t) + q_{22}^{(10)}(t) \otimes A_2(t) + q_{23}^{(11)}(t) \otimes A_3(t) + q_{24}^{(12)}(t) \otimes A_4(t) + q_{25}^{(13)}(t) \otimes A_5(t) + q_{26}^{(14)}(t) \otimes A_6(t) + q_{27}^{(15)}(t) \otimes A_7(t) + q_{28}^{(16)}(t) \otimes A_8(t) \quad (60)$$

$$A_3(t) = q_{30}(t) \otimes A_0(t) \quad (61)$$

$$A_4(t) = q_{40}(t) \otimes A_0(t) \quad (62)$$

$$A_5(t) = q_{50}(t) \otimes A_0(t) \quad (63)$$

$$A_6(t) = q_{60}(t) \otimes A_0(t) \quad (64)$$

$$A_7(t) = q_{70}(t) \otimes A_0(t) \quad (65)$$

$$A_8(t) = q_{80}(t) \otimes A_0(t) \quad (66)$$

Here,

$$M_0(t) = e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t}$$

$$M_2(t) = e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} \overline{G_2(t)}$$

Taking Laplace transforms of (58) to (66) and solving them for $A_0^{**}(s)$, the steady state availability is given by:

$$A_0 = \lim_{s \rightarrow 0} s A_0^{**}(s) = \lim_{s \rightarrow 0} \frac{sN(s)}{D(s)} = \frac{N_1}{D_1} \quad (67)$$

Where,

$$N_1 = p_{02}p_{20} + (1 - p_{22}^{(10)})u_0$$

$$D_1 = m_{22}^{(10)}p_{20} + m_{10}p_{01} + p_{02}(m_{20} + m_{21}^{(9)} + m_{23}^{(11)} + m_{24}^{(12)} + m_{25}^{(13)} + m_{26}^{(14)} + m_{27}^{(15)} + m_{28}^{(16)}) + m_{30}p_{03} + m_{40}p_{04} + m_{50}p_{05} + m_{60}p_{06} + m_{70}p_{07} + m_{80}p_{08} + (1 - p_{22}^{(10)})(m_{01} + m_{02} + m_{03} + m_{04} + m_{05} + m_{06} + m_{07} + m_{08}) + m_{10}p_{02}p_{21}^{(9)} - p_{22}^{(10)}(m_{10}p_{01} + m_{30}p_{03} + m_{40}p_{04} + m_{50}p_{05} + m_{60}p_{06} + m_{70}p_{07} + m_{80}p_{08}) + p_{02}(m_{30}p_{23}^{(11)} + m_{40}p_{24}^{(12)} + m_{50}p_{25}^{(13)} + m_{60}p_{26}^{(14)} + m_{70}p_{27}^{(15)} + m_{80}p_{28}^{(16)})$$

C. BUSY PERIOD ANALYSIS OF REPAIRMAN

Using the probabilistic arguments and defining $B_0^*(s)$ as probability that the repairman is busy for repair at instant t , given that the unit entered in regenerative state i at $t = 0$, the following recursive relations are obtained:

$$B_0(t) = q_{01} \otimes B_1(t) + q_{02} \otimes B_2(t) + q_{03} \otimes B_3(t) + q_{04} \otimes B_4(t) + q_{05} \otimes B_5(t) + q_{06} \otimes B_6(t) + q_{07} \otimes B_7(t) + q_{08} \otimes B_8(t) \quad (68)$$

$$B_1(t) = W_1(t) + q_{10}(t) \otimes B_0(t) \quad (69)$$

$$B_2(t) = W_2(t) + q_{20}(t) \otimes B_0(t) + q_{21}^{(9)}(t) \otimes B_1(t) + q_{22}^{(10)}(t) \otimes B_2(t) + q_{23}^{(11)}(t) \otimes B_3(t) + q_{24}^{(12)}(t) \otimes B_4(t) + q_{25}^{(13)}(t) \otimes B_5(t) + q_{26}^{(14)}(t) \otimes B_6(t) + q_{27}^{(15)}(t) \otimes B_7(t) + q_{28}^{(16)}(t) \otimes B_8(t) \quad (70)$$

$$B_3(t) = W_3(t) + q_{30}(t) \otimes B_0(t) \quad (71)$$

$$B_4(t) = W_4(t) + q_{40}(t) \otimes B_0(t) \quad (72)$$

$$B_5(t) = W_5(t) + q_{50}(t) \otimes B_0(t) \quad (73)$$

$$B_6(t) = W_6(t) + q_{60}(t) \otimes B_0(t) \quad (74)$$

$$B_7(t) = W_7(t) + q_{70}(t) \otimes B_0(t) \quad (75)$$

$$B_8(t) = W_8(t) + q_{80}(t) \odot B_0(t) \quad (76)$$

Here,

$$W_1(t) = \overline{G_1(t)}; W_2(t) = e^{-(\lambda_1 + \lambda_2 + \dots + \lambda_8)t} \overline{G_2(t)}; W_3(t) = \overline{G_3(t)}; W_4(t) = \overline{G_4(t)}; W_5(t) = \overline{G_5(t)}; W_6(t) = \overline{G_6(t)}; W_7(t) = \overline{G_7(t)} \text{ and } W_8(t) = \overline{G_8(t)}$$

Taking Laplace transform of (68) to (76) and solving them for $B_0^*(s)$, the busy period of the repairman is given by:

$$B_0 = \lim_{s \rightarrow 0} s B_0^*(s) = \frac{N_2}{D_1} \quad (77)$$

Where;

$$N_2 = p_{01} W_1^*[0](1 - p_{22}^{(10)}) + p_{02}(p_{21}^{(9)} W_1^*[0] + W_2^*[0] + p_{23}^{(11)} W_3^*[0] + p_{24}^{(12)} W_4^*[0] + p_{25}^{(13)} W_5^*[0] + p_{26}^{(14)} W_6^*[0] + p_{27}^{(15)} W_7^*[0] + p_{28}^{(16)} W_8^*[0]) + p_{03} W_3^*[0](1 - p_{22}^{(10)}) + p_{04} W_4^*[0](1 - p_{22}^{(10)}) + p_{05} W_5^*[0](1 - p_{22}^{(10)}) + p_{06} W_6^*[0](1 - p_{22}^{(10)}) + p_{07} W_7^*[0](1 - p_{22}^{(10)}) + p_{08} W_8^*[0](1 - p_{22}^{(10)});$$

and D_1 is already specified in (68)

D. EXPECTED NUMBER OF VISIT BY THE REPAIRMAN FOR REPAIRS

Let $V_i(t)$ be defined as the expected number of visits for repairs in $(0, t]$, given that the plant initially starts from the regenerative state i . Using the probabilistic arguments, the following recursive relations are obtained for $V_i(t)$:

$$V_0(t) = Q_{01}(t) \odot (1 + V_1(t)) + Q_{02}(t) \odot (1 + V_2(t)) + Q_{03}(t) \odot (1 + V_3(t)) + Q_{04}(t) \odot (1 + V_4(t)) + Q_{05}(t) \odot (1 + V_5(t)) + Q_{06}(t) \odot (1 + V_6(t)) + Q_{07}(t) \odot (1 + V_7(t)) + Q_{08}(t) \odot (1 + V_8(t)) \quad (78)$$

$$V_1(t) = Q_{10}(t) \odot V_0(t) \quad (79)$$

$$V_2(t) = Q_{20}(t) \odot V_0(t) + Q_{21}^{(9)}(t) \odot (1 + V_1(t)) + Q_{22}^{(10)}(t) \odot (1 + V_2(t)) + Q_{23}^{(11)}(t) \odot (1 + V_3(t)) + Q_{24}^{(12)}(t) \odot (1 + V_4(t)) + Q_{25}^{(13)}(t) \odot (1 + V_5(t)) + Q_{26}^{(14)}(t) \odot (1 + V_6(t)) + Q_{27}^{(15)}(t) \odot (1 + V_7(t)) + Q_{28}^{(16)}(t) \odot (1 + V_8(t)) \quad (80)$$

$$V_3(t) = Q_{30}(t) \odot V_0(t) \quad (81)$$

$$V_4(t) = Q_{40}(t) \odot V_0(t) \quad (82)$$

$$V_5(t) = Q_{50}(t) \odot V_0(t) \quad (83)$$

$$V_6(t) = Q_{60}(t) \odot V_0(t) \quad (84)$$

$$V_7(t) = Q_{70}(t) \odot V_0(t) \quad (85)$$

$$V_8(t) = Q_{80}(t) \odot V_0(t) \quad (86)$$

Taking Laplace stieltje's transform of (78) to (86) and solving them for $V_0^{**}(s)$, the busy period of the repairman is given by;

$$V_0 = \lim_{s \rightarrow 0} s V_0^{**}(s) = \frac{N_3}{D_1} \quad (87)$$

Where, $N_3 = (1 - p_{22}^{(10)}) + p_{02}(1 - p_{20})$;

and D_1 is already specified in (68).

PARTICULAR CASE

For this particular case, the following have been considered:

$$g_1(t)dt = \alpha_1 e^{-\alpha_1 t}, g_2(t)dt = \alpha_2 e^{-\alpha_2 t}, g_3(t)dt = \alpha_3 e^{-\alpha_3 t}, g_4(t)dt = \alpha_4 e^{-\alpha_4 t}, g_5(t)dt = \alpha_5 e^{-\alpha_5 t}, g_6(t)dt = \alpha_6 e^{-\alpha_6 t}, g_7(t)dt = \alpha_7 e^{-\alpha_7 t}, g_8(t)dt = \alpha_8 e^{-\alpha_8 t}$$

Using the data as summarized in table I, the expressions of reliability measures as in (57), (67), (77), and (87), the following values of the plant effectiveness are obtained:

- Mean time to plant failure = 29.2721 hrs.
- Availability = 0.714265 hrs.
- Busy period of repairman = 0.19397
- Expected number of repairs = 0.0395482

CONCLUSIONS

Estimated reliability results facilitate the plant engineers in understanding the plant behavior and thereby built up a scope for improvement in terms of plant performance by reviewing maintenance strategies. Mean time to plant is about 29 hours which shows, there is a failure almost every 29 hours and the company should really look into the reasons for such frequent failures and probably review the existing maintenance strategies. Other measures could further be looked into for better results from optimization perspective, but are somewhat within the tolerable limits. As a future direction, the analysis could further be explored for real complex situation as multiple components failure with single repairman.

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