

# Design and Simulation of Linear and Non-linear Controllers for dc Motor Drives Application

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## Abstract

This paper is dedicated to control the speed of separately excited dc motor using linear and non-linear controllers. For the response analysis, the linear controller is considered namely Proportional integral derivative controller (PID) and non-linear controller called Sliding mode controller (SMC). The SMC controller is based on variable structure systems (VSS) which aim at reducing the settling time, peak overshoot and steady state error of a separately excited dc motor. In the first stage, PID controller is used to control the speed, torque, angle and armature current of separately excited dc motor. A model is developed and simulated using MATLAB/SIMULINK. Later on the same is done with sliding mode controller. The speed control of dc motor using both PID and sliding mode controller is studied and results are compared. The simulation results show that sliding mode controller is superior to PID for speed control of dc motor. Therefore, the SMC is robust in presence of disturbances; the desired speed is perfectly tracked.

**Keywords:** Variable Structure Control, PID Controller, Sliding Mode Controller, dc motor Drives, MATLAB/SIMULINK.

## INTRODUCTION

DC machines have occupied a wide spectrum of many industrial, agriculture and domestic sector applications for variable speed drives such as electric vehicles, steel rolling mills, robotic manipulators, electric cranes and home appliances due to precise, wide continuous and simple control characteristics. Different fields in any sector require different speed ranges of the motors with different ranges of load variations. When the motor is required at low speed level such as in chemical processing application in industries, the non-linear factors do affect the output of the system that cannot be eliminated by the conventional controllers in real time. SMC is designed for avoiding such uncertainties. Generally, the non-linear factors of the motor are neglected where a higher speed

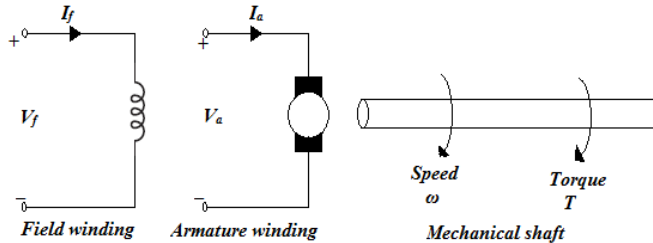
of dc motor is required because these factors do not affect much and in such cases linear controllers such as Proportional integral (PI) and Proportional integral derivative (PID) controller are effective. Since they designed easily, have low cost, inexpensive maintenance and effectiveness. Therefore, the driver modules of dc motors are cheaper and its controllers are easier. Due to easy design, they are commonly controlled by PI and PID controllers. To adjust parameters of linear controller (PI and PID), it is required to do some experiment or to determine the best mathematical model of the system. However, PI and PID controllers do not work attractively in applications of nonlinear and complex and they do not provide the desired output in desired time. For this reason, some better approach has to be taken. In this paper, SMC is that approach, a controller which has ability to work satisfactorily under linear as well as non-linear factors of the system. The results shown in the paper proves that SMC is much better than PID for the speed control of a dc motor especially when the non-linear model of the motor is considered.

## OBJECTIVE

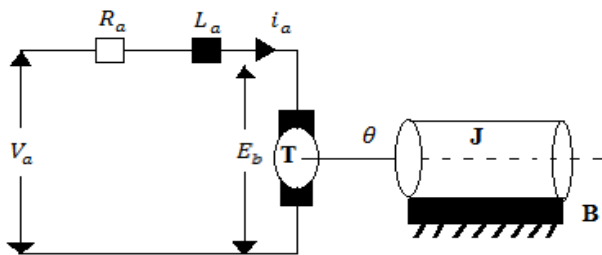
The objective is to come out with a simulation model for speed control of separately excited dc motor and analyzes its operation using PID and sliding mode control. The main reason is to select this controller in wide range area have acceptable control performance and solve two most important challenging topics in control which names, stability and robustness [4]. However, this controller second hand in wide range but, pure sliding mode controller has later disadvantages. Firstly, chattering problem; which can be source to high frequency oscillation of the controllers output. Secondly, sensitivity; this controller is extremely sensitive to the noise when the input signals very close to the zero. Last but not the least, nonlinear comparable dynamic formulation; which this problem is very important to have a good performance and it is difficult to calculation because it is depending on the nonlinear dynamic equation.

### I. STATE SPACE MODEL OF DC MOTOR

There are various types of dc motor. Depending on the type, a dc motor may be controlled by varying the input voltage or by changing the input current. In this paper, the separately excited dc motor model is chosen due to its good electrical and mechanical performances compared to other dc motor models. The separately excited dc motor is driven by applied armature voltage. A separately excited dc motor equivalent model is shown in fig. 2.



**Figure 1:** Symbols used to represent the field winding, armature winding and mechanical shaft of the dc motor



**Figure 2:** A Separately excited dc motor model

The dynamic of a separately excited dc motor may be expressed as:

$$\text{Motor Torque, } T = K_t i_a \quad \text{---(1)}$$

$$\text{Back emf, } E_b = K_b \omega_m \quad \text{---(2)}$$

$$\omega_m = \frac{d\theta}{dt} \quad \text{---(3)}$$

But using Kirchoff's voltage law,

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + E_b \quad \text{---(4)} \quad R_a i_a + L_a \frac{di_a}{dt} = V_a - K_b \omega_m$$

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = K_t i_a \quad J \frac{d\omega}{dt} + B \omega_m = K_t i_a \quad \text{---(5)}$$

Where,  $K_t$  is torque factor constant in Nm/A,  $K_b$  is the back emf constant in Vs/rad,  $i_a$  is the armature current in A,  $\omega_m$  is the angular speed in rad/s,  $V_a$  is the input terminal voltage (armature voltage) in volt,  $E_b$  is the back emf in volt,  $R_a$  is the armature resistance in ohm,  $L_a$  is the armature inductance in H,  $J$  is the moment of inertia of the motor in  $\text{kgm}^2/\text{s}^2$ ,  $T$  is the motor torque in Nm and  $B$  is the viscous friction coefficient in Nms.

Equations (4) and (5) are rearranged to obtain-

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{K_b}{L_a} \omega_m + \frac{V_a}{L_a} \quad \text{---(6)}$$

$$\frac{d\omega}{dt} = \frac{K_t}{J} i_a - \frac{B}{J} \omega_m \quad \text{---(7)}$$

In the state space model of a separately excited dc motor, equations (6) and (7) can be expressed by choosing the angular speed ( $\omega_m$ ) and armature current ( $i_a$ ) as state variables and the armature voltage ( $V_a$ ) as an input. The output is chosen to be the angular speed.

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_t}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_a} \end{bmatrix} V_a \quad \& \quad y = [0 \quad 1] \begin{bmatrix} i_a \\ \omega \end{bmatrix} \quad \text{---(8)}$$

Using Laplace Transform in equation (4) and (5),

$$J s^2 \theta(s) + b s \theta(s) = K_t I_a(s) \quad \text{---(9)}$$

$$L_a s I_a(s) + R_a I_a(s) = V_a(s) - K_b s \theta(s) \quad \text{---(10)}$$

From equation (7),

$$I_a(s) = \frac{V_a(s) - K_b s \theta(s)}{R_a + L_a s} \quad \text{---(11)}$$

where,  $s$ =Laplace operator

Substituting equation (11) in equation (9)

$$J s^2 \theta(s) + b s \theta(s) = \frac{K_t [V_a(s) - K_b s \theta(s)]}{(R_a + L_a s)}$$

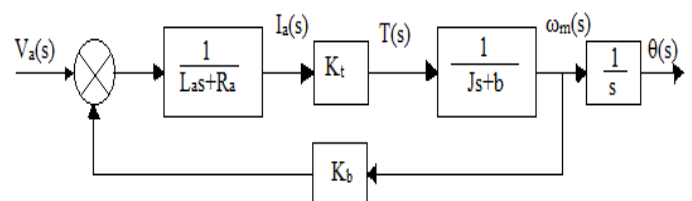
$$J s^2 \theta(s) * (R_a + L_a s) + b s * (R_a + L_a s) * \theta(s) = K_t [V_a(s) - K_b s \theta(s)]$$

$$J s^2 (R_a + L_a s) \theta(s) + [b s (R_a + L_a s) + K_t K_b s] \theta(s) = K_t V_a(s)$$

$$\theta(s) * s [(R_a + L_a s) * (J s + b) + K_t K_b] = K_t V_a(s)$$

$$G_a(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_t}{\{s[(R_a + L_a s) * (J s + b) + K_t K_b]\}} \quad \text{---(12)}$$

$$G_v(s) = \frac{s \theta(s)}{V_a(s)} = \frac{\omega(s)}{V_a(s)} = \frac{K_t}{[(R_a + L_a s) * (J s + b) + K_t K_b]} \quad \text{---(13)}$$



**Fig. 3.** Closed Loop System that Representing the dc Motor

Calculate the torque constant,  $K_t$ ;

$$\omega_m = \frac{V_t}{K_t} = \frac{2\pi N}{60} \quad \text{---(14)}$$

$$\frac{220}{K_t} = \frac{2\pi * 1500}{60} \quad K_t = 1.4 \text{ Nm/A}$$

$$\text{For } \omega = \frac{d\theta}{dt}, \quad K_t i_a(t) = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt}, \quad K_t i_a(t) = J \frac{d\omega}{dt} + B\omega$$

At the steady state, (used as analyzed data), both  $i_a(t)$  and  $\omega$  are stabilized.

$$\frac{d\omega}{dt} = 0 \quad \& \quad T = \frac{P}{\omega} = \frac{370}{157} = 2.35 \text{ Nm.}$$

$$\text{and } B = \frac{K_t i_a(t)}{\omega} = \frac{1.4 \times 2.3}{157} = 0.02 \text{ Nm/s}$$

**Table I:** Parameters of the separately excited dc motor

Parameters	Values	
	Section I	Section II
Armature resistance, Ra	13.5 Ω	0.2 Ω
Armature inductance, La	132.5 H	4 H
Moment of inertia, J	0.01 Kg.m <sup>2</sup>	0.01 Kg.m <sup>2</sup>
Viscous friction coefficient, B	0.02 Nms	0.0224 Nms
Back emf constant, Kb	1.4 Vsec/rad.	1.465 Vsec/rad.
Torque factor constant, Kt	1.4 Nm/A	1.465 Nm/A
Power, P	0.37 KW	0.43 KW
Speed, N	1500 r.p.m.	1500 r.p.m.
Supply voltage, Vt	220V	220V
Armature Current, ia	2.3 A	2.14

Table I lists the numerical values for the parameters of the separately excited dc motor studied in this paper.

### SLIDING MODE DESIGN IN VSC

From the sixties Emel'yanov and Taran 1962; Emel'yanov, 1970; Utkin, 1974 were first to discuss about the sliding mode control of variable structure control (VSC). The basic idea about the sliding mode control in unstable structure system did not appear outside the book of Itkis (Itkis, 1976) and a survey paper of Utkin (Utkin 1977) in english version. Including non-linear system, MIMO systems, discrete time models, large scale and infinite-dimensional system, sliding mode control has developed into a general design control method applicable to a wide range of system types. In recent years, uncertain dynamical systems have the dispute of controlling in increasingly form. There always is a difference in the actual plant and its mathematical model used for the controller design in the practical control problem. The mismatches are come from unknown external disturbances, plant parameters and unmodeled dynamics. The control law is designed that provides the desired performance to the control system in presence of these disturbances/uncertainties is a very challenging task for a

control engineer. This has led to an intense interest in the development of so called robust control methods which are supposed to solve this problem. One particular approach to robust controller design is called sliding mode control technique.

In control theory, sliding mode control or SMC is a non-linear control method that modify the dynamics of a non-linear system by the application of an irregular control signal that forces the system to 'slide' along a cross-section of the system usual behavior. The state feedback control law is not a continuous function of time rather it can switch from one continuous structure to another based on the current position in the state space. Hence sliding mode control is a form of variable structure control. The multiple control structures are designed in such a way that the trajectory always move towards a switching condition so that system does not remain confined within one system structure, instead it slides along the boundary of different control structures. The motion of the system as it slides along the boundary of these systems is sliding mode and the geometrical locus consisting of these boundaries is called the sliding surface or hyper surface [6].

Consider a linear time invariant  $n^{\text{th}}$  order plant with scalar control

$$\dot{x} = Ax(t) + Bu(t) \quad \text{---(15)}$$

where, matrix A of size  $n \times n$  defines the system transformation matrix and vector B of size  $n \times 1$  input vector. The sliding surface is defined as

$$s = Cx \quad \text{---(16)}$$

$$s = [c_1 \quad c_2 \quad c_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The vector 'C' ( $C > 0$ ) consist of coefficients that describe the sliding surface in terms of the state vector x. The sliding surface 's' is defined such a way called a hype surface, i.e., it is one dimension lesser than the system order. The surface need not be a plane (or line in case of second order system) always, the surface can be of any shape.

If the sliding surface is a plane then the gradient of the matrix is the matrix itself. The value of 's' specifies the distance of the point from the sliding surface. Hence  $s = 0$  implies the point is on the sliding surface. Differentiating Eq. (16) and the substituting in equation (15),

$$\dot{s} = CAx(t) + CBu(t) \quad \text{---(17)}$$

We get for the sliding to exist when  $s = 0$ , this gives us the equivalent input. Assuming that CB is invertible we get

$$u_{eq} = -(CB)^{-1}CAx(t) \quad \text{---(18)}$$

According to concept of equivalent control, substituting the equivalent input into the system equation (5), and we get an autonomous system that describes the motion of the describing

point on the sliding surface.

$$\dot{x} = \{I - B(CB)^{-1}C\}Ax(t) \quad \text{---(19)}$$

To check the stability of the sliding surface, one can use the Lyapunov second method of determining stability is commonly taken. In order to provide the asymptotic stability of Eq. (12) about the equilibrium point  $s=0$ .

$$V = \frac{1}{2}s^2 \quad \text{---(20)}$$

The following conditions must be satisfied.

$$(a) \dot{V} < 0 \quad (b) \lim_{|s| \rightarrow \infty} V = \infty$$

The condition (b) is obviously satisfied. In order to achieve finite time convergence, condition (a) can be modified to be

$$\dot{V} \leq -\alpha V^{1/2}, \quad \alpha > 0 \quad \text{---(21)}$$

Integrating inequality of Eq. (14), over the time interval  $0 \leq \tau \leq t$

$$V^{1/2}(t) \leq -\frac{1}{2}\alpha t + V^{1/2}(0) \quad \text{---(22)}$$

Consequently,  $V(t)$  reaches zero in a finite time  $t_r$  that can be bounded by,

$$t_r \leq \frac{2V^{1/2}(0)}{\alpha} \quad \text{---(23)}$$

The derivative of V is

$$\dot{V} = s\dot{s} = s(cx_2 + f(x_1, x_2, t) + u) \quad \text{---(24)}$$

Assuming

$u = -cx_2 + v$ , substituting it into Eq. (24). Therefore,

$$\dot{V} = s(f(x_1, x_2, t) + v) = sf(x_1, x_2, t) + sv \leq sL + sv \quad \text{---(25)}$$

Selecting  $v = -\rho \text{sign}(s)$ ,  $\rho > 0$  and substituting into Eq. (25)

$$\dot{V} \leq |s|L - |s|\rho = -|s|(\rho - L) \quad \text{---(26)}$$

Taking Eq. (26) in Eq. (21), condition Eq. (24) can be rewritten as-

$$\dot{V} \leq -\alpha V^{1/2} = -\frac{\alpha}{\sqrt{2}}|s|, \quad \alpha > 0 \quad \text{---(27)}$$

Combining of Eqs. (26) and (27)

$$\dot{V} \leq -|s|(\rho - L) = -\frac{\alpha}{\sqrt{2}}|s| \quad \text{---(28)}$$

Finally, the control gain  $\rho$  is-

$$\rho = L + \frac{\alpha}{\sqrt{2}} \quad \text{---(29)}$$

Consequently a control law  $u$  that drives  $s$  to zero in finite time of Eq. (17) is-

$$u = -cx_2 - \rho \text{sign}(s) \quad \text{---(30)}$$

$s\dot{s} \leq -\frac{\alpha}{\sqrt{2}}|s|$  and is named as reachability condition and

could be used for sliding mode controller design. There is no specific method to find the Lyapunov function candidate however V.I. Utkin [1] has discussed the method of using quadratic forms to find the sliding domain.

## II. SIMULATION RESULTS

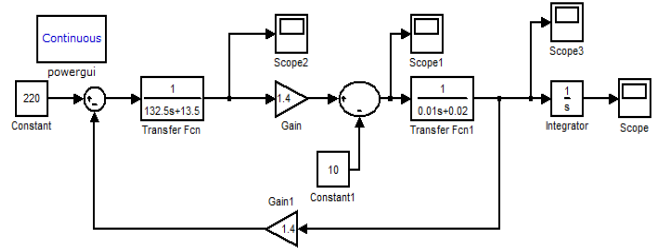


Figure 4: Simulink Model of dc Motor without PID Controller

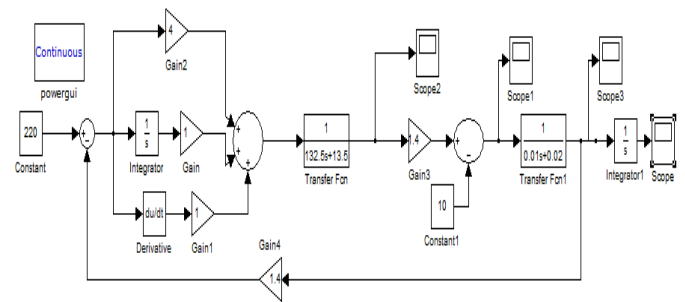


Figure 5: Simulink Model of dc Motor with PID Controller

### Section I:

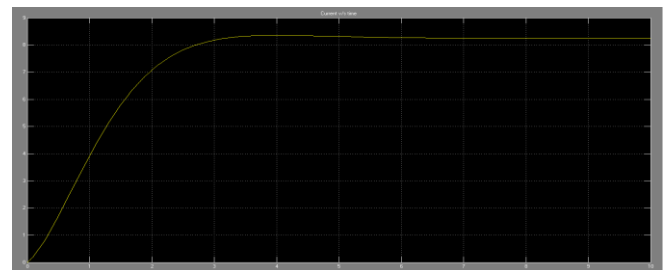


Figure 6: Current v/s time response for dc motor without controller

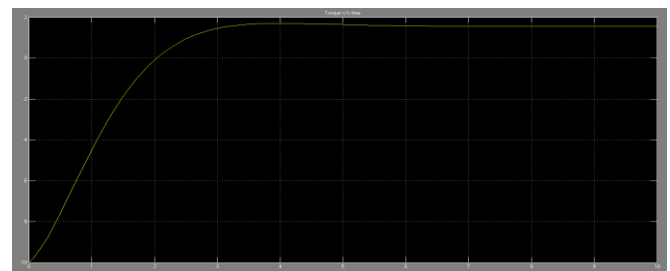
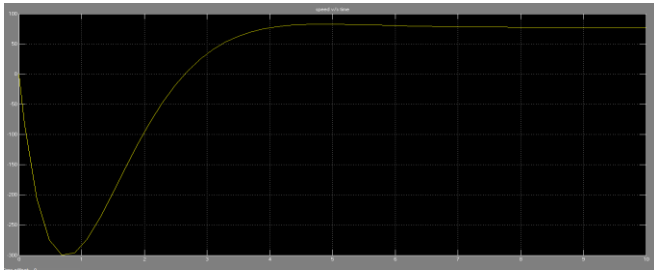
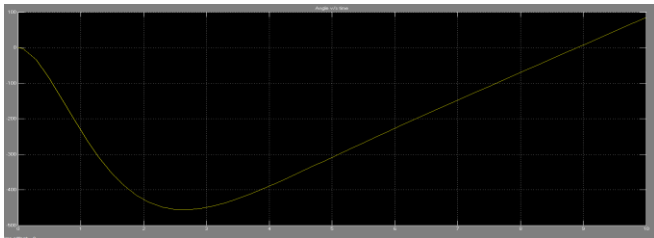


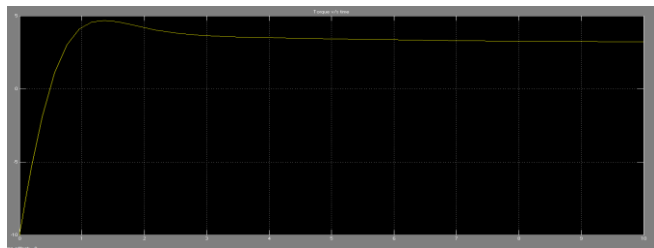
Figure 7: Torque v/s time response for dc motor without controller



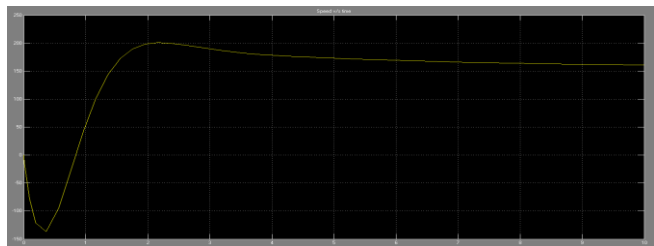
**Figure 8:** Speed v/s time response for dc motor without controller



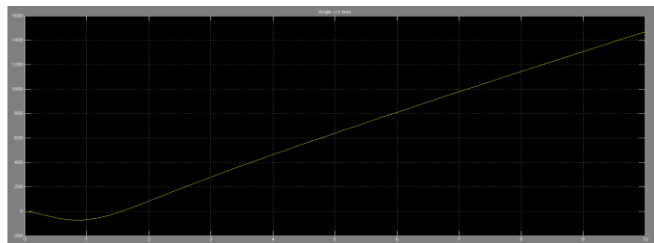
**Figure 9:** Angle v/s time response for dc motor without controller



**Figure 11:** Torque v/s time for dc motor with PID controller

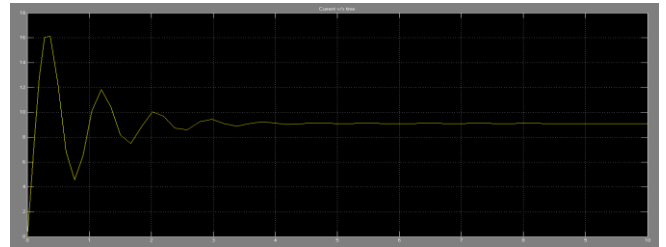


**Figure 12:** Speed v/s time for dc motor with PID controller

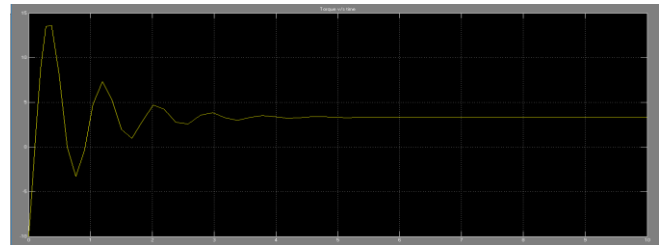


**Figure 13:** Angle v/s time for dc motor with PID controller

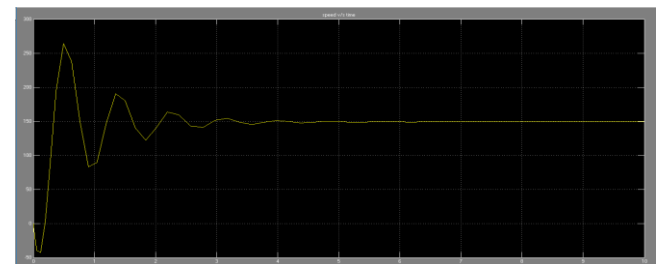
## Section II:



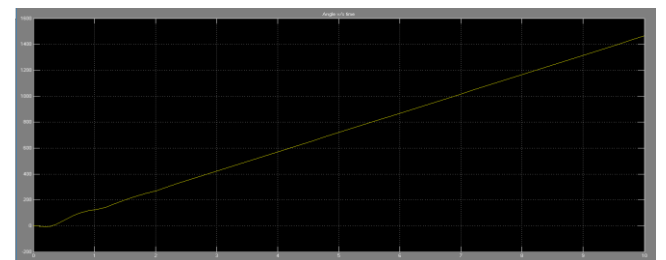
**Figure 14:** Current v/s time response for dc motor without controller



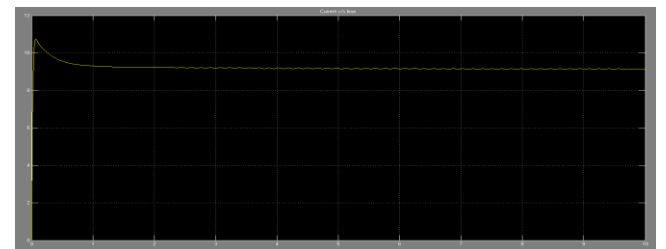
**Figure 15:** Torque v/s time response for dc motor without controller



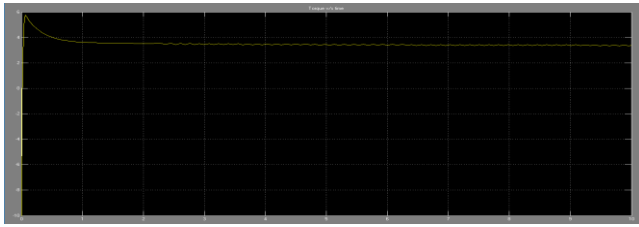
**Figure 16:** Speed v/s time response for dc motor without controller



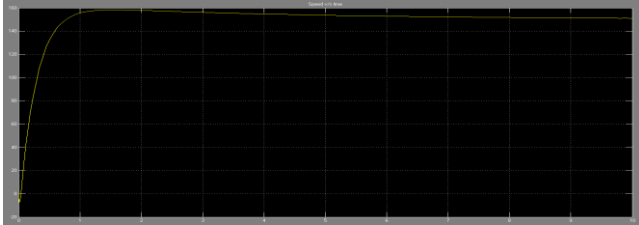
**Figure 17:** Angle v/s time response for dc motor without controller



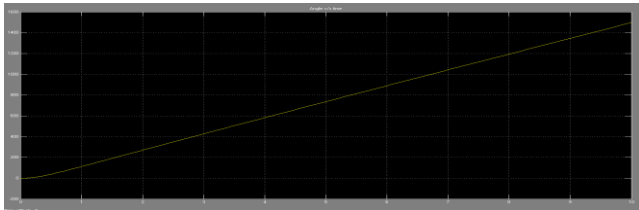
**Figure 18:** Current v/s time for dc motor with PID controller



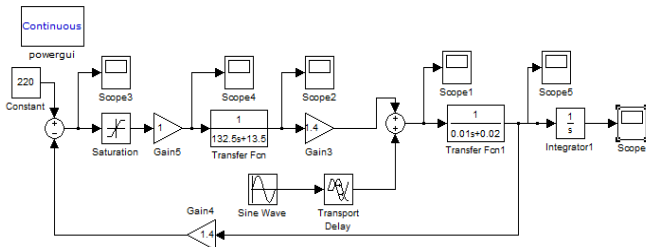
**Figure 19:** Torque v/s time for dc motor with PID controller



**Figure 20:** Speed v/s time for dc motor with PID controller

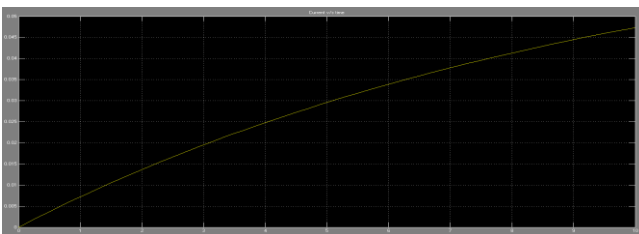


**Figure 21:** Angle v/s time for dc motor with PID controller

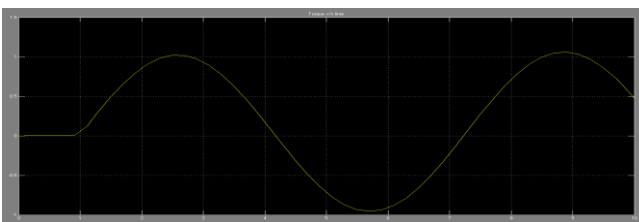


**Figure 22:** Simulink Model of dc Motor with SMC Controller

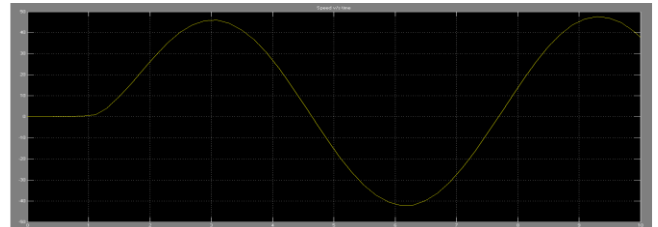
**Section I:**



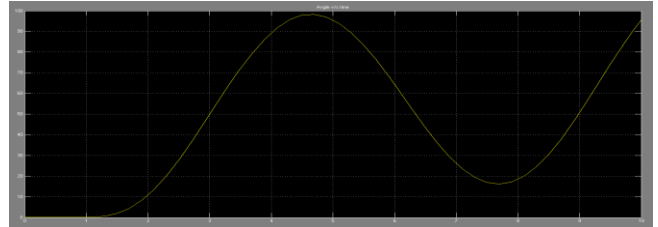
**Figure 23:** Current v/s time for dc motor with SMC



**Figure 24:** Torque v/s time for dc motor with SMC

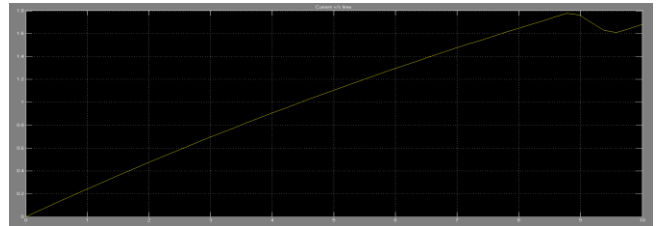


**Figure 25:** Speed v/s time for dc motor with SMC

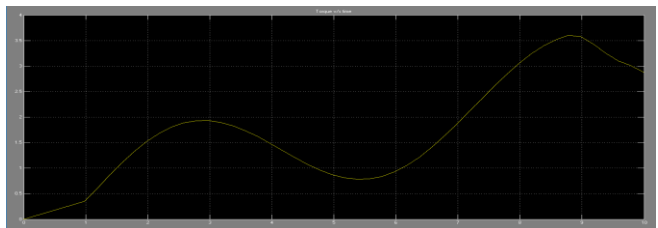


**Figure 26:** Angle v/s time for dc motor with SMC

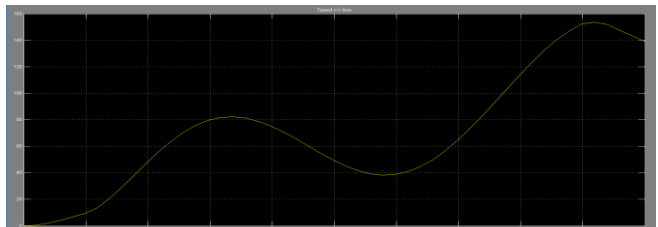
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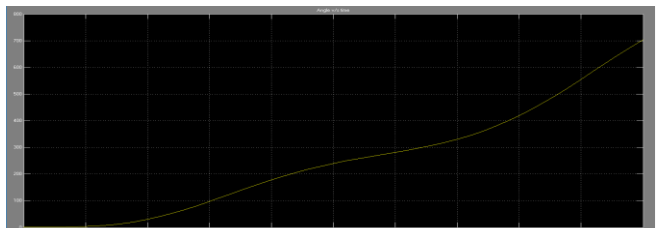
**Figure 27:** Current v/s time for dc motor with SMC



**Figure 28:** Torque v/s time for dc motor with SMC



**Figure 29:** Speed v/s time for dc motor with SMC



**Figure 30:** Angle v/s time for dc motor with SMC

## CONCLUSION

In this paper, we have considered the PID controller and sliding mode control for controlling the angle, speed, torque, armature current of a separately excited dc motor. The performance of the controllers was validated through simulations. From the comparative simulation results, one can conclude that the two controllers demonstrated nearly the same dynamic behavior under nominal condition. However, simulation results show that the sliding mode controller realized a good dynamic behavior of the motor with a rapid rise time and settling time and had better performance than the PID controller. But the comparison between the speed control of a separately excited dc motor by the sliding mode controller and PID controller showed clearly that the sliding mode controller gives better performance than the PID controller against parameter variations.

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