

High Dimensional Quality Control Chart: A Case Study of “Baju Kurung” Manufacturing Industry

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Abstract

There is a need for quality control chart that can cater for high dimension data set ($n < p$) in a real world application due to either limited number of productions (n) but high number of measurements of a product (p) or to minimize the cost for products quality testing. Therefore, monitoring the quality process control in the case of dimension (p) is large relative to the number of sample (n) is a crucial part in statistical process control study. In this paper we propose S* control chart to do such analysis in multivariate process variability monitoring for “baju kurung” manufacturing industry in Malaysia. A case study where $p=7$ and $n=5$ is presented via control chart to illustrate the advantage of the proposed method. The findings reveal that the high dimensional quality control chart can assist in differentiating between in-control and out-of-control signals easily via multivariate statistical process control chart.

Keywords: control chart, covariance test, garment industry, high dimension, case study.

INTRODUCTION

The concept of quality has existed for many years. From statistical point of view, quality is defined as the reciprocal of variance. The smaller the variance is the higher the quality and the larger the variance is the lower the quality^{1,2}. In order to reduce the variance, the statistical process quality control can be applied in our production process³.

In classical theory, Box’s M statistics, Jennrich statistics and generalized variance statistics are among statistical tools to monitor the statistical quality control on the basis of variance covariance matrix. These methods are relevant when concerning large sample size ($n > p$)⁴⁻⁷.

Yet, in multivariate statistical quality control, p is not always small as compared to sample size, n ($n < p$). The details of discussion can be clearly seen in Sharif⁸, Sharif⁹, and Djauhari and Herwindiati¹⁰.

What will happen if $n > p$? This condition makes the computation of those statistical tests burdensome because the

computational efficiency of the determinant and inverse of covariance matrix becomes low^{6,7,12-14}

In manufacturing industry, the issue ($n < p$) closely related with small number of productions (n) but high number of measurements of a product (p) or the aims of the factory to minimize the cost for products quality testing. In this case, the quality engineer has to minimize the sample, n chosen for checking the defect (or non-defect) but large number of measurement, p has to be considered. One of the examples is to determine the quality of rubber glove where it involves many types of measurement (p) such as the size of each finger, the strength, number of holes and etc. Another example, in garment industry, there is five measurements (p) of trousers; waist girth, hip girth, thigh, lowest part, and inseam. In both examples, we have to be aware that there is multi-number of quality characteristics involved for checking the defect (or non-defect) of products.

CASE STUDY OF “BAJU KURUNG” QUALITY TESTING

In Malaysia, “baju kurung” is one of the traditional clothing that must be owned by all the Malay women. It is not only used for office wear but “baju kurung” is also the preferred choice by women to look stylish for any ceremonies such as wedding receptions and celebrations.

In the present day, “baju kurung” is not only favored by Malay, but by Chinese and Indians as well. Consequently, the increasing demand for “baju kurung” has triggered many factories to produce ready-made “baju kurung”. In order to monitor the production process so that the quality of the garments at their best, it is essential to perform quality testing on several samples of “baju kurung”.

However, the challenges of conducting quality testing for “baju kurung” is due to many measurements (or dimensions, p) but limited samples (n). This constraint is imposed in order to minimize the cost and time. In this paper, we propose a new control chart based on S* test to handle high dimension data set ($n < p$)¹⁵ known as S* control chart.

The outline the remainder of this paper is as follows. In section 3, the data preparation for analysis is defined. In section 4, an empirical results based on S* control chart proposed by Sharif¹⁵ is discussed. Finally, the concluding remarks are presented in section 5 .

DATA PREPARATION

In this section, a case study on production process of “baju kurung” at one of the garment industries is presented. There are 7 variables ($p = 7$) that determine the quality of “baju kurung” namely, Shoulder (X_1), Sleeve Length (X_2). Cuff Width (X_3), Armhole (X_4), Underarm(X_5), Width (X_6) and Length (X_7), see Figure 1.

In order to test the advantage of S* control chart, Five-week data were collected in illustrating the advantage of S* control chart. Each week, there are two batches of productions. Therefore, the number of independent subgroups is $m = 10$ (5 weeks \times 2 batches). For every subgroup, we selected five samples of “baju kurung” ($n = 5$).

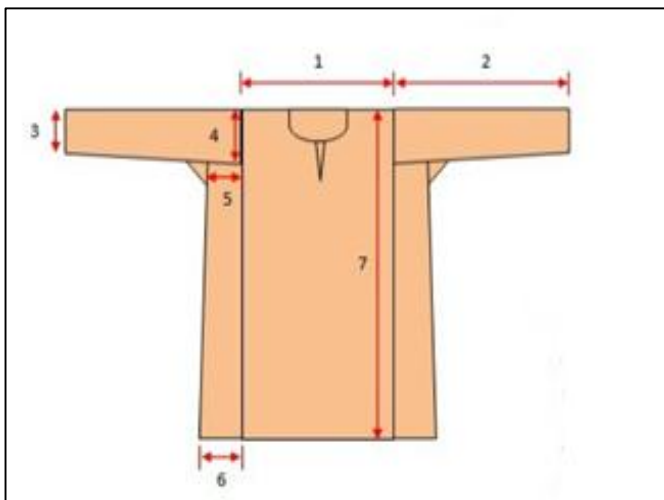


Figure 1: Technical drawing of “baju kurung”

EXPERIMENTAL RESULTS

The initial step to obtain the experimental results is computing the covariance matrix of each subgroup where the first covariance is the reference sample to the others. As a result, the following reference sample covariance matrix of 7×7 is attained.

0.027						
0.000	0.250					
-0.005	0.063	0.027				
0.002	-0.025	-0.011	0.004			
0.004	-0.025	-0.003	0.001	0.005		
0.001	0.000	0.001	0.000	0.001	0.000	
-0.005	0.038	0.013	-0.006	-0.003	0.000	0.017

Next, based on the multivariate control chart approach, we conducted a repeated test

$$H_0: \Sigma_i = \Sigma_R \text{ versus } H_1: \Sigma_i \neq \Sigma_R.$$

The above hypothesis implies that we have to compare the second, third and the rest with the first covariance matrix using the statistical test defined as below

$$S^* = A^t S_p^{-1} A \tag{1}$$

where

$$A = [vec(S_{i,L}) - vec(S_{R,L})]$$

$$S_p = M^t (I_{p^2} + K) (S_{R,L} \otimes S_{R,L}) M$$

$$M = (m_{ij}) = \begin{cases} 1; & (i, j) = (C_2^a + b, b) \text{ for } b = 1, 2, \dots, a \\ 0; & \text{elsewhere} \end{cases}$$

To compute the equation (1), we begin with identifying M. In general, the duplication matrix M of size $(k \times p^2)$ can be presented in matrix form as a block matrix.

$$M = (M_1 | M_2 | \dots | M_p)$$

where $k = \frac{1}{2}p(p + 1)$ and this matrix can be partition into p blocks. M_1 is a matrix with the first element is equal to 1 and the other elements are zero.

After that, we transform all the covariance elements into the vector space, and then we choose only the lower element of that matrix for next computational $vec(S_{1,L})$. Generally, covariance matrix is a symmetric matrix where the lower and the upper element of matrix consists the same elements (duplication). By removing the redundant elements, Sharif show that singularity problem can be solved⁷. All the statistical result is then computed for the remainder of 9 sub-groups.

Table 1 and Figure 2 present the results. Based on Table 1, the S* test values varies among sub groups. The upper control limit

is calculated using chi-square upper control limit (UCL) with $[7 \times (7 - 1)/2] = 21$ degree of freedom. Since it is a chi-square the lower control limit (LCL) is zero (0). Figure 2 displays the S* control chart which easier to visualize the results rather than in table form.

Table 1: Statistical test using S* test

Subgroup	S* Test	Upper Control Limit
1	60.4	73.3
2	101.5	73.3
3	65.4	73.3
4	30.0	73.3
5	27.2	73.3
6	24.7	73.3
7	77.5	73.3
8	38.7	73.3
9	181.9	73.3

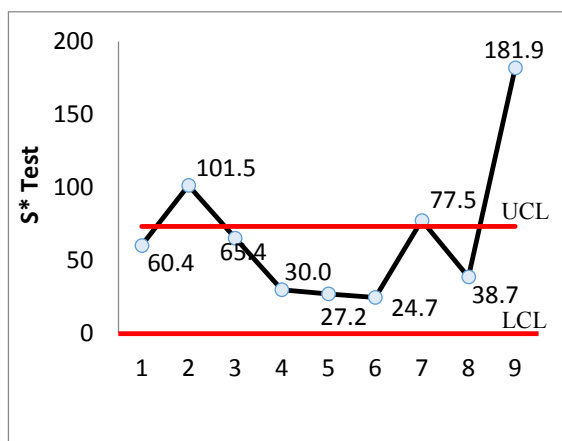


Figure 2: S* control chart for “baju kurung”

From Figure 2, the second, seventh and ninth subgroups are out-of-control (defect signal). This indicates all the “baju kurung” in these subgroups needs to be measured in identifying the defects and also investigate the cause of the defects in ensuring only quality “baju kurung” are produced.

CONCLUDING REMARKS

In this paper, we have successfully shown that S* control chart can be employed in detecting defect items with high dimension data set ($n < p$) particularly in monitoring the quality of “baju kurung”.

One of the significant strengths of S* control chart is its ability of using small sample size in constructing the chart. It is also

easy to interpret either in-control or out-of-control when all the statistical results can be visualized. Thus, these advantages display the crucial role of S* control chart as high dimensional quality control chart.

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