

# Optimal Power Allocation and Performance Analysis of Cooperative Multicast Systems Using Non-Orthogonal Multiple Access

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**Abstract:** For cooperative multicast systems using non-orthogonal multiple access, we propose optimal power allocation to maximize the achievable rate, and present its performance analysis in terms of average achievable rate and outage probability assuming Rayleigh fading channels. To perform the optimal power allocation, the source requires instantaneous channel information for a subset of the links. In the performance analysis, we provide approximate expressions for the average achievable rate and outage probability of the optimal power allocation using high signal-to-noise ratio (SNR) approximation. From the high SNR performance analysis, we show that the performances for the optimal power allocation are not affected by the number of destinations, while those for the fixed power allocation become worse as the number of destinations increases, which can be a remarkable finding although the optimal power allocation requires the overhead for the channel information feedback and the complexity for computing the power allocation coefficients.

**Keywords:** Cooperative multicast system, non-orthogonal multiple access, decode-and-forward relaying, power allocation, Rayleigh fading channels.

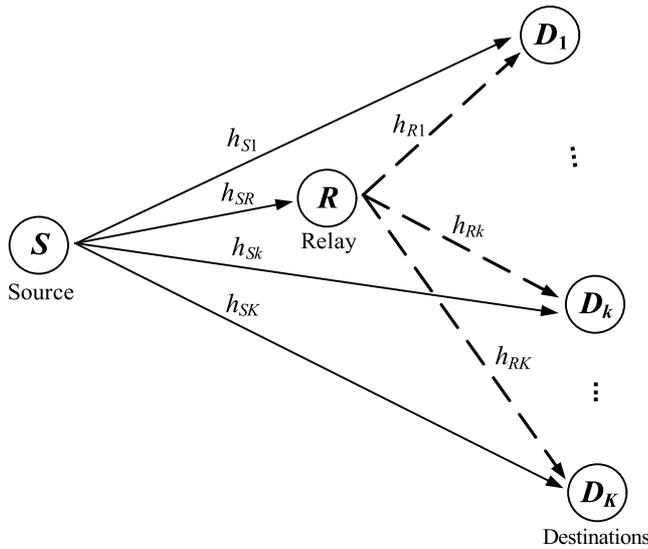
## INTRODUCTION

Multimedia broadcast and multicast service has attracted great attention in cellular systems [1]. Recently, cooperative multicast transmission with the help of relays has been

focused to cope with fading, shadowing, and path loss effects in wireless channels [2], [3]. In [2], the outage probability and power allocation of cooperative multicast systems (CMSs) with amplify-and-forward relaying have been investigated for Rayleigh fading channels, and in [3] the outage probability of decoded-and-forward based CMSs using the best relay selection has been studied for Rayleigh fading channels.

Non-orthogonal multiple access (NOMA) allowing the simultaneous transmission of multiple data signals [4] has been recently introduced into cooperative communication systems to improve their spectral efficiency, e.g., coordinated multi-point systems using NOMA [5], cooperative NOMA systems using maximum ratio combining (MRC) [6], and cooperative relaying systems using NOMA [7], [8].

In this paper, we introduce the NOMA scheme into CMSs, which may be a natural extension to enhance the spectral efficiency, and propose optimal power allocation to maximize the achievable rate of the CMSs using NOMA. We also present its performance analysis in terms of average achievable rate and outage probability for independent Rayleigh fading channels, where the source requires instantaneous channel information for a subset of the links (i.e., the source-to-relay and relay-to-destination links) to perform the optimal power allocation. In the performance analysis, we provide approximate expressions for the average achievable rate and outage probability of optimal power allocation using high signal-to-noise ratio (SNR) approximation. From the high SNR performance analysis, it is shown that the performances for optimal power allocation are the functions of only SNR and the average channel power for the source-to-relay link and are not affected by



**Figure 1.** A cooperative multicast system using NOMA, where the solid and dashed lines represent signal transmissions for the first and the second time slots, respectively.

the number of destinations. Further, through numerical investigation, the average rate and outage probability of CMS using NOMA with optimal power allocation are compared to those of CMS using NOMA with fixed power allocation as well as the conventional CMS, respectively.

## SYSTEM MODEL

We consider the CMS with  $K$  destinations ( $D_1, D_2, \dots, D_K$ ), as shown in Fig. 1, where a source (S) transmits multicast signals to the  $K$  destinations directly and through a relay (R). In the CMS, the NOMA protocol [4] is employed to enhance the spectral efficiency. In the CMS using NOMA, the  $K$  destinations receive two independent multicast data symbols during two time slots and achieve the sum rate for the two data symbols, whereas in the conventional CMS, the destinations obtain only a single multicast data symbol during two time slots. It is noted that, in CMS using NOMA, all the destinations desire the same multicast data, and the multicast data transmissions succeed only when every destination successfully decodes the received signals from the source and the relay. In this paper, we assume that all the destinations are simple receivers that do not require complicated processing for decoding such as MRC and successive interference cancellation (SIC).

In the CMS using NOMA, the source sends  $\sqrt{P_t a_1} s_1 + \sqrt{P_t a_2} s_2$  to the relay and the  $K$  destinations for the first time

slot, where  $P_t$  denotes the total transmit power,  $s_i$  denotes the  $i$ -th multicast data symbol with a unit average power, and  $a_i$  represents the power allocation coefficient for symbol  $s_i$ . We assume that  $a_1 + a_2 = 1$  and  $a_1 > a_2$ . Then, the relay decodes and cancels symbol  $s_1$  with SIC, and hence can decode symbol  $s_2$ . On the other hand, the destinations decode symbol  $s_1$  regarding symbol  $s_2$  as noise. For the second time slot, then only the relay sends the decoded symbol  $s_2$  with power  $P_t$  to the  $K$  destinations.

The channel coefficients of S-to-R, S-to- $D_k$ , and R-to- $D_k$  links are denoted as  $h_{SR}$ ,  $h_{Sk}$ , and  $h_{Rk}$ , respectively, and assumed to be independent complex Gaussian random variables with mean zero, i.e., their magnitudes are independent Rayleigh random variables. Then, the received signals at the relay and the  $k$ -th destination during the first time slot are respectively given as

$$r_{SR} = h_{SR}(\sqrt{P_t a_1} s_1 + \sqrt{P_t a_2} s_2) + n_{SR}, \quad (1)$$

and

$$r_{Sk} = h_{Sk}(\sqrt{P_t a_1} s_1 + \sqrt{P_t a_2} s_2) + n_{Sk}, \quad (2)$$

where  $n_{SR}$  and  $n_{Sk}$  represent additive white Gaussian noise with variance  $\sigma^2$  at the relay and the  $k$ -th destination, respectively. Using (1), the received SNRs of symbol  $s_1$  and  $s_2$  at the relay are respectively

$$\gamma_{SR}^{s_1} = \frac{|h_{SR}|^2 \rho a_1}{|h_{SR}|^2 \rho a_2 + 1}, \quad (3)$$

and

$$\gamma_{SR}^{s_2} = |h_{SR}|^2 \rho a_2, \quad (4)$$

where  $\rho = P_t / \sigma^2$ , and  $\gamma_{SR}^{s_2}$  in (4) is obtained after cancellation of symbol  $s_1$ . Using (2), the received SNR of symbol  $s_1$  at destination  $k$  is

$$\gamma_{Sk}^{s_1} = \frac{|h_{Sk}|^2 \rho a_1}{|h_{Sk}|^2 \rho a_2 + 1}. \quad (5)$$

For the second time slot, the signal received by destination  $k$  from the relay is given as

$$r_{Rk} = h_{Rk} \sqrt{P_t} s_2 + n_{Rk}, \quad (6)$$

where  $n_{Rk}$  is additive white Gaussian noise with variance  $\sigma^2$  at the destination, and thus the received SNR for symbol  $s_2$  is

$$\gamma_{Rk}^{s_2} = |h_{Rk}|^2 \rho. \quad (7)$$

The achievable rate of multicast transmission is dominated by the worst link to minimize outage and retransmission [9].

Using (3) and (5), the achievable rate for symbol  $s_1$  in the CMS using NOMA is thus obtained as [7, eq. (8)]

$$C_M^{s_1} = \frac{1}{2} \min\{\log_2(1+\gamma_{SR}^{s_1}), \log_2(1+\gamma_{S1}^{s_1}), \dots, \log_2(1+\gamma_{SK}^{s_1})\}, \quad (8)$$

where it is noted that the relay should decode symbol  $s_1$  for SIC. As the achievable rate for decode-and-forward relaying is determined by the weakest link between S-to-R and R-to- $D_k$  links [10], using (4) and (7), the achievable rate for symbol  $s_2$  in the CMS using NOMA is obtained as [7, eq. (9)]

$$C_M^{s_2} = \frac{1}{2} \min\{\log_2(1+\gamma_{SR}^{s_2}), \log_2(1+\gamma_{R1}^{s_2}), \dots, \log_2(1+\gamma_{RK}^{s_2})\}, \quad (9)$$

where the achievable rate of multicast transmission between the relay and the destinations is determined by the worst link. Using (8) and (9), the achievable sum rate in the CMS using NOMA is obtained as

$$C_M^{pro} = C_M^{s_1} + C_M^{s_2}. \quad (10)$$

## POWER ALLOCATION AND PERFORMANCE ANALYSIS

Let the average power of  $h_{SR}$  be denoted as  $\beta_{SR}$ , and the average powers of  $h_{Sk}$  and  $h_{Rk}$  be denoted as  $\beta_{Sk}$  and  $\beta_{Rk}$  for  $k = 1, 2, \dots, K$ , respectively. Mostly, the relay is located between the source and the destinations, and thus the S-to-R link may be better than the S-to- $D_k$  link in terms of path loss and shadowing effects. In this paper, hence, it is reasonably assumed that  $\beta_{Sk} < \beta_{SR}$  for all  $k$ . Hereafter, let  $\lambda_{SR} \triangleq |h_{SR}|^2$ ,  $\lambda_{Sk} \triangleq |h_{Sk}|^2$ , and  $\lambda_{Rk} \triangleq |h_{Rk}|^2$ .

Using (3)-(5) and (7),  $C_M^{pro}$  in (10) is expressed as

$$C_M^{pro} = \frac{1}{2} \log_2 \left( 1 + \frac{\min\{\lambda_{SR}, \lambda_{S1}, \dots, \lambda_{SK}\} \rho a_1}{\min\{\lambda_{SR}, \lambda_{S1}, \dots, \lambda_{SK}\} \rho a_2 + 1} \right) + \frac{1}{2} \log_2 \left( 1 + \min\{\lambda_{SR} a_2, \lambda_{R1}, \dots, \lambda_{RK}\} \rho \right) \quad (11)$$

Letting  $X \triangleq \min\{\lambda_{SR}, \lambda_{S1}, \dots, \lambda_{SK}\}$  and  $Y \triangleq \min\{\lambda_{R1}, \dots, \lambda_{RK}\}$ ,  $C_M^{pro}$  in (11) is rewritten as

$$C_M^{pro} = \frac{1}{2} \log_2(1 + X\rho) - \frac{1}{2} \log_2(1 + X\rho a_2) + \frac{1}{2} \log_2(1 + \min\{\lambda_{SR} a_2, Y\} \rho), \quad (12)$$

where we use that  $a_1 = 1 - a_2$ .

## Optimal Power Allocation

In this section, we derive an optimal power allocation coefficient,  $a_2^*$ , in order to maximize the achievable sum rate in (12). If  $\lambda_{SR} a_2 \leq Y$ , then  $C_M^{s_2} = \frac{1}{2} \log_2(1 + \lambda_{SR} \rho a_2)$ . Thus, when  $\lambda_{SR} a_2 \leq Y$ , the derivative of  $C_M^{pro}$  in (12) with respect to  $a_2$  is obtained as

$$\frac{dC_M^{pro}}{da_2} = -\frac{1}{2 \ln 2} \left( \frac{X\rho}{1 + X\rho a_2} \right) + \frac{1}{2 \ln 2} \left( \frac{\lambda_{SR} \rho}{1 + \lambda_{SR} \rho a_2} \right) = \frac{(\lambda_{SR} - X)\rho}{2 \ln 2 (1 + X\rho a_2)(1 + \lambda_{SR} \rho a_2)}. \quad (13)$$

As  $\lambda_{SR} \geq X$ ,  $\frac{dC_M^{pro}}{da_2} \geq 0$  when  $\lambda_{SR} a_2 \leq Y$ . If  $\lambda_{SR} a_2 > Y$ , then  $C_M^{s_2} = \frac{1}{2} \log_2(1 + Y\rho)$ . Hence, when  $\lambda_{SR} a_2 > Y$ , the derivative of  $C_M^{pro}$  in (12) with respect to  $a_2$  is obtained as

$$\frac{dC_M^{pro}}{da_2} = -\frac{1}{2 \ln 2} \left( \frac{X\rho}{1 + X\rho a_2} \right). \quad (14)$$

Thus,  $\frac{dC_M^{pro}}{da_2} < 0$  when  $\lambda_{SR} a_2 > Y$ . It means that  $C_M^{pro}$  is an increasing function of  $a_2$  for  $a_2 \leq \frac{Y}{\lambda_{SR}}$  but a decreasing function of  $a_2$  for  $a_2 > \frac{Y}{\lambda_{SR}}$ . Therefore, an optimal power allocation coefficient can be obtained as  $a_2^* = \frac{Y}{\lambda_{SR}}$ . However,  $a_2^* = 0.5$  when  $\frac{Y}{\lambda_{SR}} \geq 0.5$  because  $0 < a_2 \leq 0.5$  and  $C_M^{pro}$  increases with  $a_2$  for  $a_2 \leq \frac{Y}{\lambda_{SR}}$ . Thus, the optimal power allocation coefficient for the CMS using NOMA is obtained as

$$a_2^* = \begin{cases} \frac{Y}{\lambda_{SR}} & \text{for } \frac{Y}{\lambda_{SR}} < 0.5 \\ 0.5 & \text{for } \frac{Y}{\lambda_{SR}} \geq 0.5 \end{cases}. \quad (15)$$

From (15), it is noteworthy that the instantaneous channel information for the S-to-R and R-to- $D_k$  links, except for the S-to- $D_k$  link, for all  $k$  are required at the source for the optimal power allocation.

## Performance Analysis for Optimal Power Allocation

As the statistical analysis of (12) with the optimal power allocation in (15) may be intractable, using high SNR approximation, (12) is approximated as follows:

$$C_M^{pro} \underset{\rho \rightarrow \infty}{\sim} \frac{1}{2} \log_2 \left( \frac{1}{a_2} \right) + \frac{1}{2} \log_2 \left( \min\{\lambda_{SR} a_2, Y\} \rho \right) \triangleq A_C. \quad (16)$$

Inserting (15) into (16), the approximate sum rate for the optimal power allocation is obtained as

$$A_C^* = \begin{cases} \frac{1}{2} \log_2 \left( \frac{\lambda_{SR}}{Y} \right) + \frac{1}{2} \log_2(Y\rho) & \text{for } \frac{Y}{\lambda_{SR}} < 0.5 \\ \frac{1}{2} \log_2(2) + \frac{1}{2} \log_2 \left( \frac{\lambda_{SR}\rho}{2} \right) & \text{for } \frac{Y}{\lambda_{SR}} \geq 0.5 \end{cases}$$

$$= \frac{1}{2} \log_2(\lambda_{SR}) + \frac{1}{2} \log_2(\rho). \quad (17)$$

From (17), it is noted that the sum rate for the optimal power allocation is dependent on only the channel for the S-to-R link and the transmit SNR,  $\rho$ . Using the probability density function (PDF) of  $\lambda_{SR}$ ,  $f_{\lambda_{SR}}(x) = \frac{1}{\beta_{SR}} e^{-x/\beta_{SR}}$ , and [11, eq. (4.331.1)], the approximate average sum rate for the optimal power allocation is obtained as

$$\bar{A}_C^* = \frac{1}{2} \log_2(\rho) + \int_0^\infty \frac{1}{2} \log_2(x) f_{\lambda_{SR}}(x) dx$$

$$= \frac{1}{2} \log_2(\rho) - \frac{\log_2 e}{2} \left\{ E_C + \ln \left( \frac{1}{\beta_{SR}} \right) \right\}, \quad (18)$$

where  $E_C$  denotes the Euler constant. In addition, letting  $R_t$  be a rate threshold of the outage, and using the PDF of  $\lambda_{SR}$ , the approximate outage probability for the optimal power allocation is derived as

$$A_O^* = \Pr \{ A_C^* < R_t \} = \Pr \left\{ \lambda_{SR} < \frac{2^{2R_t}}{\rho} \right\} = 1 - e^{-\frac{2^{2R_t}}{\rho\beta_{SR}}}. \quad (19)$$

From (18) and (19), it is noted that in the high SNR regime, the performances for the optimal power allocation are dependent upon only  $\rho$  and  $\beta_{SR}$ , and become better as those increase.

### Performance Analysis for Fixed Power Allocation

In order to compare with the average rate and outage performance for the optimal power allocation, we derive the approximate average sum rate and outage probability for the fixed power allocation. Letting  $Z \triangleq \min\{\lambda_{SR}a_2, Y\}$  and using the PDF of  $\lambda_{Rk}$ ,  $f_{\lambda_{Rk}}(x) = \frac{1}{\beta_{Rk}} e^{-x/\beta_{Rk}}$ , the PDF of  $Z$  is given as

$$f_Z(x) = \left( \frac{1}{\beta_{SR}a_2} + \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right) e^{-x \left( \frac{1}{\beta_{SR}a_2} + \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right)}.$$

Using the approximate expression in (16) and the PDF of  $f_Z(x)$ , the approximate average sum rate for the fixed power allocation is then obtained as

$$\bar{A}_C = \frac{1}{2} \log_2 \left( \frac{1}{a_2} \right) + \frac{1}{2} \log_2(\rho) + \int_0^\infty \frac{1}{2} \log_2(x) f_Z(x) dx$$

$$= \frac{1}{2} \log_2(\rho) - \frac{\log_2 e}{2} \left\{ E_C + \ln \left( \frac{1}{\beta_{SR}} + a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right) \right\}, \quad (20)$$

where [11, eq. (4.331.1)] is used. Using (16) and the PDF of  $Z$ , the approximate outage probability with the outage threshold  $R_t$  for the fixed power allocation is obtained as

$$A_O = \Pr \{ A_C < R_t \} = \Pr \left\{ Z < \frac{2^{2R_t} a_2}{\rho} \right\}$$

$$= 1 - e^{-\frac{2^{2R_t}}{\rho} \left( \frac{1}{\beta_{SR}} + a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right)}. \quad (21)$$

From (20) and (21), it is noted that in the high SNR regime, the performances for the fixed power allocation rely on  $\rho$ ,  $K$ ,  $a_2$ ,  $\beta_{SR}$ ,  $\beta_{Rk}$  for all  $k$ , and become better as  $\rho$ ,  $\beta_{SR}$ , and  $\beta_{Rk}$ 's increase but worse as  $K$  and  $a_2$  rise. Hence, the optimal power allocation achieves more performance gain over the fixed power allocation as  $K$  and  $a_2$  (used for the fixed power allocation) increase and  $\beta_{Rk}$ 's decrease since the performances for the optimal power allocation depend on only  $\rho$  and  $\beta_{SR}$ , as seen in (18) and (19).

### Performance Gain Analysis of Optimal Power Allocation

**Theorem 1.** In high SNR regime, when  $K$  increases, the rate gain achieved by optimal power allocation over fixed power allocation increases with the scaling of  $\frac{1}{2} \log_2 K$ .

*Proof.* From (18) and (20), the rate gain becomes

$$\bar{A}_C^* - \bar{A}_C = \frac{1}{2} \log_2 \left( 1 + \beta_{SR} a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right)$$

$$\stackrel{(a)}{=} \frac{1}{2} \log_2 \left( 1 + K \frac{\beta_{SR} a_2}{\beta_{RD}} \right)$$

$$\sim \frac{1}{2} \log_2 K, \quad (22)$$

where (a) follows from the assumption that  $\beta_{Rk} = \beta_{RD}$  for all  $k$ , that is, the second hop channels are identically distributed. ■

**Theorem 2.** Let a target average rate be given as  $T_C$ , and the SNR to achieve the target average rate be  $\mu_O$  for optimal power allocation and  $\mu_F$  for fixed power allocation. Then, in high SNR regime, the SNR gain achieved by optimal power allocation over fixed power allocation is obtained as  $\frac{\mu_F}{\mu_O} =$

$$\left( 1 + \beta_{SR} a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right).$$

*Proof.* (18) and (20) are respectively rewritten as

$$\bar{A}_C^* = \frac{1}{2} \log_2 \left( \rho e^{-E_C} \beta_{SR} \right), \quad (23)$$

$$\bar{A}_C = \frac{1}{2} \log_2 \left( \rho e^{-E_c} \left( \frac{1}{\beta_{SR}} + a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right)^{-1} \right). \quad (24)$$

Using (23) and (24), the SNRs,  $\mu_O$  and  $\mu_F$ , to achieve a given target average rate,  $T_C$ , are respectively obtained as

$$\mu_O = \frac{2^{2T_C} e^{E_c}}{\beta_{SR}}, \quad (25)$$

$$\mu_F = 2^{2T_C} e^{E_c} \left( \frac{1}{\beta_{SR}} + a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right). \quad (26)$$

Using (25) and (26), the SNR gain is then obtained as

$$\frac{\mu_F}{\mu_O} = 1 + \beta_{SR} a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}}. \quad (27)$$

**Theorem 3.** Let a target outage probability be given as  $T_O$ , and the SNR to achieve the target outage probability be  $\nu_O$  for optimal power allocation and  $\nu_F$  for fixed power allocation. Then, in high SNR regime, the SNR gain achieved by optimal power allocation over fixed power allocation is obtained as  $\frac{\nu_F}{\nu_O} = \left( 1 + \beta_{SR} a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right)$ .

*Proof.* Using (19) and (21), the SNRs,  $\nu_O$  and  $\nu_F$ , to achieve a given outage probability,  $T_O$ , are respectively obtained as

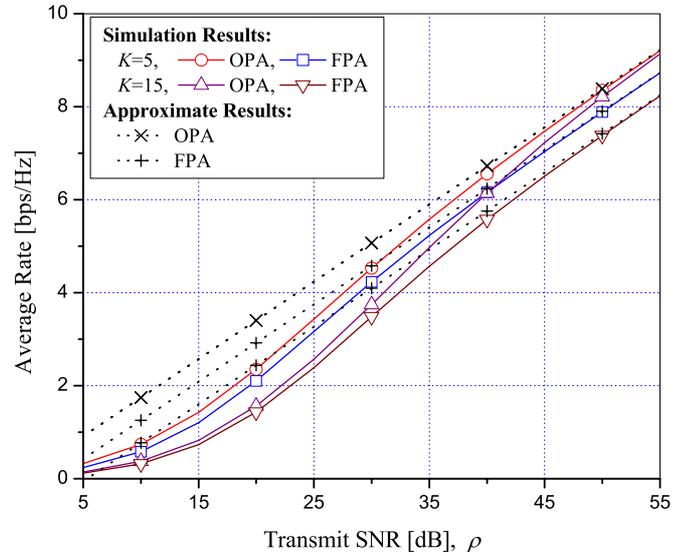
$$\nu_O = -\frac{2^{2R_t}}{\beta_{SR}} \left( \ln(1 - T_O) \right)^{-1}, \quad (28)$$

$$\nu_F = -2^{2R_t} \left( \frac{1}{\beta_{SR}} + a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}} \right) \left( \ln(1 - T_O) \right)^{-1}. \quad (29)$$

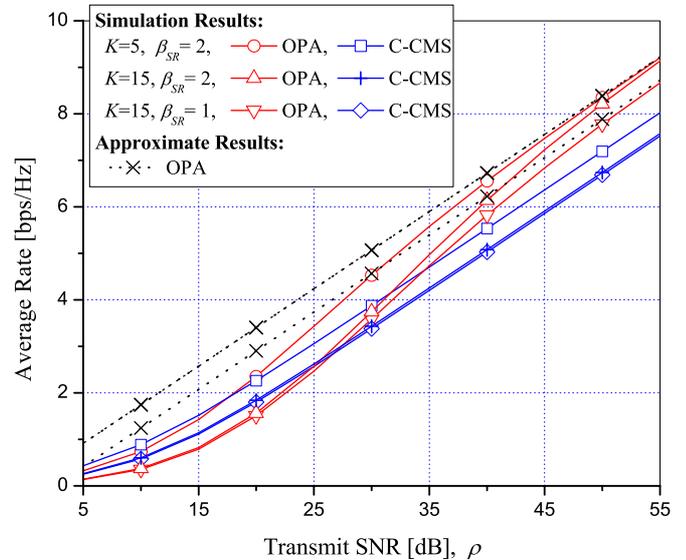
Using (28) and (29), the SNR gain is then obtained as

$$\frac{\nu_F}{\nu_O} = 1 + \beta_{SR} a_2 \sum_{k=1}^K \frac{1}{\beta_{Rk}}. \quad (30)$$

The results of Theorem 2 and Theorem 3 consistently show that, for high SNR, the SNR gain of optimal power allocation in comparison with fixed power allocation increases as  $\beta_{SR}$ ,  $K$ , and  $a_2$  (used for the fixed power allocation) increase and  $\beta_{Rk}$  decreases.



**Figure 2.** Average rates of N-CMS with OPA and FPA when  $\beta_{SR} = 2$ , where  $a_1 = 0.9$  and  $a_2 = 0.1$  for FPA.



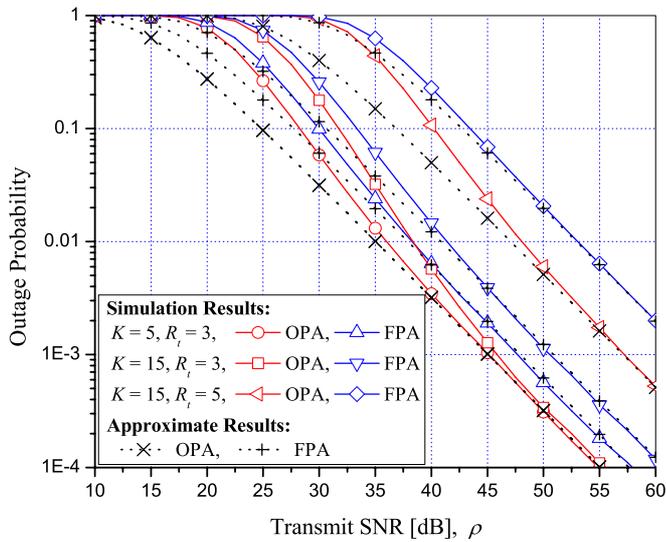
**Figure 3.** Average rates of N-CMS with OPA and C-CMS when  $\beta_{SR} = 1, 2$ .

## NUMERICAL RESULTS

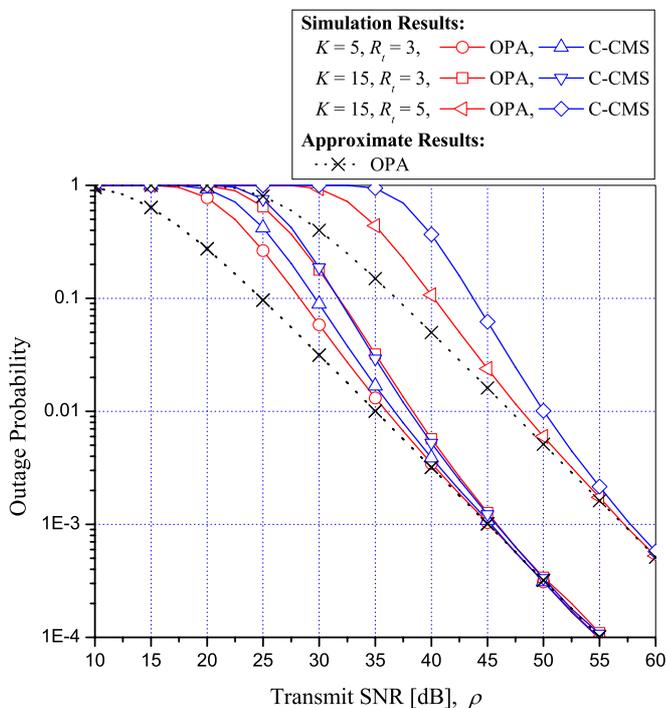
From [12, eq. (15)] and [3, eq. (2)], the achievable rate for the conventional CMS (C-CMS) with decode-and-forward relaying is given as

$$C_M^{con} = \frac{1}{2} \log_2 \left( 1 + \min\{\lambda_{SR}, \lambda_{S1} + \lambda_{R1}, \dots, \lambda_{SK} + \lambda_{RK}\} \rho \right). \quad (31)$$

To compare with the NOMA-based CMS (N-CMS), the average rate and outage probability of C-CMS are obtained by (31) through Monte Carlo simulation. For simulations, the values



**Figure 4.** Outage probabilities of N-CMS with OPA and FPA when  $\beta_{SR} = 2$ , where  $a_1 = 0.9$  and  $a_2 = 0.1$  for FPA.



**Figure 5.** Outage probabilities of N-CMS with OPA and C-CMS when  $\beta_{SR} = 2$ .

of  $\beta_{Sk}$  and  $\beta_{Rk}$  for all  $k$  are uniformly generated between 0.1 and 0.2, and between 1.0 and 1.1, respectively.

Fig. 2 shows the average rates of N-CMS with optimal power allocation (OPA) and fixed power allocation (FPA) with  $a_1 = 0.9$  and  $a_2 = 0.1$  when  $\beta_{SR} = 2$ . The figure demonstrates that the approximate results for OPA and

FPA well match their simulated ones for high SNR, respectively. As seen in (18), the approximate results for OPA are not affected by  $K$ . Also, the simulated results for OPA with  $K = 5$  and 15 become closer as SNR increases although OPA with  $K = 5$  has better rate performance than that with  $K = 15$  for low SNR. OPA provides better average rate compared with FPA as SNR rises. The rate gain achieved by OPA over FPA increases with  $K$  at high SNR. Fig. 3 compares the average rates of N-CMS with OPA and C-CMS when  $\beta_{SR} = 1$  and 2. The figure illustrates that the rate gain achieved by N-CMS with OPA over C-CMS increases as  $\beta_{SR}$  goes up for high SNR, but C-CMS attains better rate performance than N-CMS with OPA for low SNR. For large  $K$ , N-CMS with OPA achieves considerably better rate compared with C-CMS when SNR is high.

Fig. 4 shows the outage probabilities of N-CMS with OPA and FPA with  $a_1 = 0.9$  and  $a_2 = 0.1$  when  $\beta_{SR} = 2$ . In the figure, the approximate and simulated results are in good agreement for high SNR. Analogous to the average rate results in Fig. 2, when  $R_t = 3$ , the outage performances for OPA with  $K = 5$  and 15 become similar for high SNR. However, the outage performance for FPA diminishes as  $K$  increases. Thus, OPA attains higher performance gain than FPA when  $K$  is large. In addition, OPA has better outage performance than FPA as  $R_t$  rises. Fig. 5 compares the outage probabilities of N-CMS with OPA and C-CMS when  $\beta_{SR} = 2$ . In the figure, it is observed that when  $K = 5$  and  $R_t = 3$ , N-CMS with OPA achieves better outage performance in comparison with C-CMS at low SNR, but has similar performance to C-CMS at high SNR. For  $R_t = 3$ , when  $K$  is large, N-CMS with OPA has similar performance to C-CMS for all SNR regime. However, as  $R_t$  goes up, N-CMS with OPA works better than C-CMS.

It is noted that N-CMS with OPA requires instantaneous channel state information (CSI) for the S-to-R and R-to- $D_k$  links for all  $k$  at the source, whereas N-CMS with FPA and C-CMS require no CSI at the source. Therefore, OPA achieves the performance gain at the cost of CSI feedback overhead.

## CONCLUSIONS

In this paper, we present OPA for CMS using NOMA to maximize the achievable rate, and investigate its average rate and outage probability assuming independent Rayleigh fading channels, where the source should know instantaneous CSI for a subset of the links for OPA. Numerical results show that for high SNR, the rate and outage performances for the

OPA are not affected by the number of destinations, whereas those for the FPA are degraded as the number of destinations increases, which is a remarkable finding. On the other hand, as compared to the conventional CMS, CMS using NOMA with OPA attains better outage performance as the outage threshold increases, and achieves better rate performance for high SNR as the number of destinations. However, the performance gains of OPA are achieved at the expense of CSI feedback overhead. Therefore, we may determine whether to adopt the OPA or not, by considering the CSI feedback overhead and the computational complexity at the source in addition to the achievable performance gain.

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