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Generalized Analytic Solutions of The Unsteady Krook Kinetic Model for a Rarefied Gas Affected by a Nonlinear Thermal Radiation Field by using Mean Value Theorem

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Abstract

In this paper, the general and unrestricted solutions for unsteady Krook kinetic model, which are introduced in [8], are demonstrated. Also, the non-stationary Krook kinetic equation model for a rarefied gas affected by nonlinear thermal radiation field, instead of the stationary equation is solved. Furthermore, the positive solutions of unsteady Krook kinetic model, for $0 \le y \le y^*$, where $y^* = \int_1^z F(x)dx$ and z is the largest zero of F with $z<1$,are proved.In a frame co-moving with the fluid, analytically the BGK (Bhatnager-Gross-Krook) model kinetic equation is applied. A new approach depending on the intermediate value theorem is used to get the general solution of the nonlinear ordinary differential equations which are produced from applying the moment method to the unsteady Boltzmann equation. Hence, the unsteady problem solutions will provide a great generality and will be applicable in many fields.Finally, analyticalstudy for the gas microscope behavior such as the temperature and the concentration is showed.

Keywords: Rarefied gases; Thermal radiation field; BGK model;Unsteady Boltzmann kinetic equation.

INTRODUCTION

All matter emits thermal radiation (TR) continuously, and consequently TR is an inherent part of our environment. Radiative heat transfer is important in system analysis particularly when high temperatures are involved, cryogenic systems are also considered, when radiation is being utilized as a source flux, or when radiative transfers the primary mode of heat rejection. Some application examples where TR transfer is of primary importance include solar collectors, boilers and furnaces, spacecraft cooling systems, and cryogenic fuel storage systems [1]. The radiative processes play a major role in the thermodynamics of the Earth system. For this purpose, researchers have used simple blackbody (BB) types of planetary models to theoretically estimate planetary entropy production rates. The analysis of simple radiative models of the Earth system provides insight into its thermodynamic behavior even though it iscomplex. From a thermodynamic perspective, thermal radiation (TR) exchange, i.e., incoming sunlight and outgoing TR, is the only significantform of energy transfer between the Earth and the universe. Further, processes such as absorption and emission dominate planetary entropy production, and the non-uniform absorption of solar radiation (SR) on the Earth causes circulation of the atmosphere and oceans [2]. They have analyzed simple blackbody type radiative models to investigate the thermodynamic behavior of the Earth's system and to estimate planetary temperature and entropy production rates. It is more accurate to model the Earth system as a gray-body because absorption of sunlight and emission of TR are substantially less than that of a blackbody [2].Some authors in both linearized and non-linearized radiation heat flux formulas [3-7] investigated the gas, influenced by a thermal radiation field. Usually, they consider that the gas is dense, so that it obeys Navier-Stokes equations. However, to the best of my knowledge, the situation when a nonlinear thermal radiation force acting on a rarefied neutral gas has not yet been investigated within the framework of the molecular gas dynamics and the unsteady kinetic Boltzmann equation. Harmonious with this great importance of studying the effect of thermal radiation field on gases, the enhancement and the development of the previous paper [8] are introduced in this paper.

In this paper, we solve the non-stationary Krook kinetic equation model for a rarefied gas affected by nonlinear thermal radiation field, instead of the stationary equation. In section 2, we introduce the unsteady approach for studying the influence of thermal radiation field on a rarefied neutral gas, using the unsteady kinetic Boltzmann equation instead of the

Navier–Stokes equations, which are satisfied only for the dense gases.In section 3, we find the general solutions without any conditions. In aframe co-moving with the fluid, analytically the BGK (Bhatnager-Gross-Krook) model kinetic equation is applied. We use a new approach depending on the intermediate value theorem to get the general solution of the nonlinear ordinary differential equations. In section 4, we give the conclusion of the results applied to the Helium gas for various plate temperatures.

MATHEMATICAL FORMULATION

Let us assume that the upper half of the space $(y \geq 0)$, which is bounded by an infinite immobile flat plate $(y = 0)$, is filled with a monatomic neutral dilute gas with a uniform pressure P_S [9-13] and the plate is heated suddenly to produce heat radiationfield. The flow is considered unsteady, one-dimensional and compressible. In a frameco-moving with the fluid the behavior of the gas is studied under the assumptions that:

(i) At the rest plate boundary, the velocities of the incident and reflected particles are equal; but of opposite sign. This is happened according to Maxwell formula of momentum defuse reflection. On the other hand the exchange will be due to only the temperature difference between the particles and the heated plate, taking the form of full energy accommodation [10].

(ii) The gas is considered gray absorbing-emitting but not a scattering medium.

(iii) A thermal radiation force is acting from the plate on the gas in vector notation [14-16]as

$$\vec{F} = \frac{-4\sigma_s}{3n_s c} \vec{\nabla} T^4(y) \Rightarrow F_y = \frac{-16\sigma_s T^3}{3n_s c} \frac{dT(y)}{dy} \tag{1}$$

For unsteady motion, the process in the system under study subject to a thermal

radiation force F_y can be expressed in terms of the Boltzmann kinetic equation [17-22] in the BGK model written in the form:

$$C_y \frac{\partial f}{\partial y} + \frac{F_y}{m} \frac{\partial f}{\partial C_y} = \frac{(f_0 - f)}{\tau} \tag{2}$$

where

$$f_0 = \frac{n}{(2\pi RT)^{\frac{3}{2}}} \exp\left(\frac{-C^2}{2RT}\right), C^2 = C_x^2 + C_y^2 + C_z^2 \tag{3}$$

Lee's moment method [23-29] for the solution of the Boltzmann's equation is employed here. One of the most important advantages of this method is that the surface boundary conditions are easily satisfied. Maxwell converted the Maxwell-Boltzmann equation into an integral equation of transfer, or moment equation, for any quantity Q that is a function only of the molecular velocity. The distribution function used there should be considered as a suitable weighting function which is not the exact solution of the Maxwell-Boltzmann equation in general. Lees found that the distribution function employed in Maxwell's moment equation must satisfy the following basic requirements:

(i) It must have the "two-sided" character that is an essential feature of highly rarefied gas flows.

(ii) It must be capable of providing a smooth transition from free molecule flows to the continuum regime.

(iii) It should lead to the simplest possible set of differential equations and boundary conditions consistent with conditions (i) and (ii). When the application of heat to a gas causes it to expand, it is thereby rendered rarer than the neighboring parts of the gas; and it tends to form an upward current of the heated gas, which is of course accompanied with a current of the more remote parts of the gas in the opposite direction. The fresh portions of gas are brought into the neighborhood of the source of heat, carrying their heat with them into other regions [30]. We assume the temperature of the upward going gas particles is T_1 while the temperature of the downward going gas particles is T_2. The corresponding concentrations are n_1 and n_2. Making use of the Liu-Lees model of the two-stream Maxwellian distribution function near the plate suggested by Kashmarov [31] in the form:

$$f = \begin{bmatrix} f_1 = \dfrac{n_1}{(2\pi RT_1)^{\frac{3}{2}}} \exp\left(\dfrac{-C^2}{(2RT_1)}\right), & for\ C_y > 0 \uparrow \\ f_2 = \dfrac{n_2}{(2\pi RT_2)^{\frac{3}{2}}} \exp\left(\dfrac{-C^2}{(2RT_2)}\right), & for\ C_y < 0 \downarrow \end{bmatrix} \tag{4}$$

The velocity distribution function f is not directly of interest to us, in this stage, but the moments of the distribution function are of interest. Therefore we derive the Maxwell's moment equations by multiplying the Boltzmann equation by a function of velocity $Q_i(C)$ and integrating over the velocity space. How many and what forms of Q_i will be used is dependent on how many unknown variables need to be determined and how many equations need to be solved. Multiplying equation (2) by some functions of velocity

$Q_i = Q_i(C)$, and integrating with respect to C taking into consideration the discontinuity of the distribution function caused by the cone of influence [31]. Jeans [32] and Chapman and Cowling [33] showed that the resulting equation can then be written as:

$$\frac{d}{dy}\left(\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} Q_i C_y f_2 d\underline{C} + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} Q_i C_y f_1 d\underline{C}\right) - \frac{1}{m}(F_y)\left(\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} \frac{dQ_i}{dC_y} f_2 d\underline{C} + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} \frac{dQ_i}{dC_y} f_1 d\underline{C}\right)$$

$$= \frac{1}{\tau}\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q_i f_0 d\underline{C} - \left(\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} f_2 + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} f_1\right) Q_i d\underline{C}\right) \quad (5)$$

where $d\underline{C} = dC_x dC_y dC_z$

where F_y is the external force defined by Eq. (1). The previous equation is called the general equation of transfer or the transfer equation. We can obtain the dimensionless forms of the variables by taking:

$$y = \bar{y}\left(\frac{5\sqrt{\pi}}{4}\frac{\mu_s}{n_s T_s}\sqrt{2RT_s}\right), C = \bar{C}\sqrt{2RT_s}, f_1 = \frac{\bar{f_i}\left(2\pi RT_s\right)^{\frac{3}{2}}}{n_s}, i = 0,1,2$$

$$T_1 = \bar{T}_1 T_s, n_1 = \bar{n}_1 n_s, n_2 = \bar{n}_2 n_s \text{ and } dU = d\bar{U} K_B T_s. \quad (6)$$

Once the expressions for f_0, f_1 and f_2 are introduced, macroscopic quantities such as density, velocity, temperature, etc…, can be computed from the appropriate weighted integral of the distribution functions as follows:

Number density [31]:

$$n(y) = \int f(y, C_y)\, d\underline{C} = \frac{n_1 + n_2}{2} \quad (7)$$

Hydrodynamic (bulk) velocity:

$$V(y) = \frac{1}{n}\int C_y f(y, C_y)\, d\underline{C} = \frac{n_1\sqrt{T_1} - n_2\sqrt{T_2}}{n_1 + n_2} \quad (8)$$

Temperature:

$$T(y) = \frac{1}{3n}\int C^2 f(y, C_y)\, d\underline{C} = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2} \quad (9)$$

The static pressure normal to the plate:

$$P_{yy} = \frac{1}{3n}\int C_y^2 f(y, C_y)\, d\underline{C} = \frac{1}{2}(n_1 T_1 + n_2 T_2) \quad (10)$$

The heat flux component:

$$Q_y(y) = \frac{1}{3n}\int C_y C^2 f(y, C_y)\, d\underline{C} = n_1 T_1^{\frac{3}{2}} - n_2 T_2^{\frac{3}{2}} \quad (11)$$

PHYSICAL PROBLEM

In Eq. (4) there are four unknown functions T_1, T_2, n_1 and n_2 needed to be determined. Thus, we need four equations to solve our problem. We make two moment equations by taking $Q_i = 1, C_y, C^2$ and $\frac{1}{2}C_y C^2$ and substitute by Eq. (4) into Eq. (5). After dropping the bars, we get the following four equations:

The conservation of mass:

$$\frac{d}{dy}\left(n_1 T_1^{\frac{1}{2}} + n_2 T_2^{\frac{1}{2}}\right) = 0 \quad (12)$$

The conservation of y -momentum

$$\frac{d}{dy}\left(n_1T_1+n_2T_2\right)=N\left[\left(\frac{n_1T_1+n_2T_2}{n_1+n_2}\right)^3\frac{d}{dy}\left(\frac{n_1T_1+n_2T_2}{n_1+n_2}\right)\right]\cdot\left(n_1+n_2\right) \tag{13}$$

The conservation of energy:

$$\frac{d}{dy}\left(n_1T_1^{\frac{3}{2}}+n_2T_2^{\frac{3}{2}}\right)=-N\left[\left(\frac{n_1T_1+n_2T_2}{n_1+n_2}\right)^3\frac{d}{dy}\left(\frac{n_1T_1+n_2T_2}{n_1+n_2}\right)\right]\cdot\left(n_1T_1^{\frac{1}{2}}-n_2T_2^{\frac{1}{2}}\right) \tag{14}$$

The heat flux in the y - direction:

$$\frac{5}{4}\frac{d}{dy}\left(n_1T_1^2+n_2T_1^2\right)=-\frac{3}{2}N\left[\left(\frac{n_1T_1+n_2T_2}{n_1+n_2}\right)^3\frac{d}{dy}\left(\frac{n_1T_1+n_2T_2}{n_1+n_2}\right)\right]\left(n_1T_1+n_2T_2\right)$$
$$-\mu\left(n_1T_1^{\frac{3}{2}}-n_2T_2^{\frac{3}{2}}\right) \tag{15}$$

where

$$\mu=\frac{2}{\sqrt{\pi}K_n},\tau=\frac{5\sqrt{\pi}}{4}\frac{\mu_s}{n_sT_s}K_n$$

where K_n is the hydrodynamic Knudsen number defined by

$$K_n=\frac{\text{Mean free path}}{\text{Hydrodynamic}}=\frac{l}{\left(\frac{\mu_s}{n_sT_s}V_T\right)} \quad \text{and} \quad N=\frac{16\sigma_sT_s^3}{3n_sc\,m\,R}$$

is a non-dimensional constant.We study our system under the following boundary conditions

$$\begin{gathered}\frac{n_1(0)+n_2(0)}{2}=1,\\ \frac{n_1(0)T_1(0)+n_2(0)T_2(0)}{n_1(0)+n_2(0)}=1,\\ \left(n_1(0)T_1(0)^{\frac{1}{2}}-n_2(0)T_2(0)^{\frac{1}{2}}\right)=0,\\ T_2(0)=\kappa T_1(0):0<\kappa\leq 1,\end{gathered} \tag{16}$$

Where κ is the temperature between the downward going gas particles and the upward going gas particles after reflection from heated plate surface.Using $x_1=T_1^{\frac{1}{2}}\, x_2=-T_2^{\frac{1}{2}}$, we consider the following system:

$$\frac{d}{dy}\left(n_1x_1+n_2x_2\right)=0$$

$$\frac{d}{dy}\left(n_1x_1^2+n_2x_2^2\right)=N\left[\left(\frac{n_1x_1^2+n_2x_2^2}{n_1+n_2}\right)^3\frac{d}{dy}\left(\frac{n_1x_1+n_2x_2}{n_1+n_2}\right)\right]\cdot\left(n_1+n_2\right)$$

$$\frac{d}{dy}\left(n_1x_1^3+n_2x_2^3\right)=-N\left[\left(\frac{n_1x_1^2+n_2x_2^2}{n_1+n_2}\right)^3\frac{d}{dy}\left(\frac{n_1x_1^2+n_2x_2^2}{n_1+n_2}\right)\right]\cdot\left(n_1x_1+n_2x_2\right)$$

$$\frac{5}{4}\frac{d}{dy}\left(n_1x_1^4+n_2x_2^4\right)=-\frac{3}{2}N\left[\left(\frac{n_1x_1^2+n_2x_2^2}{n_1+n_2}\right)^3\frac{d}{dy}\left(\frac{n_1x_1+n_2x_2}{n_1+n_2}\right)\right]\left(n_1x_1^2+n_2x_2^2\right)-\mu\left(n_1x_1^3+n_2x_2^3\right)\quad \mu=\frac{2}{\sqrt{\pi}K_n}$$

(17)

Where N,μ are constants. We introduce new functions

$$\alpha=n_1+n_2 \tag{18}$$

$$\beta=n_1x_1+n_2x_2 \tag{19}$$

$$\gamma=n_1x_1^2+n_2x_2^2 \tag{20}$$

$$\delta=n_1x_1^3+n_2x_2^3 \tag{21}$$

$$\varepsilon=n_1x_1^4+n_2x_2^4 \tag{22}$$

$$\text{and } q=\frac{n_1x_1^2+n_2x_2^2}{n_1+n_2}=\frac{\gamma}{\alpha} \tag{23}$$

using$'=\frac{d}{dy}$, we have then

$$\beta'=0,\gamma'=Nq^2q'\gamma,\delta'=-Nq^3q'\beta,\frac{5}{4}\varepsilon'=-\frac{3}{2}q^3q'\gamma-\mu\delta.$$

Now the initial conditions are translated as follows,

$$\alpha(0)=2\ ,\gamma(0)=2,\beta(0)=0 \text{ and } x_1(0)=-\lambda x_2(0) \text{ where } \lambda=\kappa^{\frac{1}{2}}.$$

Recall that,

$$\alpha\gamma-\beta^2=n_1n_2\left(x_1-x_2\right)^2 \text{ and } \beta\delta-\gamma^2=n_1n_2x_1x_2\left(x_1-x_2\right)^2,$$

so at $y=0,$ we have

$$4=n_1n_2\left(x_1-x_2\right)^2,\ -4=n_1n_2x_1x_2\left(x_1-x_2\right)^2,$$

Which yields $x_1(0)\,x_2(0)=-1.$

Together with $x_1(0)=-\lambda x_2(0)$, we have $x_1(0)=\lambda^{\frac{1}{2}},x_2(0)=-\lambda^{-\frac{1}{2}}$.

Finally we have $\alpha\delta-\beta\delta=n_1\,n_2\left(x_1-x_2\right)^2\left(x_1+x_2\right)$. At $y=0,$ this now is

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$$2\delta(0) = n_1 n_2 \left(x_1 - x_2\right)^2 \left(\lambda^{\frac{1}{2}} - \lambda^{-\frac{1}{2}}\right) = \alpha(0)\gamma(0)\left(\lambda^{\frac{1}{2}} - \lambda^{-\frac{1}{2}}\right) = 4\left(\lambda^{\frac{1}{2}} - \lambda^{-\frac{1}{2}}\right).$$

Therefore, $\delta(0) = 2\left(\lambda^{\frac{1}{2}} - \lambda^{-\frac{1}{2}}\right)$. Now our differentional equation says that $\beta' = 0$, so $\beta = 0$ identically. Then also $\delta' = 0$, so $\delta = 2\left(\lambda^{\frac{1}{2}} - \lambda^{-\frac{1}{2}}\right)$ for all y. Our differentional equation for γ implies $\gamma = \exp\left(\frac{Nq^3}{3}\right)C_2$. From (18) we have $q(0) = \frac{\gamma(0)}{\alpha(0)} = 1$, we also have $C_2 = 2e^{\frac{-N}{3}}$ and then $\gamma = 2\exp\left(\frac{1}{3}N\left(q^3 - 1\right)\right)$. We also have $\alpha = \frac{1}{q}\gamma$. We still need to find out, how q depends upon y. Also using $\beta = 0$, we have $\varepsilon = \frac{\delta^2}{\gamma} + q\gamma$.

Hence,

$$\varepsilon = \frac{\delta^2}{2}\exp\left(\frac{1}{3}N\left(q^3 - 1\right)\right) + 2q\exp\left(\frac{1}{3}N\left(q^3 - 1\right)\right)$$

(Substitute $E = \exp\left(\frac{1}{3}N\left(q^3 - 1\right)\right)$), now

$$F(q)\frac{dq}{dy} = -\mu\delta,$$

where

$$F(q) = -\frac{5}{8}N\delta^2 q^2 E^{-1} + E\left(\frac{5}{2} + \frac{11}{2}Nq^3\right).$$

We integrate from $y = 0$ to y and find the formula determine $q(y)$,

$$\int_1^{q(y)} F(x)dx = -\mu\,\delta\,y \tag{24}$$

Observe that $-\mu\delta$ is a positive constant. We can deduce some properties of the function $\int_1^{q(y)} F(x)dx = G(q)$ and then for the function $q(y)$. All we can use is that $\frac{dG}{dq} \equiv F(q)$ and properties of F. For example, we have $F(0) = \frac{5}{2}E > 0$ and $F(q) \to +\infty$ as $q \to +\infty$, the first distinction of cases with $\frac{dG}{dq}(1) = F(1)$. Recall that $F(1)\frac{dq}{dy}(0) = -\mu\,\delta$ (Evaluate at $y = 0$). Now $F(1) = -\frac{5N}{8}\delta^2 + \frac{5}{2} + \frac{11}{2}N$ (as $E(1) = 1$) can be positive or negative. (Recall $\delta = 2\left(\lambda^{\frac{1}{2}} - \lambda^{-\frac{1}{2}}\right)$ and λ is between 0 and 1 so δ can be between 0 and $-\infty$).

Now we study two cases:

The first case when $F(1) = 0$: In this case, there is no solution. This is because $q(0) = 1$ and therefore if the solution existed, it would have to satisfy $F(q(y))\frac{dq}{dy} = -\mu\delta$ also at the point $y = 0$ but this gives a contradiction $0.\frac{dq}{dy} = -\mu\delta$ with the fact that $\mu\delta$ is not zero.

The second case when $F(1) < 0$, we have $q'(0) < 0$ and q' cannot vanish according to the equation $F(q)q' = -\mu\delta$, this is because if $q'(y)$ vanished for some y in the interval of the definition of q, then this equation again gives a contradiction $F(q(y)).0 = -\mu\delta$ with $\mu\delta \neq 0$. As it doesn't vanish, it has a constant sign and by our assumption $F(1) < 0$, we have $q'(0) = \frac{-F(1)}{\mu\delta} < 0$. Therefore, $q'(y)$ is always negative. This implies that $q(y) = T(y)$ is a decreasing function of y (as shown in Fig. 1)As $F(0) > 0$, there must be zeros of F between 0 and 1. Let us denote the largest of these zeros. Then $F(q) > 0$ if $0 < q < z$, $F(q) < 0$ if $z < q < 1$. Since, $\frac{dG}{dq} = F$, G has a maximum at

$q=z$. The corresponding value of y (according to Eq. (24)) is $y^*=\frac{1}{-\mu\delta}G(z)$. Hence $q(y)$ can be determined by $G(q(y))=-\mu\delta y$ for $0\le y\le y^*, q(y^*)=z$ and as $F(q)q'=-\mu\delta, F(z)q'(y^*)=-\mu\delta$. More precisely $q'(y^*)=\lim_{y\to y^*} q'(y)=-\infty$. This is because $F(q)q'=-\mu\delta$, that $q'=\frac{-\mu\delta}{F(q(y))}$, therefore, as $y\to y^*$, one has $q(y)\to z$ and $F(q(y))\to F(z)=0$. On the other hand if $y\to y^*$ from below, then $F(q(y))$ tends to zero from below, $F(q(y))<0$. As $-\mu\delta$ is a positive constant, this mean that $q'(y)\to-\infty$. This means that a solution of the system exists only for positive y, if $0\le y<y^*$. Now for $0\le y<y^*$, we can find a function $q(y)$ and then:

$\gamma(y)=2\exp\left(\frac{1}{3}l\left(q^3(y)-1\right)\right)$ as shown in Fig. 2.

$\alpha(y)=\frac{1}{q(y)}\gamma(y), \beta=0, \delta=2\left(\lambda^{\frac{1}{2}}-\lambda^{-\frac{1}{2}}\right)$ as shown in Fig. 3.

This necessarily means that we can find positive $n_1, n_2, x_1, -x_2$ from these values, this is always possible as we will see:

Recall again

$$A_1=\alpha\gamma-\beta^2=\alpha\gamma=n_1n_2(x_1-x_2)^2,$$
$$A_2=\beta\delta-\gamma^2=-\gamma^2=n_1n_2(x_1x_2)(x_1-x_2)^2,$$
$$A_4=\alpha\delta-\beta\gamma=\alpha\delta=n_1n_2(x_1+x_2)(x_1-x_2)^2.$$

x_1 and x_2 can be determined using $x_1x_2=\frac{A_2}{A_1}$ and $x_1+x_2=\frac{A_4}{A_1}$ provided the quadratic equation $x^2-\frac{A_4}{A_1}x+\frac{A_2}{A_1}=0$ has two real solutions. For this to happen we used that the discriminate $\left(\frac{A_4}{A_1}\right)^2-4\frac{A_2}{A_1}>0$. This is equivalent to $A_4^2-4A_1A_2>0$ and hence to $\alpha^2\delta^2+4\alpha\gamma^3>0$. For our solutions this is always as α,γ are positive, this is because $q(y)$ is positive function of y, because it is given by $\gamma=2\exp\left(\frac{1}{3}N\left(q^3-1\right)\right)$ and this is positive as it is an exponential and $\alpha(y)=\frac{\gamma(y)}{q(y)}$, is positive because $\gamma(y)$ and $q(y)$ are positive. Once x_1 and x_2 positive have been found because their product is $x_1x_2=\frac{A_2}{A_1}$ and this is negative so this implies one solution is positive and the other is negative, we can choose the positive solution to be by x_1 and the negative solution to be x_2, so we have

$$x_1=\frac{1}{2}\left(\frac{\delta}{\gamma(y)}+\sqrt{\left(\frac{\delta}{\gamma(y)}\right)^2+4\frac{\gamma(y)}{\alpha(y)}}\right),$$
$$x_2=\frac{1}{2}\left(\frac{\delta}{\gamma(y)}-\sqrt{\left(\frac{\delta}{\gamma(y)}\right)^2+4\frac{\gamma(y)}{\alpha(y)}}\right).$$

It remains to solve $n_1+n_2=\alpha, n_1x_1+n_2x_2=0$ to find n_1, n_2. Her we obtain $n_1=\frac{-\alpha x_2}{x_1-x_2}$, $n_2=\frac{\alpha x_1}{x_1-x_2}$ and they are positive.

Finally we have:

- $n(y)=\frac{n_1+n_2}{2}$ as shown in Fig. 4,
- $P(y)=\frac{n_1x_1^2+n_2x_2^2}{2}$ as shown in Fig. 5,
- and $Q(y)=n_1x_1^3+n_2x_2^3$ as shown in Fig. 6.

DISCUSSION AND CONCLUSION

In a frame co-moving with the gas, we have investigated the behavior of the gas under the influence of a thermal radiation field in the unsteady state of a plane heat transfer problem in the system (gas + heated plate). The thermal radiation is introduced in the force term in the Boltzmann equation for the case of a neutral gas. In all calculations and Figures, we take the following parameters values for the Helium gas:

$$\sigma_s = 5.6705\times10^{-8}\, Wm^{-2}K^{-4} : K_n = 5;$$
$$R = 8.3145\, JK^{-1}\, mol^{-1} : \rho_s = mn_s = 7.344\times10^{-11}\, kg/m^3;$$
$$c = 2.9979\times10^{8} m/\sec : n_s = 10^{16}\, m^{-3};$$
$$N(1000K) = 1.56203$$

Note that $N(1000K) = 1.56203$ is the only value, which satisfies $F(1) < 0$ .i.e. the only value that the positive solutions exist. We prove that the solution of the system exists only for positive y , where $0 \le y < y^*$. Also we determine the value of y^* by using the Newton method, and find that $y^* = 10.01335946$ .So the positive solutions of the system only exist in the interval $[0, 10.01335946]$, also we discuss the behavior of the gas particles in the non-equilibrium state. The number density $n(y)$ increases with increasing the distance from the plate, on the contrary, the temperature $T(y)$ (see Figs. 4 and 1) decreases, this is because application of heat to a gas causes it to expand due to that when the application of heat to a gas causes it to expand, it is thereby rendered rarer than the neighboring parts of the gas. It tends to form an upward current of the heated gas, which is of course accompanied with a current of the more remote parts of the gas in the opposite direction.

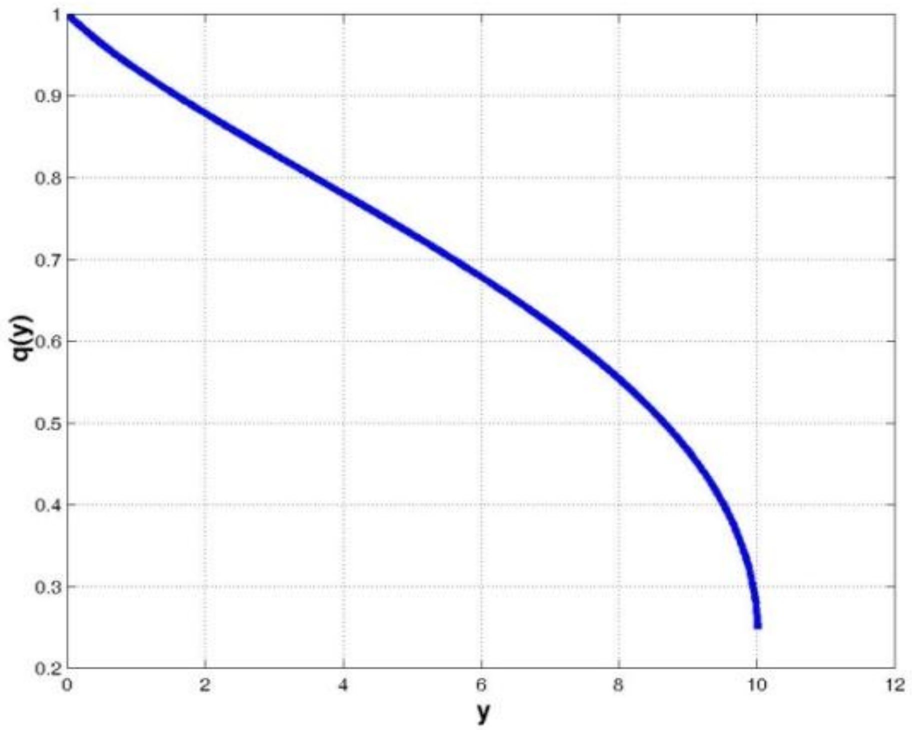

Figure 1: Temperature $T(1000K)$ at $N = 1.65203, \mu = 0.1, \kappa = 0.03$

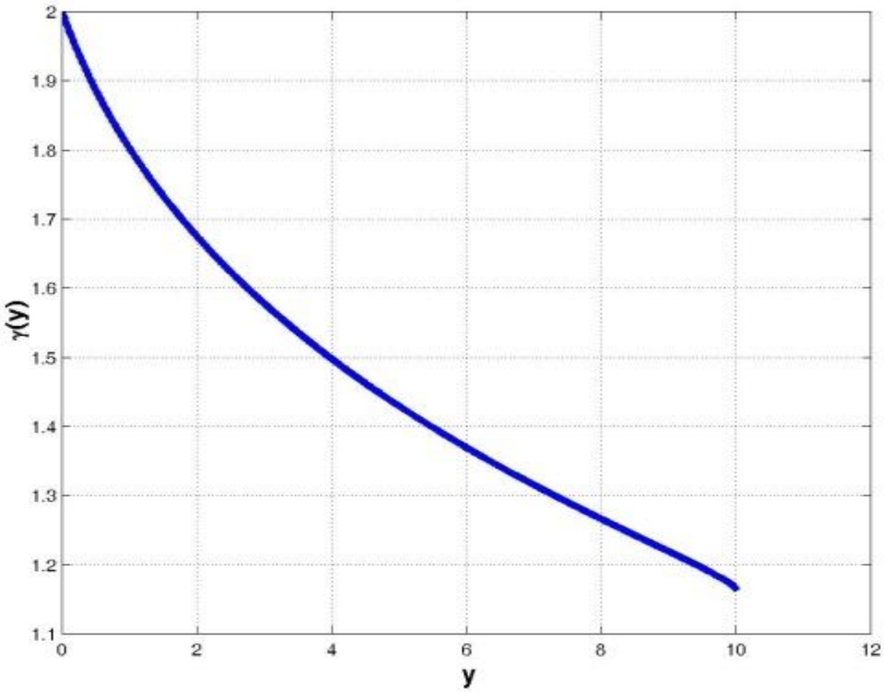

Figure 2: Solution of $\gamma(y)$ at $N=1.65203, \mu=0.1, \kappa=0.03$

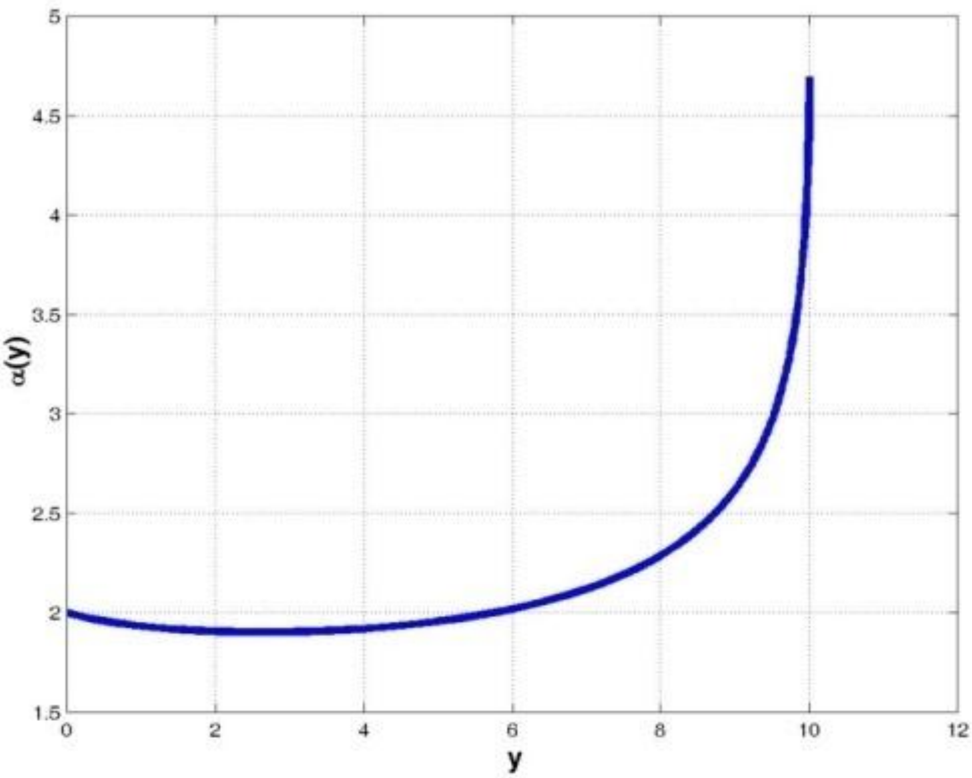

Figure 3: Solution of $\alpha(y)$ at $N=1.65203, \mu=0.1, \kappa=0.03$

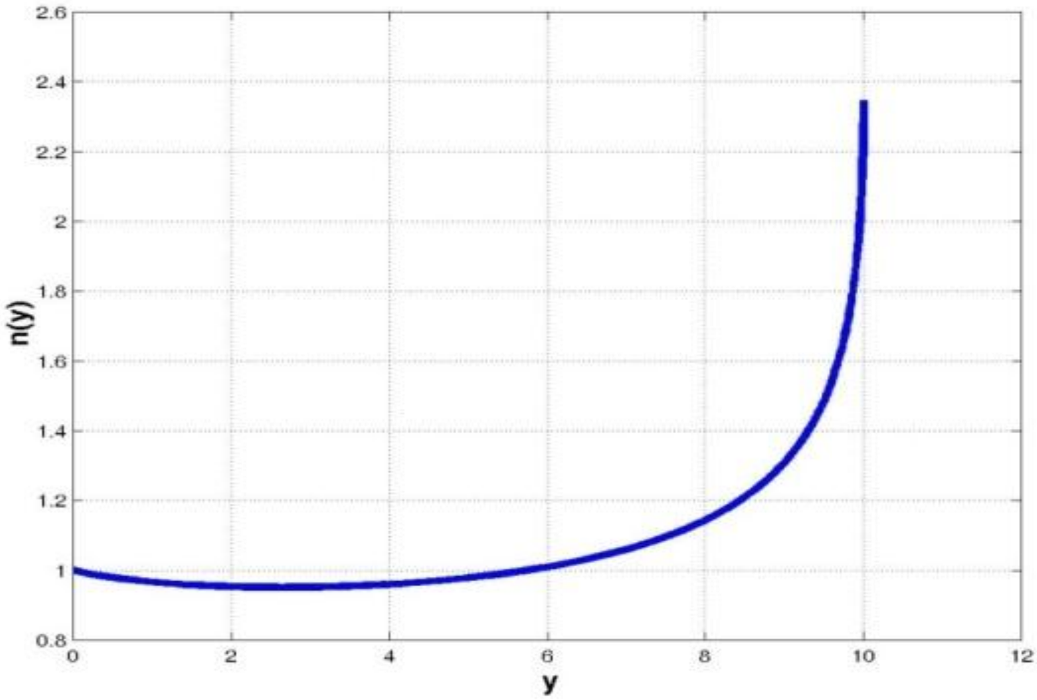

Figure 4: Concentration $n(1000K)$ at $N = 1.65203, \mu = 0.1, \kappa = 0.03$

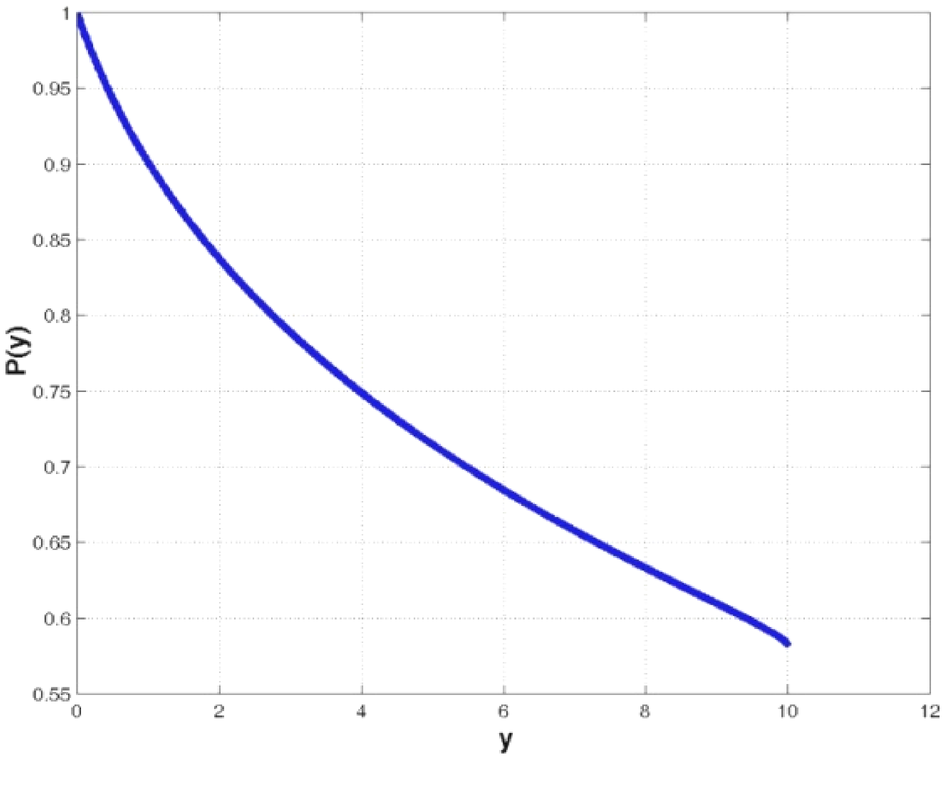

Figure 5: Static Pressure $P(1000K)$ at $N = 1.65203, \mu = 0.1, \kappa = 0.03$

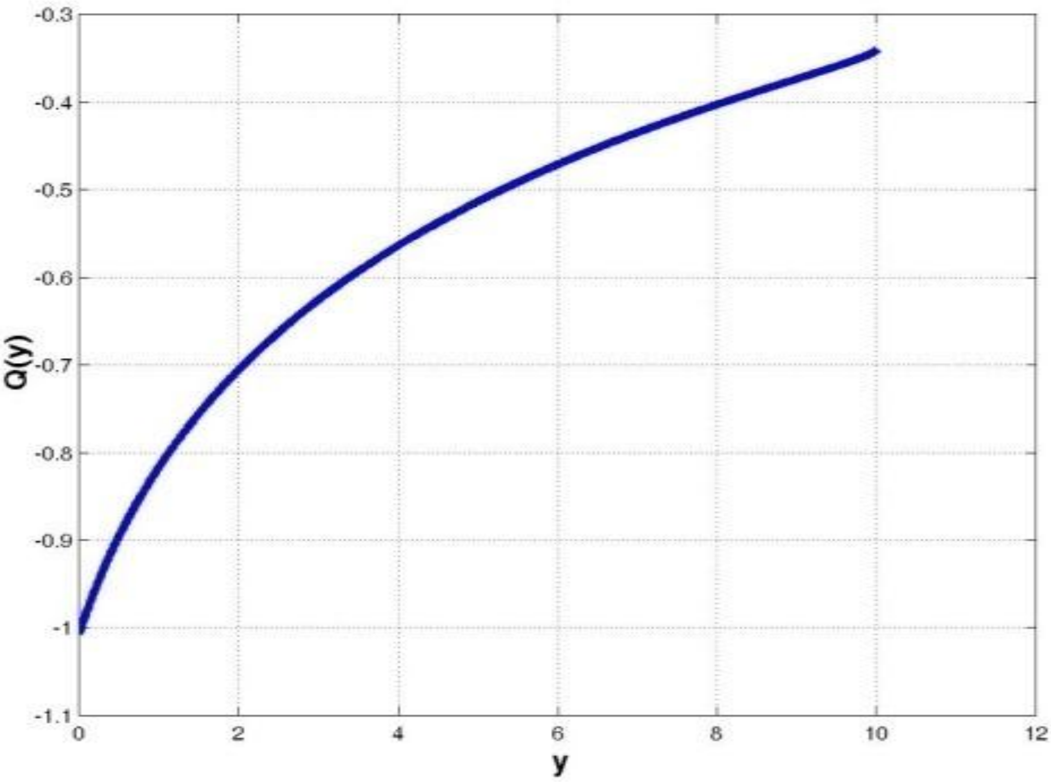

Figure 6: Heat Flux Component $Q(1000K)$ at $N=1.65203, \mu=0.1, \kappa=0.03$

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