

# Reliability Based Design Optimization of BGA Electronic Packages using the Kriging Substitution Model

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## Abstract

Fatigue of the solder joints constitutes the main failure mechanism of BGA (Ball Grid Array) packages due to the thermal stress induced by the different thermal expansion coefficients (CTE) of the materials constituting it. The study of this failure mode via the finite element method using a global model and a local sub-model based on Anand's viscoplastic law and Darvaux's model, has some disadvantages. These disadvantages are due to the complexity of the model which is accompanied by an increase in the number of parameters. And all of these parameters are subject to uncertainties causing thereby a negative impact on the lifetime of solder joints. This situation may lead to the failure of the package. For this reason, the quantification of uncertainties should be taken into consideration to assess performance requirements. And then conducted a study with the method of reliability based design optimization (RBDO), in order to design an improved structure with a higher confidence level. However, this method requires an enormous computational effort. To remedy to this constraint, we will use the kriging model, which will allow us to build a substitution model that combines efficiency and precision. As a result, an analysis with the reliability based design optimization coupled with the kriging model was conducted giving very accurate results and in short time.

**Keywords:** BGA package, thermal fatigue, Thermal cycling Finite element method, Reliability analysis, of reliability based design optimization, Kriging model.

## INTRODUCTION

The BGA packages are widely used in electronic products, as they can meet several requirements in particular in terms of their high density I/O, reduced size and great performance [1]. The structure of the BGA is shown in figure 1.

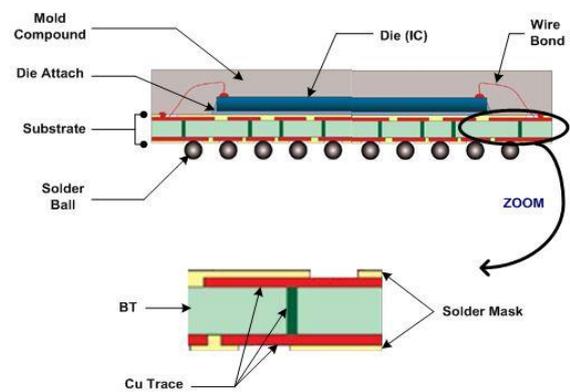


Figure 1: The structure of the BGA.

In a BGA package, solder joints perform the function of mechanical supports and allowing, in the same time, an electrical interconnection. Solder joints are important elements for the proper functioning of the electronic package. However, BGAs are exposed to various thermomechanical constraints, generated by temperature changes, due to power on / off cycles, or changes in operational loads. In conjunction with the differences in thermal expansion coefficients, these factors can expose the solder joints to loads that can cause their cracking and the failure of the package. This failure is not an accidental fact, but occurs very frequently. This is why many studies have been carried out in this field, in order to predict the lifetime of solder joints and improve their performance.

Indeed, several authors, including in particular A. Deshpande and Y. Wan [1,2] have developed a model based on the finite element method (FEM) method and on the mechanisms of solder joints failure when subjected to a thermal load. The initiated simulations are coupled to a multi-parameter numerical optimization tool, based on a Sequential Quadratic Programming Method, in order to optimize the solder joints thermal reliability by minimizing the thermal stress.

However, simulation tools developed by those authors are based on a deterministic approach and don't take into account the uncertainties in the geometry, material properties, boundary conditions or loads. These uncertainties influence the solder joints lifetime. This is why it is necessary to formulate the problem in probabilistic terms in order to calculate the failure probability and to conduct a reliability based design optimization (RBDO) on the package.

Nevertheless, RBDO methods present a disadvantage which is the enormous computational cost necessary to apply them. To overcome this constraint, we have used the kriging model, which has enabled us to build a substitution model that combines efficiency and precision. Therefore, reliability analysis can be performed accurately and in an extremely short time, using FORM / SORM [4-7] simulation methods coupled with the kriging model. At the end, reliability analysis is associated in the optimization process, to improve the solder joint structural design performance and reliability.

The methodology used in this work is organized around 3 stages described as follows:

- First stage: Develop a finite element model of the BGA package using a global and local model based on Anand viscoplastic law and Darvaux model to predict the lifetime of the solder.
- Second stage: Focused on reliability analysis, in order to calculate the failure probability. However, this reliability analysis proved to be constraining in terms of computation time due to the complexity of finite element model. To remedy this situation, we will use the substitution model of kriging, in order to approximate our finite elements models [8-11].
- Third and final stage: Describe RBDO methodology to optimize solder joint performance by minimizing the probability of system failure under cost constraints. The resolution of this problem is carried out using a sequential RBDO algorithm [12-14].

### Lifetime prediction model, subjected to thermal cycle

Fatigue of the solder joints is the main failure mechanism under BGA packages thermal cycle. Several models have been developed in order to predict the fatigue life of solder joint in power modules and other types of electronic packages. The model proposed by Darvaux [15] is one of the most widely used failure criteria for low cycle thermal fatigue prediction.

Indeed, this model describes the relationship between the volume-averaged inelastic work density increment  $\Delta W$  and the number of cycles to crack initiation  $N_0$  and the crack propagation rate  $da/dN$  :

$$N_0 = K_1 \Delta W^{K_2} \quad (1)$$

$$\frac{da}{dN} = K^3 \Delta W^{K_4} \quad (2)$$

**Table 1:** Values of empirical constants used in the prediction of mechanical fatigue life.

$K_1$	7100 cycles/psi <sup>K<sub>2</sub></sup>
$K_2$	-1.62
$K_3$	$2.76 \times 10^{-7}$ in./cycle/psi <sup>K<sub>4</sub></sup>
$K_4$	1.05

Where  $K_1, K_2, K_3$  and  $K_4$  are the empirical constants as shown in Table 1 and  $a$  is the characteristic crack length. Thus, the lifetime  $N_t^f$  can be obtained as follows:

$$N_t^f = N_0 + \frac{a}{da/dN} \quad (3)$$

The parameter  $\Delta W$  is defined as follows:

$$\Delta W = \frac{\sum_{i=1}^n \Delta W_i \cdot V_n}{\sum_{i=1}^n V_n} \quad (4)$$

Where  $\Delta W_i$  denotes the inelastic work density in the  $it$  element in FEM, whose volume is denoted  $V_n$ .

To calculate with precision  $\Delta W$  in Equation 4, a finite element model with a precise description taking into account the time and temperature-dependent deformation of the solder joints is therefore necessary. Among the different constituent time and temperature-dependent deformation behavior models for solder joint in power modules is the viscoplastic model introduced by Anand which is widely used [16].

The Anand model consists of two coupled differential equations that relate the inelastic strain rate to the rate of deformation resistance. The strain rate equation is:

$$\dot{\epsilon}_p = A \left[ \sinh \left( \frac{\xi \sigma}{S} \right) \right]^{\frac{1}{m}} e^{-Q/RT} \quad (5)$$

Where  $\dot{\epsilon}_p$  is the inelastic strain rate,  $A$  is a constant,  $\xi$  is the stress multiplier,  $\sigma$  is the stress,  $S$  is the deformation resistance,  $R$  is the gas constant,  $m$  is the strain rate sensitivity,  $Q$  is the activation energy, and  $T$  is the absolute temperature. The equation that defines the rate of deformation resistance is:

$$\dot{S} = \left\{ h_0 (|B|)^\alpha \frac{B}{|B|} \right\} \dot{\epsilon}_p \quad (6)$$

Where

$$B = 1 - \frac{S}{S^*} \quad (7)$$

$$S^* = \hat{S} \left[ \frac{1}{A} \dot{\epsilon}_p e^{-Q/RT} \right]^n \quad (8)$$

Where  $S^*$  is the saturation value of  $S$ ,  $\hat{S}$  is the coefficient for deformation resistance saturation value, and  $n$  is the strain

rate sensitivity. From the development of the previous equations, there are nine material parameters that need to be defined in the Anand model. Table 2 presents these parameters for the SAC305 (Sn96.5Ag3Cu0.5) alloy used in this work [17].

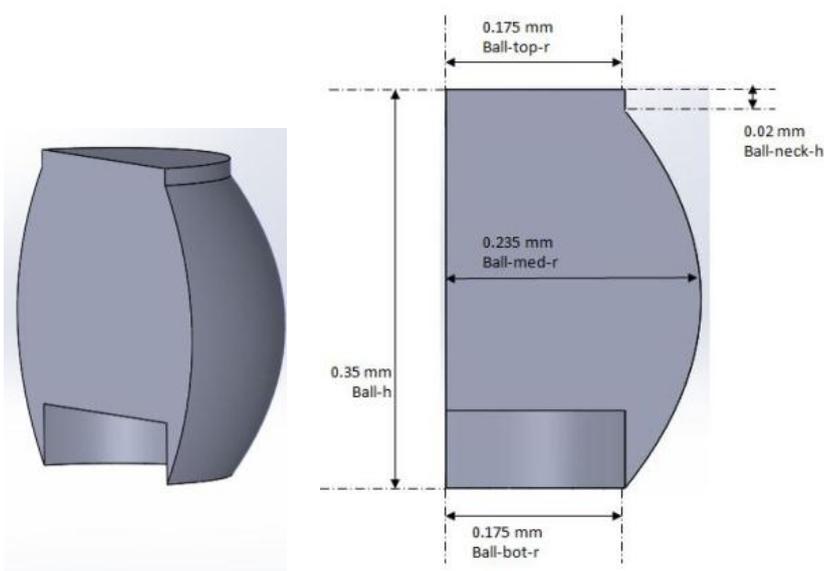
**Table 2:** Anand Model Parameters of the SAC305.

Parameter	Description	Value
$S_0$ (MPa)	Initial value of deformation resistance	45.9
$Q/R$ ( $K^{-1}$ )	Activation energy/Boltzmann's constant	7460
$A$ ( $s^{-1}$ )	Preexponential factor	$5.87 \times 10^6$
$\xi$	Stress multiplier	2.0
$m$	Strain rate sensitivity of stress	0.0942
$h_0$ (MPa)	Hardening/softening constant	9350
$\hat{S}$ (MPa)	Coefficient for saturation value of deformation resistance	58.3
$n$	Strain rate sensitivity of the saturation value	0.015
$a$	Strain rate sensitivity of the hardening/softening	1.5

### Finite element model

As in any FEM analysis, this is initiated by model generation, followed by thermal cycles simulation of and the transition from global model to local sub-model, ending with the calculation of the lifetime.

To do this, a 3D global FEM model of BGA  $10 \times 10 \text{ mm}^2$  was developed under ANSYS APDL 16 whose structural parameters and materials properties, are respectively, shown in Table 3, figure 2 and table 4 [18].



**Figure 2:** Solder joint dimensions

**Table 3:** BGA dimensions

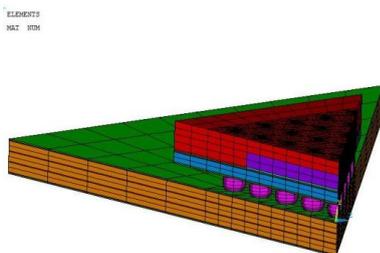
Layer	Size (mm)	Thickness (mm)
Substrate.	10 × 10	0.2
Die attach	7 × 7	0.3
Die	7 × 7	0.29
Overmold	10 × 10	0.265
Top & Bot. Cu	10 × 10	2 × 0.018
Top & Bot Solder Mask	10 × 10	2 × 0.02
Opening / Pads	0.225/0.254	0.018

**Table 4:** BGA material mechanical properties.

Material	Poisson Coefficient	Tg (°c)	Young Modulus (Mpa)	C.T.E. (ppm/°c)
Substrate	0.19	185	28000	CTE1x,y= 14, CTE1z= 35 --- CTE2x,y= 5, CTE2z= 140
Die attach	0.4	42	410/60/40/70/120 @ 25/100/150/200/250°c	CTE1,2 = 48 / 140
Die	0.279	--	131000 / 130000 / 129000 @ 20 / 50 / 227 °c	2.36 / 2.89 / 3.3 / 3.61 @ -73 / 52 / 152 / 227 °c
Mold compound	0.35	120	20000 / 500 @ 25 / 215 °c	CTE1,2 = 11 / 45
Cu	0.344	--	128900	16.7
SMask	0.467	104.8	3500	CTE1,2 = 60 / 160
Solder joint	0.35	--	34300	25

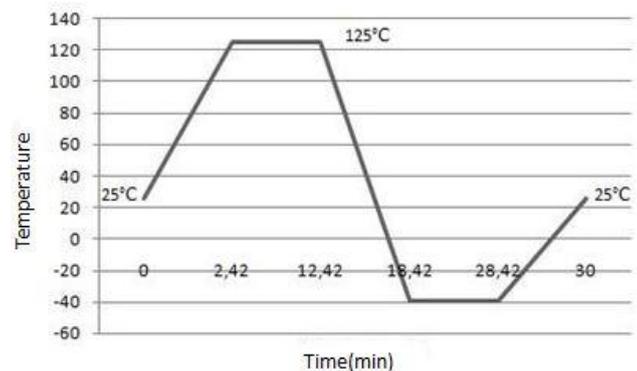
Thereafter, a parametric mesh was used in order to increase results robustness and precision. To reduce calculation time, only 1/8 of the package is modeled, due to the symmetry. The studied BGA is composed of 8 materials: Substrate, Die attach,

Die, Over mold, Cu, Solder Mask, FR4 and solder joints. The solder joints were considered as a viscoplastic material and modeled by element SOLID185. As for the other materials, these are considered to be elastic and modeled by element SOLID45.



**Figure 3:** Global finite element model of the BGA

The cyclic temperature loads vary between  $-40^{\circ}\text{C}$  and  $125^{\circ}\text{C}$  with a 4 minute rise, a 6 minute descent, and 10 minute downtime at maximum and minimum temperature. The reference temperature (without sollicitation) is  $25^{\circ}\text{C}$ . Four thermal cycles are performed in order to obtain a stable stress-strain hysteresis cycle.



**Figure 4:** Thermal cycle description

Once the calculation of the global model has been completed, the most critical solder joint must be identified on the basis of the maximum value of the plastic work which constitutes an identification criterion.

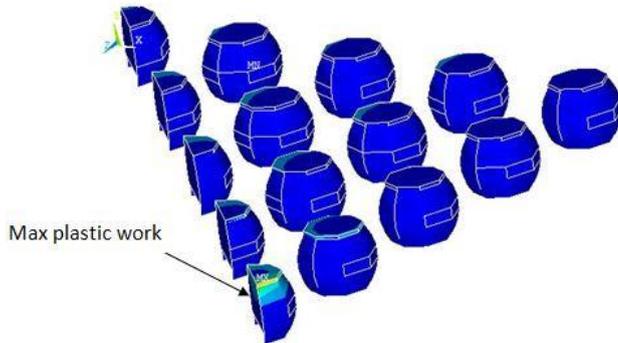


Figure 5: Plastic work at the end of the thermal cycle

It results that the external solder joint on the diagonal of the package is clearly the most critical (figure 5).

Then we created a local sub-model that allowed us to have more accurate results of the most critical solder joint (figure 6). The boundary conditions of the sub-model are determined from solution of the global model and applied to sub-model by the use of interpolation method. Once the displacement field is interpolated to the sub-model, the thermal cycles have been applied.

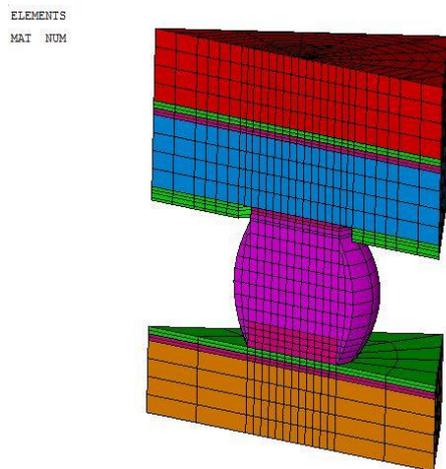


Figure 6: Local sub-model

After obtaining the result of the sub-model, a graph of the solder joint was drawn at the end of the thermal cycle. It has been observed that the package side is the most critical and therefore the lifetime prediction will be performed for this side (figure 7).

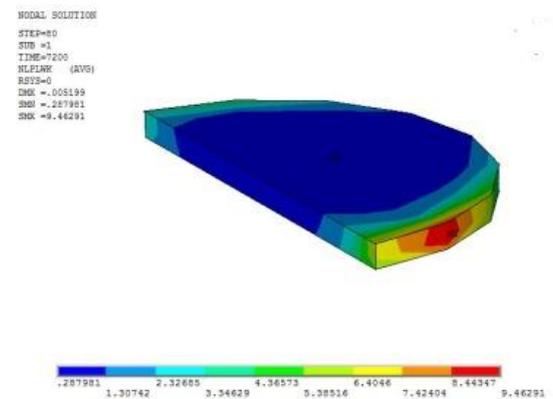
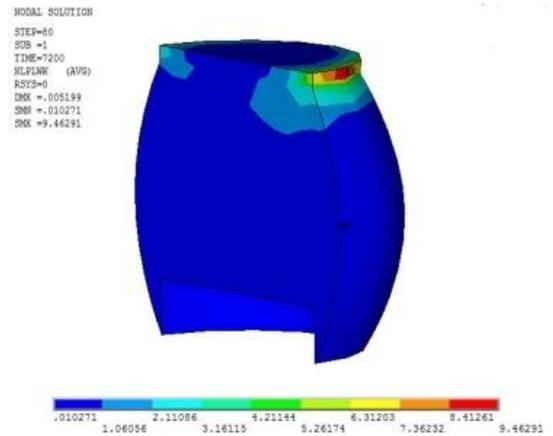


Figure 7: Plastic work in the sub-model

The results from the simulations are shown in Table 5. The number of cycles to crack initiation and lifetime are calculated using Darveaux methodology described above.

Table 5: Simulation results.

number of cycles to crack initiation	111 cycles
lifetime	745 cycles

### Reliability analysis

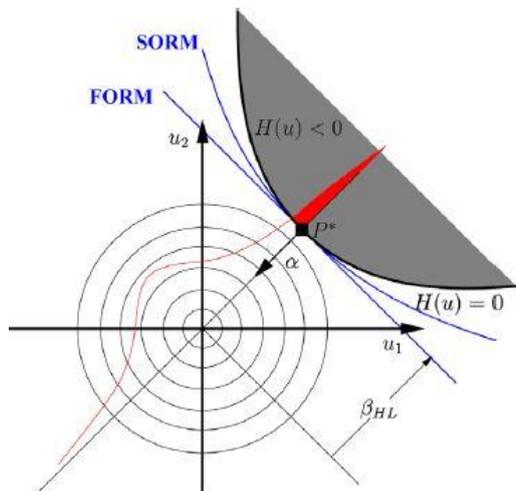
In principle, the number of cycles to crack initiation which is "111cycles" should not be less than a predefined value. However, uncertainties exert a considerable influence on the lifetime of solder joints. And the result due to the uncertainties may be less than the required threshold, causing the system failure. For this reason the purely deterministic analysis has been replaced by a probabilistic analysis that takes into account these uncertainties. The goal is to perform a reliability analysis and calculate the failure probability.

**Reliability Analysis Methods**

The essential problem in analysis of structural reliability resides in calculation of the following integral that provides the failure probability:

$$P_f = \int_{G(X) \leq 0} f_X dx_1 \dots dx_n \quad (9)$$

Where  $X$  is the vector of random variables representing the uncertain parameters of the structure,  $f(X)$  is the probability density function of  $X$ ,  $G(X)$  is the limit state function,  $G(X) \leq 0$  is the integral domain which constitutes the set of failure. Solving this integral analytically is sometimes impossible, because the limit state function is not explicit and its evaluation is the result of a call to the finite elements script. Hence the need to make approximations. Among the methods of approximation, there are the First Order Reliability and second methods (FORM/SORM) which consist in replacing the function limit state by a linear or second order hyper plane.



**Figure 8:** Approximation methods FORM/SORM in standard space

To apply FORM / SORM, we need a transformation from a random vector  $X$  with the continuous distribution function  $f(X)$  to a standard Gaussian vector  $U$  (mean zero, unit variance and independent components) [26], expressed by  $u = T(x)$ . Where  $x = T^{-1}(u)$ ,  $T(\cdot)$  is the operator of probabilistic transformation. The limit state function in the standardized space is written as follows:

$$H(U) \equiv G(X) = 0 \quad (10)$$

Search reliability index  $\beta$  in standardized space which is defined as the distance between the origin  $O$  and the point  $P^*$  closest to the origin in the limit state surface.  $P^*$  is the most probable breaking point.

$$\beta = \min \left( \sqrt{U^T U} \right) \text{ subject to } H(X, U) \leq 0 \quad (11)$$

The failure probability is calculated by:

$$P_f = \varphi(-\beta) \quad (12)$$

$\varphi$  is the cumulative Gaussian distribution of the normal distribution.

Concerning the second approximation Order Reliability (Sorm), this consists to replace the surface of failure by a tangent quadratic hyper-surface and having the same curvature of the real surface at the design point. The probability of failure is calculated by:

$$P_f = \varphi(-\beta) \left( \prod_{i=1}^{n-1} \frac{1}{1 + \beta k_i} \right) \quad (13)$$

Such a problem requires a very high computational cost, which is mainly due to the repetitive call of the finite element model. To solve this problem, the concept of kriging methodology is applied in order to build a high-quality substitution model that combines precision and efficiency.

**Kriging model**

The kriging is a semiparametric interpolation technique, characterizing the real response function  $G$  in two parts: the linear regression part and nonparametric part.

$$g(X) = f^T \beta + z(X) \quad (14)$$

Where the first term  $f(X) = [f_1(X), \dots, f_m(X)]^T$  represents the basis function,  $\beta = [\beta_1(X), \dots, \beta_m(X)]^T$  is the vector of regression coefficients to be calculated and  $m$  represents the number of basis functions.  $Z(X)$  is a Gaussian process with zero mean and constant variance  $\sigma^2$ . The covariance can be defined as follows:

$$Cov[Z(X_i), Z(X_j)] = \sigma^2 R(X_i, X_j), \quad i, j = 1, \dots, N \quad (15)$$

Where  $N$  is the number of experimental points,  $\sigma^2$  is the progress variance and  $R(\cdot, \cdot)$  is the correlation function that is given by:

$$R(X_i, X_j) = \exp \left( - \sum_{i=1}^n \theta_l |X_{ik} - X_{jk}|^2 \right) \quad (16)$$

Where  $n$  is the dimension of input vector  $X$  and  $\theta$  is the

correlation parameter.  $X_{ik}$  and  $X_{jk}$  are respectively the  $k$ th components of vectors  $X_i$  and  $X_j$ . The correlation matrix is then defined as follows:

$$R = \begin{bmatrix} R(X_1, X_2) & \cdots & R(X_1, X_N) \\ \vdots & \ddots & \vdots \\ R(X_N, X_2) & \cdots & R(X_N, X_N) \end{bmatrix} \quad (17)$$

The estimation of  $\beta$  and  $\sigma$  is given by:

$$\hat{\beta} = (F^T R^{-1} F)^T F^T R^{-1} g \quad (18)$$

$$\hat{\sigma} = \frac{1}{N} (g - F \hat{\beta})^T R^{-1} (g - F \hat{\beta}) \quad (19)$$

Where  $F$  is a vector of  $f$  and  $g$  is the vector of experimental points of output responses. Finally, at a given  $X$ , the expected value of  $\mu_{\hat{g}}$  and variance  $\sigma_{\hat{g}}^2$  are predicted by kriging model which is defined as follows:

$$\mu_{\hat{g}}(X) = f^T(X) \mu_{\hat{\beta}} + r^T(X) R^{-1} (g - F \hat{\beta}) \quad (20)$$

$$\sigma_{\hat{g}}(X) = \sigma^2 - [f^T(X) \quad r^T(X)] \begin{bmatrix} 0 & F^T \\ F & R \end{bmatrix}^{-1} \begin{bmatrix} f(X) \\ r(X) \end{bmatrix} \quad (21)$$

With  $r^T(X) = [R(X, X_1), \dots, R(X, X_N)]^T$  is the correlation vector between the unknown points  $X$  and the known experimental points.

The leave-one-out technique (equation 22) of the mean squared error is used as a stopping criteria to refine kriging model.

$$PRESS = \frac{1}{m} \sum_{i=1}^m \left( \frac{\hat{y}_{X/X_i}(X_i) - g(X_i)}{g(X_i)} \right)^2 \quad (22)$$

Where  $\hat{y}_{X/X_i}(X_i)$  is the  $i$ th leave-one-out Kriging prediction of the limit state function constructed from the experiment points without the  $i$ th sample  $X_i$ .

The leave-one-out error is an important estimator of the performance of a learning algorithm, it is a process that does not require additional evaluations of the limit state function and only uses the available observations in the experiment points obtained to build the Kriging model. We can obtain a surrogate model with an acceptable level of accuracy when leave-one-out error is set to 0.1.

### Probability of failure

A sensitivity study must first be made to select the most significant variables that affect the system status. Other variables that play a small role are assumed to be

deterministic. As a result of this sensitivity study, a reliability analysis will be conducted.

The sensitivity analysis relating to the number of cycles to crack initiation of the solder joint, Which concerned material properties, showed that the thermal expansion coefficient (CTE) of the solder joint, the stress multiplier ( $\xi$ ),  $k_2$  and initial value of deformation resistance  $S_0$  impact number of cycles to crack initiation of the solder joints

The failure is based on the fact that the number of cycles to crack initiation must not be less than  $N_0 = 100$  cycles. The limit state function can then be written as follows:

$$G(X) = 1 - \frac{N_0}{N_{0calc}} \quad (23)$$

Where  $N_{0calc} = K_1 \Delta W^{K_2}$ . Probability distributions and the parameters of the most influential random variables are listed in the table 6.

**Table 6:** Statistical description of random variables

Variable	Variable description	Distribution	Mean	Standard deviation
CTE	thermal expansion coefficient	normal	2.50E-05	0.15E-6
$S_0$	initial value of deformation resistance	normal	45.9	0.3
$\xi$	stress multiplier	normal	2	0.02
K2	K2	normal	-1.62	0.045

The results of design points and estimation of the reliability indices and failure probability are shown in the table 7:

**Table 7:** Reliability Analysis Results

Variables	Design point(Form)	Design point(Sorm)
CTE	0.0000249	0.0000249
$S_0$	45.9	45.9
$\xi$	2.005	2.005
K2	-1.659	-1.66
$\beta$	0.889	0.861
$P_f$	18,69%	19,44%
Reliability level	81.31%	80.56%

It can be concluded that the results obtained by the two Form/Sorm methods proved to be consistent. Similarly, it is

specified that the calculated failure probability is equal to 18.69% and the reliability level is 81.31%.

And

**Reliability based design optimization**

Two cases can be the object to a structural optimization based on the reliability either [19]. The cost (volume, weight, etc.) or the reliability can be optimized. Regarding the cost, it is minimized, subject to a given minimum reliability as well as other performance requirements (CRP). Whereas reliability is maximized subject to a given maximum cost as well as other performance requirements (RCP). These two problems can be formulated as follows:

CRP:  $\min C(d, \beta(d))$   
 s.t. constraints on design and cost parameters  
 constraints for reliability simple bounds

RCP:  $\min P_f(d)$   
 s.t. constraints on design and cost parameters  
 constraints for reliability simple bounds

In this article, the aim is to minimize the failure probability or to maximize the reliability of solder joints under the structural volume constraint. In this problem, we have 8 optimization variables: 4 random variables  $X_r$  (CTE,  $S_0$ ,  $\xi$  and K2) and 4 design variables  $X_d$  ( ball-h, ball-neck, ball-med-r et ball-top-bot-r). Using the classical model, the optimization problem can be written in two sub-problems described in figure 9.

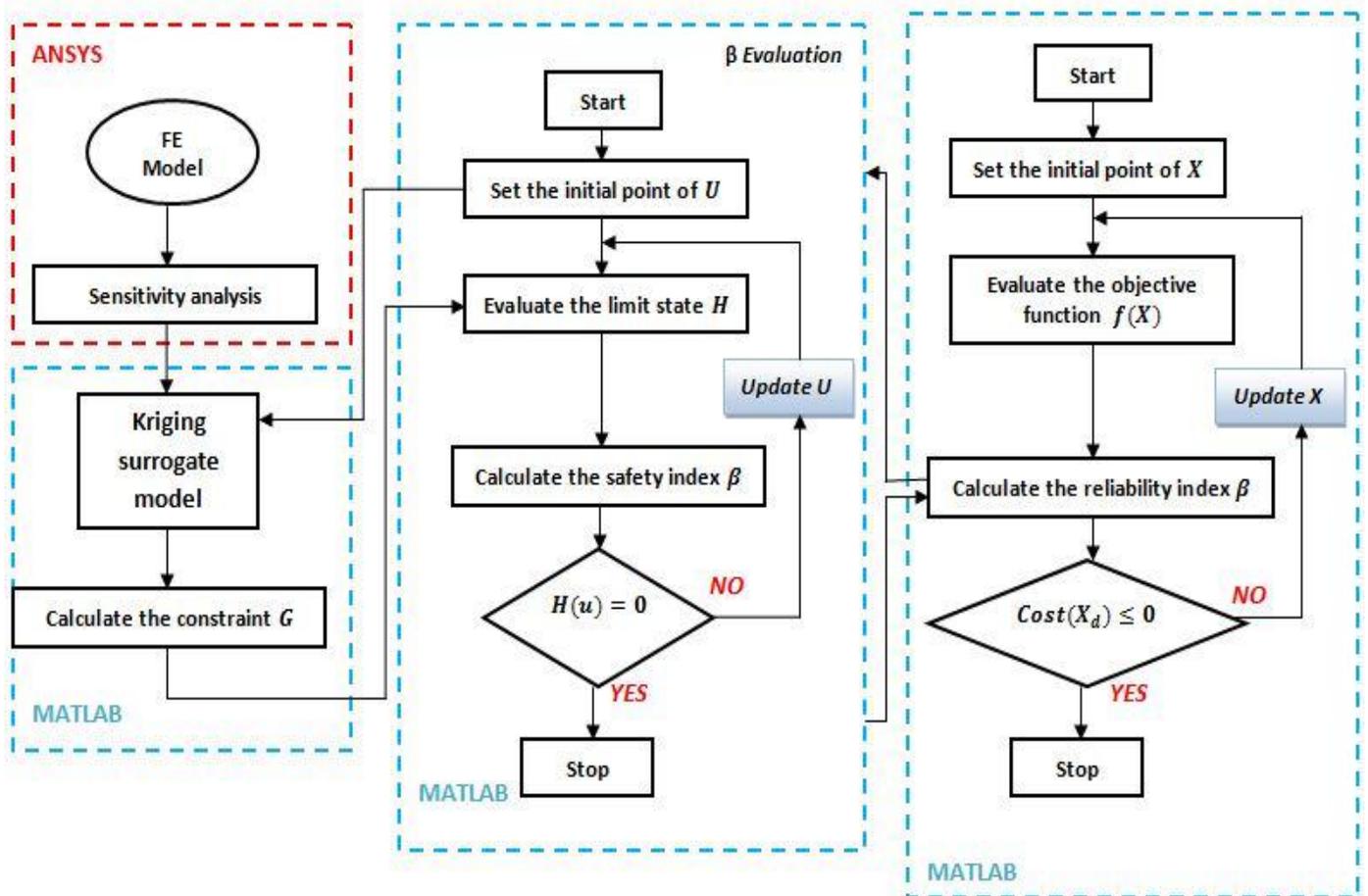


Figure 9: RBDO algorithm

1. Optimization problem subject to cost constraints:

Find  $X_d = [X_{d1} \dots X_{dn}]$  and  $X_r = [X_{r1} \dots X_{rn}]$

Such that to minimize  $P_f = PR[G(X_d, X_r) \leq 0]$

Subjected to cost  $cost(X_d) \leq C_0;$   
 $X^{lb} \leq X_d \leq X^{ub}$

2. Calculation of Reliability Index

$$\min: d(u) = \sqrt{\sum_j u_j^2} \text{ subject to } 1 - \frac{N_0}{N_{ocalc}}$$

Where  $C_0$  is the allowable volume, which is a function of vector of design variables  $X_d$ .  $Pr[.]$  is the probability operator and  $P_f$  is the failure probability corresponding to the limit state function  $G$ . The problem RBDO consists in finding the vector design  $X_d$  which minimizes the failure probability of the solder joint under the cost constraint corresponding to the structural volume.

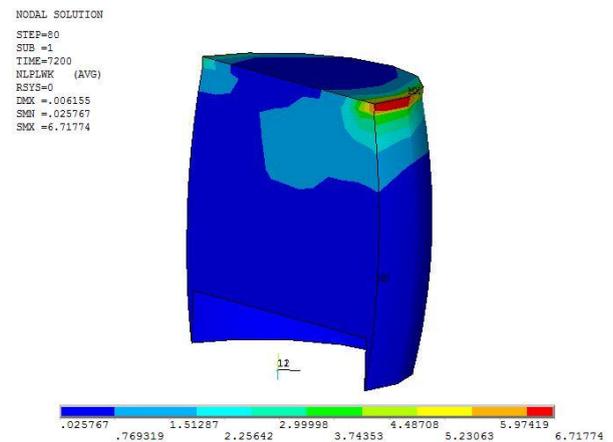
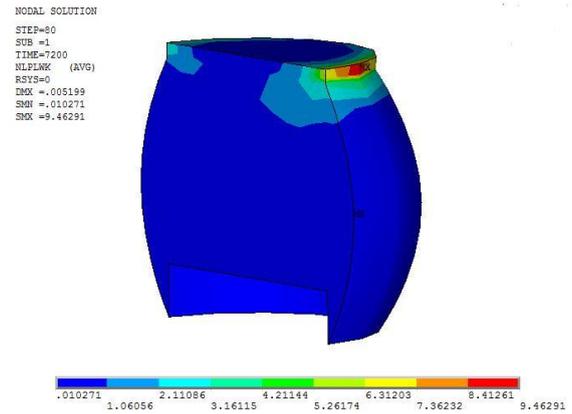
The solution of these two problems consists in using the following optimization technique "sequential quadratic programming SQP".

The table 8 shows the optimal and initial design. Similarly, it is apparent from the table that the results obtained show that the probability of failure is minimized. It increased from 18.69% to 0.21%. Whereas the solder joint volume is reduced from 0.045 to 0.038, a reduction rate of 15.5%. As for the level of reliability, this one rises to 99.79% after being initially at 80.31% and this due to the new design. As for the number of cycles to crack initiation and the lifetime these have respectively increased from 111 to 169 cycles and from 745 cycles to 1000 cycles, a progression rate of 52% and 34% respectively.

In addition, Figure X shows the new design and the plastic work distribution after application of the reliability-based design optimization.

**Table 8:** RBDO optimization results

variables	Initial point	optimal point
ball-h	0.35	0.3736
ball-neck	0.02	0.0165
ball-med-	0.235	0.2
ball-top-bot-r	0.175	0.18
Volume	0.045	0.038
N0	111	169
Nf	745	1000
$P_f$	18.69%	0.21%
Reliability level	80.31%	99.79%



**Figure 10:** Plastic work before and after RBDO.

**CONCLUSION**

In this paper, we have followed a reliability based design optimization methodology, combined with an analysis via the finite element method, in order to minimize the failure probability under the solder joint volume constraint. This finite element analysis was carried out on the basis of two 3D models: A Global one and a Local one. The Global model is made to identify the position of the most critical solder joint and to calculate the displacement field. Whereas the local model was developed to predict lifetime. Also considering the enormous computational time generated by this methodology, we used the substitution model of kriging as it provides different advantages in terms of accuracy and significant gain of computation time.

The reliability analysis thus carried out proved to be very useful in predicting the effects of uncertainties related to material properties on the number of cycles to crack initiation and the lifetime of the solder joints. In addition, the use of the reliability based design optimization allowed us not only to obtain an improved design with a High level of reliability compared to the original design (ie 99.79% versus 80.31%), but also to minimize the probability of failure and the volume of solder joints.

## ACKNOWLEDGMENTS:

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