Connection of Odkvist Parameter and Values of Microhardness when Hardening by Plastic Deformation

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Abstract

In the work the authors put the emphasis on the fact that development of new strengthening technologies, in particular by methods and ways of metal forming, requires finding of connections between the practical measured parameters of hardening and results of mathematical modeling of technological processes. As results of pilot studies the authors have chosen the parameter of microhardness and its distribution on the section of products. As results of theoretical and mathematical modeling of real productions they have chosen Odkvist parameter. With use of various mathematical transformations and physical assumptions the authors reveal the connection between Odkvist parameter and one of mechanical parameters of products – microhardness. They show that creation of the specified dependence requires carrying out two elementary experiments with sample material. By the results of these works one can define the corresponding coefficients which are the parts of the equation of connection between Odkvist parameter and microhardness. As an example, the authors show the connection of Odkvist parameter with microhardness parameter for gradient-reinforced products processed by the method of strengthening complex local loading of the center of deformation. They give a short review of the technology for receiving details like plugs considered in the example. At the same time in a body of a product (plug) one gets the strengthened structure, gradient from an external surface, which conforms to operational requirements better, than received when processing by traditional ways of hardening. As a result in the work the authors show that Odkvist parameter is the important characteristic of any strengthening technology and one can use it as a universal criterion when comparing results of theoretical and pilot studies.

Keywords: metal forming, Odkvist parameter, microhardness, hardening, complex local loading of the center of deformation.

INTRODUCTION

Modern requirements to a complex of operational characteristics of details of different functions demand qualitatively new approaches to technologies of their production. The most successful and perspective are the technologies based on the metal forming (MF) as they allow receiving the necessary product (or a billet) with the minimum waste at simultaneous improvement of structure and mechanical properties of the material.

The development of new MF technologies, as a rule, demands carrying out considerable volume of researches in which two main directions are allocated: a theoretical one – by mathematical modeling and an experimental one –based on this or that physical model with reasonable criteria of similarity. At the same time it is necessary to compare the results of theoretical and experimental studies. And there appears a problem: theoretical researches (mathematical modeling) allow receiving data on distribution of tension and deformations and their derivatives on the considered model, and the results of an experiment allow receiving data on displacements (for example, by drawing a coordinate grid) and distribution of hardness (microhardness) on some sections and surfaces.

Thus, in the results of mathematical and physical modeling there is no general indicator which allows comparing the results correctly in the same units within the limits of interval uncertainty [8]. There are options of comparison of indicators of hardness and resistance to deformation [1–4]; however, they have no general character and at some difficult schemes of the intense deformed state may not be true. So, a conclusion follows that the task of development of a certain uniform criterion for comparison of results of experimental and theoretical studies is of current interest.

MATERIALS AND METHODS

Generally for each point of a body in the course of deformation it is possible to define, with the known external forces, components of a tensor of tension. The latter are connected with components of a tensor of deformations through certain dependences. After removal of loadings the body generally has form and structure different from initial (which defines change of some properties). It is easy to see the change of a form in practice – it is necessary to measure a sample before
deformations and later. Structural transformations are more difficult – they are connected with the change of sizes and orientation of crystals; increase or reduction of number of dislocations, Franck-Read's sources, etc. The specified internal changes in the deformed body can be estimated, having led a number of standard tests: for example, stretching of a sample and measurement of hardness or microhardness.

In practice as a result of carrying out tests on stretching (or compression) of the samples deformed with various degrees curves of hardening are built. Curves of hardening can be of the first (dependence of resistance of plastic deformation \( \sigma_s \) on relative lengthening \( \varepsilon \) ) and of the second sort (dependence \( \sigma_s \) on relative reduction of cross-sectional area \( \psi \) ), and they also may have a form \( \sigma_s = f(\varepsilon) \), where \( \varepsilon \) means true (logarithmic) deformations. All representations and expressions for deformations are interconnected: \( \varepsilon = \ln(\varepsilon - 1) \), therefore the choice of this or that curve of hardening depends only on convenience of use. Let’s consider curves of hardening of the first sort:

\[
\sigma_s = f(\varepsilon). 
\]

These curves can be approximated through various dependences, in particular, in the form of a binomial [2]:

\[
\sigma_s = \sigma_T + A \cdot \varepsilon^n, \tag{2}
\]

where \( A \) and \( n \) are the coefficients received at approximation; \( \sigma_T \) – is the value of tension, corresponding to the beginning of a plastic current, i.e. a fluidity limit.

Approximation in the form (2) is also often used in the analysis of curves \( \sigma_s = f(\varepsilon) \) where \( \sigma_s \) is the tension corresponding to the greatest loading preceding destruction of a sample or temporary resistance.

Value \( \sigma_s \) essentially depends on the scheme of tension and size of a spherical tensor of tension. When drawing curves of hardening they use the results of tests which are close to linear tension, i.e. \( \sigma_s = \sigma_i \); on the other hand, for the specified state \( \sigma_i = \sigma_i \) is fair. At the same time intensity of tension is functionally dependent on the second invariant of a tensor of tension. Thus, it is possible to allow equality of intensity of plastic deformations and sizes of resistance to plastic deformation, i.e.

\[
\sigma_i = \sigma_s \tag{3}
\]

From (3) and (2) follows:

\[
\sigma_i = \sigma_T + A \cdot \varepsilon^n \tag{4}
\]

When carrying out tests on measurement of the hardness (microhardness), single value is often received (in any point) that reflects requirements of technical documentation. For many processes of MF (rolling, drawing, pressing) it is considered that hardness doesn't change at all length of a sample and at its section. At the same time, the strengthening MF technologies have a purpose to change hardness of certain areas of a product (for example, superficial plastic deformation). As a result the received products generally have some distribution of hardness at all volume, first of all at the section of a detail.

Figure 1. The graphic explanation to the choice of the system of coordinates

For simplicity we will introduce the law of distribution of hardness (microhardness) in functions from one coordinate \( x \) (see fig. 1). We will choose the direction from an internal surface to an external one (point A in fig. 1), and the beginning of coordinates will be a transition point from not strengthened surface to strengthened (point B in Figure1).

We will accept that for the accepted system of coordinates microhardness changes under the square law:

\[
H_\mu = a \cdot x^2 + b \cdot x + c, \tag{5}
\]

where \( x \) – is the coordinate of a point in which microhardness has been measured, a transition point from not strengthened state to strengthened corresponds to zero value, the point on a product surface corresponds to the maximum value; \( H_\mu \) – is value of microhardness in a point with the current coordinate \( x \); \( a, b, c \) – are the coefficients received as a result of approximation of distribution of microhardness in function from one coordinate \( x \).
From (5) it is obvious that to the greatest change of coordinate \( x \) there corresponds value: \( h_{\text{max} \mu} \) is maximum deepness of hardening. We notice that (5) can be elementary transformed to the linear law of distribution of microhardness by acceptance \( a = 0 \).

In work [1] it is shown that hardness parameters according to Brinell and Vikkers can be connected with intensity of tension \( \sigma_1 \), so that the specified dependence for engineering practice has a linear character:

\[
\sigma_i = k \cdot H_\mu
\]

where \( k \) is a proportionality coefficient.

Having substituted in (6) (4), we will receive:

\[
\sigma_i = k \cdot H_\mu.
\]

In (7) we will substitute (4) and (5) and we will receive:

\[
k \cdot (a \cdot x^2 + b \cdot x + c) = \sigma_i + A \cdot \varepsilon^n.
\]

At \( \varepsilon = 0 \) and \( x = 0 \) from (8) we will receive:

\[
\sigma_i = k \cdot x.
\]

Expression (9) reveals physical sense of a coefficient of proportionality in (7). We pay attention that value \( c \) represents the initial hardness of a sample; \( H_{\mu0} = c \), then:

\[
k = \frac{\sigma_i}{H_{\mu0}}
\]

Taking into account (9) we transform expression (8) to

\[
k \cdot (a \cdot x^2 + b \cdot x) = A \cdot \varepsilon^n
\]

and, having expressed deformations, we receive:

\[
\varepsilon = \sqrt[2]{\frac{k \cdot (a \cdot x^2 + b \cdot x)}{A}}.
\]

We rewrite (11) in the following form taking into account equality \( H_{\mu0} = c \) and (5):

\[
\varepsilon^n = \frac{k}{A} (H_\mu - H_{\mu0}).
\]

From (12) it is clear that \( n \) and \( \frac{k}{A} \), can be received by the results of an experiment which point is in forming or stretching of two identical samples with a different level of deformation \( \varepsilon \). Further it is necessary to measure hardness \( H_\mu \) of samples. As a result it is possible to make and solve a system of equations on the basis of (12):

\[
\begin{align*}
\varepsilon_1^n &= \frac{k}{A} (H_{\mu1} - H_{\mu0}) \\
\varepsilon_2^n &= \frac{k}{A} (H_{\mu2} - H_{\mu0})
\end{align*}
\]

where \( \varepsilon_1 \) and \( \varepsilon_2 \) – the relative deformations of 1 and 2 samples, \( H_{\mu1} \) and \( H_{\mu2} \) – hardness of 1 and 2 samples after deformation.

From (13) we receive:

\[
n = \ln \left( \frac{H_{\mu1} - H_{\mu0}}{H_{\mu2} - H_{\mu0}} \right) \ln \left( \frac{\varepsilon_1}{\varepsilon_2} \right),
\]

\[
k = \frac{\varepsilon_1 \ln \left( \frac{H_{\mu1} - H_{\mu0}}{H_{\mu2} - H_{\mu0}} \right)}{\ln \left( \frac{\varepsilon_1}{\varepsilon_2} \right)}.
\]

Expression (11) can’t be used practically as actually in case of MF, in particular, in case of the hardening technologies, different elements of a product perceive deformations different in value and type. Besides, multitransitional and multi-cycle operations of MF are often used. Therefore it is necessary to calculate the parameter of Odkvist which characterizes stored deformation. Having used a curve of hardening of the first kind...
and approximation in the form (4), we actually attributed to a sample a certain level of the relative deformation on conditionally known resistance of deformation as we can consider only final process because hardness and microhardness can be measured only after the end of an experiment.

From the theory of MF it is known that when considering final processes in case of monoaxial stretching or compression of a sample with initial length \( l_0 \) to length \( l_f \), neglecting an elastic component, equality of parameter of Odkvist to logarithmic deformations [3] can be received, i.e.:

\[
q = \int \mathbf{e}^p dt = \int \mathbf{e}^x dt = \int \mathbf{e}^y dt = \int \mathbf{e}^z dt = \frac{V}{t} = \ln \frac{l_f}{l_0} = e .
\]

(16)

As a result, having accepted these assumptions and considering that \( e = \ln (e - 1) \), from (11) and (16) we receive:

\[
q = \ln \left( \frac{k \cdot (a \cdot x^2 + b \cdot x)}{A} - 1 \right) .
\]

(17)

where \( n \) and \( \frac{k}{A} \) can be calculated by (14) and (15) by the results of the experiment, or can be taken from reference books when determining \( k \) according to (10) or [1].

We note that expression \( (a \cdot x^2 + b \cdot x) \) represents change of hardness (see formula 12):

\[
\delta H_\mu = H_\mu - H_{\mu 0} = (a \cdot x^2 + b \cdot x) ,
\]

(18)

where \( H_{\mu 0} \) – value of initial microhardness; \( H_\mu \) – value of microhardness after deformation commission.

Then (17) will be transformed to

\[
q = \ln \left( \frac{k \cdot (H_\mu - H_{\mu 0})}{A} - 1 \right) \text{ or taking into account (10):}
\]

\[
q = \ln \left( \frac{\sigma_f \cdot (H_\mu - H_{\mu 0})}{H_{\mu 0}} - 1 \right) .
\]

(19)

Expression (19) should be used for processes of MF when the hardness (microhardness) after deformation is conditionally constant on section (rolling, drawing, pressing). Dependence (17) should be applied in all other cases; the main problem consists in creation of the approximating dependence in the form of (5) for each concrete method.

RESULTS

As an example of use of dependence (17) we consider MF technology with complex local loading of the center of deformation (CLL-deformation) [4-6]. One of almost realized ways of CLL-deformation is presented in fig. 3. Processing in a way [5] (see Figure 3) is carried out as follows. Billet 1 is established on rotating mandrel 4, the axial force \( P_a \) is applied to an end creating the axial tension which isn't reaching a fluidity limit by means of a clip 3. The billet is given a torque, an external surface is formed by repeated reciprocating axial movement of rollers 5 and 6 having the site of a bigger diameter to which force \( P_a \) is applied then the rollers are taken away, the clip is taken away and ready plug is taken away.

As a result we receive gradient from an external surface strengthened structure, which conforms to operational requirements better than received after traditional ways of hardening.
In work [5] by results of the analysis of data on distribution of microhardness depending on depth of hardening and use of the phenomenological description of process from the point of view of distribution of force from its source we received dependence similar to (5) and (18):

\[ \delta H_p = a \cdot x^2, \quad (20) \]

\[ a = \frac{H_{\mu \max} - H_{\mu 0}}{h_{\mu \max}^2}. \quad (21) \]

where \( H_{\mu \max} \) – the maximum value of microhardness (value in a near-surface layer); \( h_{\mu \max} \) – maximum depth of hardening.

As a result (17) taking into account (20) for CLL-deformation has the following form:

\[ q = \ln \left( \frac{k \cdot a \cdot x^2}{A} - 1 \right) \quad (22) \]

\[ h_{\mu -} ^{MM} \]

\[ B \]

\[ A \]

**Figure 4.** Dependence between the parameter of Odkvist and depth of hardening (at CLL-deformation while applying the scheme presented in Fig. 3; the location of points A and B are similar to the ones presented in Fig. 1)

In fig. 4 there can be seen the distribution of values of parameter of Odkvist depending on hardening depth for the bush \( \varnothing_{\text{exter}} = 45 \text{ mm}, \varnothing_{\text{inter}} = 30 \text{ mm} \) made of BrO5TS5S5 alloy with the initial microhardness \( H_{\mu 0} = 102.8) \text{HV} \) processed in the way presented in fig. 3 with force \( P_H = 310) \text{H} \), number of passes \( n = 38 \) and a step of feed \( s = 0.5) \text{mm} \). \[4-6\]. Values of sizes \( n = 0.8402 \) and \( m = 0.0068 \) have also been received during the experiments on the forming of samples in work [4]. Value of parameter \( a = 8,225 \) is received during the analysis of experimental data in work [5, 7, 9-10].

Values \( q \) received at mathematical modeling by means of the Stamp 2.0 program [4] are also presented in fig. 4 in the form of the points. The relative deviation of the points received as a result of mathematical modeling of CLL-deformation from the derived dependence (22) doesn’t exceed 10%.

**CONCLUSIONS**

Satisfactory coincidence of dependence (22) to data of mathematical modeling confirms admissibility of the simplifications accepted at finding (17). The shown connection between the parameter of Odkvist and hardness allows determining it practically by the results of simple mechanical tests on the available equipment. Thus, it is proved that the parameter of Odkvist is an important characteristic of any strengthening technology and can be used as a universal criterion when comparing results of theoretical and pilot studies.

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