Cost Effective High Reliability Routing in Multi-Channel Cognitive Radio Networks

Nandini K. S.\textsuperscript{1} and Dr. S.A. Hariprasad\textsuperscript{2}

\textsuperscript{1}PhD Scholar, Jain University, Bangalore, India.
\textsuperscript{1}Department of Electronics and Communication Engineering
\textsuperscript{2}Director of School of Engineering and Technology
\textsuperscript{1}RNS Institute of Technology, Bangalore, India.
\textsuperscript{2}Jain University, Bangalore, India.

Abstract

In this paper we present, a Cost Effective High Reliability Routing (CEHRR) in multi-channel multi-hop Cognitive Radio Networks. This is basically a graph based and centralized approach. In CEHRR, we choose both the overall cost and the reliability of the links as the routing metric to find the best route. In our scheme, the path reliability is maximised while keeping the overall cost minimum. The simulation results show that the performance of CEHRR is better when compared to the methods where the cost and the reliability are considered separately.

Keywords: Cognitive radio networks, Dijkstra algorithm, Cost Effective High Reliability Routing, cost per received packet.

INTRODUCTION

The greedy requirement of spectrum resource by the wireless system, inefficient allocation of frequency and underutilization of licensed bands has raised scarcity of available spectrum resource. The available spectrum resource is solely dedicated to licensed/primary users and regulated by national governments. Many allocated spectrum is wasted, as many of the licensed users like TV broadcasting are underutilized. The spectrum utilization varies between 15% to 85% for the allocated frequency. In most of the spectrum bands, 90% of the time, spectrum is not used by the user. One of the major design criteria of wireless systems is the efficient use of radio spectrum. Hence secondary users (SU) conception aroused. Secondary/unlicensed users are the opportunistic users, where the licensed bands of primary users (PU) are utilised by SU whenever the PU are not utilising the spectrum. In 1998 J.Mitola coined the concept of cognitive radio (CR). In recent times, Cognitive Radio Networks (CRN’s) have become economically attractive and popular for different applications [1], [2], [3].

CR is an intelligent system, which dynamically changes the frequency and has ability to reconfigure the transmitter parameters. The discontinuous unused frequency bands are known as white space or spectrum holes. The CR finds the white space and allows SU to use until PU become active. Spectrum sensing methods [4] are used to find the presence or absence of licensed user. Whenever PU becomes active SU depart from accessing the spectrum allocated for PU.

At present, with a few exceptions, almost all the available radio spectrum bands are regulated and allocated exclusively to licenced users who are called Primary Users (PUs). TV broadcasters/receivers, mobile communication service providers/users are some of the PUs. In general, a radio spectrum band may not be fully utilized by its PUs. The unused spectrum bands with respect to time, space and frequency are called Spectrum Holes. These Spectrum Holes are utilized by Secondary Users (SUs) for communication among themselves.

Two given SUs can have communication between them, provided at least one common spectrum hole is available for the link that connects them. However, the availability of a required spectrum hole for a specific link is a random phenomenon based on the activities of the PUs. In a multi-hop topological scenario, the overall successful communication between a source and a remote (multi-hop) destination depends on the reliability of individual links connecting them. When several multi-hop paths can be formed to connect a source to a destination, our objective is to find the optimal path that provides the highest reliability at the lowest possible cost. The effective link (edge) weights are appropriately defined to take care of both the reliability and the cost together. Then we use Dijkstra algorithm for determining the optimal path that minimizes the total weight among all the possible paths from the source to the destination.

In Section II, we give the basic CRN model, the basic assumptions, symbols/notations, and associated relations among them. Section III, describes a CRN as a random graph
and then gives the main algorithm that finds the optimal path.

**Contributions**

Dijkstra algorithm is used to find the best shortest path in the network. Our objective is to find that optimal path which minimizes Cost Per Received Packet (CPRP). The CPRP takes care of both reliability and the actual cost. The optimal path is the best route from the source to the destination. We modify CPRP to get Modified CPRP (MCPRP). The modification is done such that minimization of MCPRP implies the minimization of CPRP.

**Organisation of the report**

The rest of this paper is organized as follows. An overview of related work is presented in section II. The system model is provided in section III. The proposed algorithm is presented in section IV. Section V presents the simulation result and performance analysis, and section VI concludes this paper.

**RELATED WORKS**

In depth and extensive research work has been going on in the study of efficient routing in the CRN domain. The inherent complex nature of the routing problems in CRNs has provided abundant scope for theoretical and empirical research activities in this field. Several survey papers are on routing protocols and routing metrics in CRNs [5-10] [100-109].

**SYSTEM MODEL**

**Assumptions**

We presume that spectrum sensing is completed and channel availability is known priorly. The set of channels which are ready for communications are followed by each user in the network. Every user is set by single radio which can either transmit or receive once at a time. We consider directed graph with bidirectional links, static nodes and isotropic antenna. We also assume that each user is capable of switching from one frequency to another frequency. Whenever the user is transmitting through particular channel, other user is made inactive/silent in accessing the channel is considered.

**Basic CRN model, symbols and notations**

Consider a geographical area covered by M primary wireless subnets designated by PWSN(1), PWSN(2),..., PWSN(M). Each primary wireless subnet has an Access Point (AP) and a set of Primary Users as shown in Fig.1.

In Fig.1, AP(j) is the Access Point for PWSN(j) for j =1 to M. Primary Users are denoted by pu(j,1), pu(j,2) and so on. (Only 3 of them are shown in Fig.1.) The collection of pu(j,1), pu(j,2) etc. is represented by the Primary User set as,

\[ PU(j) = \{ pu(j,1), pu(j,2), \ldots, \} \quad \text{for } j = 1 \text{ to } N \]

The coverage area(j) is indicated by the enclosing circle. Secondary Users within coverage area(j) are su(j, 1), su(j, 2) and so on. (Only two secondary users are shown in Fig.1.) The communication channel for PWSN(j) is denoted by ch(j). We assume that each PWSN is served by a single communication channel. Thus ch(j) is used for communication within PWSN for j = 1,2,..,M. The channels are non-overlapping (orthogonal) so that there is no inter channel interference even when coverage areas overlap.

Our Cognitive Radio Network comprises of M Primary Wireless Subnets with their respective Primary Users and a total of N Secondary Users. The Primary User sets (simply called as Primary User) are identified by PU(1), PU(2),...,PU(M). The secondary Users are identified by numbers 1, 2,..,N. In this paper, Secondary Users are also called nodes). The Secondary Users are assumed to be static. The M orthogonal communication channels, one exclusively for each PWSN which are represented by symbols ch(1), ch(2),..,ch(M) form the channel set of the CRN and it is represented as,

\[ CH = \{ ch(1), ch(2), \ldots, ch(M) \} \]

**Channel Model**

A specific wireless communication channel is a radio frequency band with centre frequency fc and a bandwidth of BW. In our scheme, for the ease of understanding, we assume IEEE 802.11a standard wi-fi channels. 802.11a provides 24 non-overlapping channels of 20 MHz bandwidth. Each channel can support text, audio and video. The normal indoor communication range is about 35 meters. Modulation used is Orthogonal Frequency Division Multiplexing (OFDM). Theoretical maximum data rate is 54
Mbps. Each wireless device has an isotropic antenna and the standard transmission power is taken as 40mW with an antenna gain of 6 db.

**CRN architecture:** The basic configuration of a typical CRN is shown in Fig. 2. In Fig. 2, the number of Primary Wireless Subnets M = 4, the number of secondary users, N = 6. Hereafter, PWSN(j) and the corresponding channel ch(j) are used interchangeably since they cover the same set of users. A channel can cover a subset of SUs. The subset of SUs covered by channel ch(j) is denoted by SS(j) for j = 1 to M. The subsets of SUs covered by different channels (subnets) are shown in Table 1. A Secondary User can belong to more than one channel. Thus, in figure 2, SUs 1 and 2 belong to ch(1), ch(2) and ch(4).

<table>
<thead>
<tr>
<th>Channel</th>
<th>Boundary Colour</th>
<th>Subset of SUs</th>
</tr>
</thead>
<tbody>
<tr>
<td>ch(1)</td>
<td>Black</td>
<td>SS(1) = {1, 2, 3}</td>
</tr>
<tr>
<td>ch(2)</td>
<td>Red</td>
<td>SS(2) = {1, 2, 4, 5}</td>
</tr>
<tr>
<td>ch(3)</td>
<td>Green</td>
<td>SS(3) = {3, 6}</td>
</tr>
<tr>
<td>ch(4)</td>
<td>Blue</td>
<td>SS(4) = {1, 2, 5, 6}</td>
</tr>
</tbody>
</table>

**Table 1. Channels and the Corresponding Subsets**

A. **Channel Accessibility Matrix**

At a specified time t, a secondary user may have access to several primary channels. This information is represented by the Channel Accessibility Matrix [ref] CA(t). The size of CA(t) is NxM. The rows of CA(t) represent the SUs while the columns represent the channels. CA(t) is a binary matrix and its element ca(i, j, t) at row i and column j at t is defined as,

\[
ca(i,j,t) = \begin{cases} 
1 & \text{if SU } i \text{ is within the communication range of } ch(j) \\
0 & \text{otherwise}
\end{cases}
\]

For the configuration of Fig. 1, matrix CA(t) is given by the values as shown in Table 2. The elements CA(t) depends on the geographical locations of the SUs and the locations of the primary base stations, their power levels and the signal to noise ratio’s at the receivers. For non-mobile SUs, CA(t) is independent of t. Then we use the symbol CA instead of CA(t).

<table>
<thead>
<tr>
<th></th>
<th>ch(1)</th>
<th>ch(2)</th>
<th>ch(3)</th>
<th>ch(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>SU2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>SU3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>SU4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SU5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>SU6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 2. Elements of CA for the topology of Fig. 1**

B. **Channel Accessibility Indicator**

Consider the number of 1’s among the elements of CA matrix. If all the elements are 1’s, every channel is accessible by every secondary nodes. This corresponds to 100 per cent accessibility. On the other hand if the number 1’s in CA is zero, no channel is accessible. This means 0 per cent accessibility. If the number of 1’s in CA is high, it means the channel accessibility is also high. Therefore, for a given CRN, the Channels Accessibility Indicator (CAI) as the ratio of the total number of 1’s in CA to the total number of elements of CA. That is, we define CAI as,

\[
CAI = \frac{\text{Number of } 1's \text{ in matrix CA}}{M \times N}
\]

Here, M and N are the number of rows and columns of matrix CA. The product M*N gives the total number of elements in CA. The value of CAI can vary from 0 to 1. A high value of
CAI means the degree of channel accessibility is high.

C. Host Channels for nodes

For a given Secondary User i, those channels which are accessible to i are designated as the Host Channels of i. The set holding the host channels of SU i designated as HCN(i) which stands for Host Channels for Node i. Here, the members of set HCN(i) are the host channels of Secondary User i. We say that set HCN(i) covers SU i, in the sense that the members of HCN(i) are accessible for communication to SU i, that is node i. From the example of Table 2,

\[
\begin{align*}
HCN(1) &= \{ \text{ch}(1), \text{ch}(2), \text{ch}(4) \} \\
HCN(2) &= \{ \text{ch}(1), \text{ch}(2), \text{ch}(4) \} \\
HCN(3) &= \{ \text{ch}(1), \text{ch}(3) \} \\
HCN(4) &= \{ \text{ch}(2) \} \\
HCN(5) &= \{ \text{ch}(2), \text{ch}(4) \} \\
HCN(6) &= \{ \text{ch}(3), \text{ch}(4) \}
\end{align*}
\]

From this, we can see that, HCN(i) is the collection of the column indices of the ones of row i in matrix CA.

D. Host Channels for secondary communication links

Consider the communication between node i and node k. For the possible existence of communication between i and k, both should be covered by a common host channel ch(j). This means, ch(j) should be a host channel for both node i and node k. That is, ch(j) belongs to both HCN(i) and HCN(k). Therefore, those host channels which can provide communication between i and k are obtained by the intersection of set HCN(i) and HCN(k). Let us use the symbol HCL(i, k) to represent the set of channels which can provide communication between i and k. Then,

\[
HCL(i, k) = HCN(i) \cap HCN(k)
\]

for i, k = 1 to N with i ≠ k.

E. Communication among the members of the subset

The members (Secondary Users) of a subset can communicate among them when the corresponding channel is not occupied by that Primary User. Thus when ch(j) is not occupied by PU(j), members of SS(j) can communicate among themselves for j = 1 to M. In Fig. 1, when ch(2) is not occupied by PU(2), the SUs, 1, 2, 4, 5 who are the members of SS(2), can communicate among themselves using ch(2). But nodes (SUs) 3 and 4 cannot communicate because they belong to different channels. Therefore any node i and any node k can communicate when they belong to the same channel and when that channel is not occupied by its Primary User for i, k =1 to N and i ≠ k. When a channel is not occupied by its primary user, the channel is said to be available or free. When free, Secondary Users can use that channel.

F. Time slots and Channel States

In our time sharing cognitive radio network, we use slotted-time model. In this model, the secondary users utilize the free channel in terms of time slots, t, t+1, t+2, and so on as shown in Table 3. Here, t is the current time slot under consideration. (t+1) is the next time slot and (t−1) is the previous time slot and so on. The interval of each slot T is chosen appropriately based on the granularity of the available opportunities.

<table>
<thead>
<tr>
<th>T−2</th>
<th>T−1</th>
<th>T</th>
<th>T+1</th>
<th>T+2</th>
</tr>
</thead>
</table>

A channel may be occupied by its primary user or it may be free at a given time slot t. This information is called the Channel State. The Channel state of ch(j) at time slot t is represented by the symbol CS(j, t) which is a binary variable defined as,

\[
CS(j) = \begin{cases} 
1 & \text{if } ch(j) \text{ is free for SU's at } t \\
0 & \text{if } ch(j) \text{ is occupied by PU's at } t
\end{cases}
\]

It is understood that CS(j, t) is same as CS(ch(j), t).

Therefore, secondary communication among the subset members of SS(j) in time slot t is possible when CS(j, t) = 1.

G. Communication opportunity between secondary users

Consider two SUs (nodes) i and k covered by multiple primary channels. The host channels covering the node pair (i, k) is represented by the set HCL(i, k) as given by Eq. (3). Let the members of HCL(i, k) be designated by ch(B(i, k, u)), for u =1, 2,…,Q(i, k) where Q(i, k) is the total number of channels that cover the node pair (i, k). That is, Q(i, k) = |HCL(i, k)|. Here, B(i, k, u)’s are the channel indices of those channels which cover the node pair (i, k). Index B(i, k, u)∈ {1:M}. Using these notations, set HCL(i, k) can be expressed as,

\[
HCL(i, k) = \bigcup_{u=1}^{Q(i,k)} ch(B(i,k,u))
\]

Here, symbol U represents union of the elements forming the set. In the configuration of Fig. 1, for node pair (1, 2),
HCL(1, 2) = \{ ch(1), ch(2), ch(4) \}
In this case, Q(1, 2) = 3 and,
ch(B(1,2,1)) = ch(1)
ch(B(1,2,2)) = ch(2)
ch(B(1,2,3)) = ch(4)
Similarly, HCL(3,6) = \{ch(3)\} and Q(3,6) = 1.
But, HCL(3,4) = \{\} = empty and Q(3,4) = 0 as the node pair (3,4) is not covered by any channel. Communication between nodes 3 and 4 is not possible because no common channel is available between them. For successful communication between node pair (i, k) at time slot t, the following two conditions are to be satisfied. First, HCL(i, k) should not be empty. That is,
Q(i, k) > 0                                                             (5)
The second condition is, at least one of the channels covering the node pair should be free from the Primary user at t. Then the node pair can communicate using that channel. Let the cover set of host channels covering node pair (i, k) be HCL(i, k) as already enumerated by Eq. (4). Let the corresponding channel states of the members of HCL(i, k) at t be represented as a column vector (for spatial convenience).
\[
\text{CS}(\text{HCL}(i, k), t) = \begin{bmatrix}
\text{CS}(B(i,k,1), t) \\
\text{CS}(B(i,k,2), t) \\
\vdots \\
\text{CS}(B(i,k,Q(i,k)), t)
\end{bmatrix}
\]                                                             (6)
In this case, CS(HCL(i,k), t) is a binary vector. The second condition stated above, means at least one of the elements of this vector should be 1. This can be represented by the Boolean equation as,
\[
\bigvee_{x=1}^{Q(i,k)} \text{CS}(B(i,k,x), t) = 1
\]                                                             (7)
where x is the index of enumeration.

H. Probability of the availability of channels

In CRN, a particular channel, licensed to certain PU may or may not be available to the SUs at a specified time slot. The availability is a probabilistic event. Thus the status of channel ch(j) at time slot t, represented by CS(j, t) is a discrete binary random variable at a given t. We assume that CS(j, t)'s are independent and identically distributed random variables over j for a specific t. Let us represent the probability that CS(j, t) = 1 (which means ch(j) is available at t) by the symbol p(j, t). That is,
p(j, t) = \text{Prob}(CS(j, t) = 1)                                     (8)
for j = 1 to M.
Then, (1− p(j, t)) is the probability that CS(j, t) = 0.

I. Link Connectivity

Consider a node pair(i, k) for i, k = 1 to N and i ≠ k and t the corresponding single hop communication link(i, k, t) at time slot t. If pair(i, k) can communicate in the time slot t, then link(i, k, t) is connected (ON). Otherwise it is OFF. The link connection between i and k may or may not exist depending on whether Eqs. (5) and (7) are satisfied or not. Let us now, consider only those node pairs(i, k) for which Eq. (5) is satisfied. Then we have to consider the satisfiability of Eq. (7) only. It depends on the channel states of the covering channels of Eq. (7).

The channel states corresponding to Eq. (7) are, CS(B(i,k,x),t) for x = 1 to Q(i, k). The probability that CS(B(i,k,x),t) =1 is given by p(B(i,k,x),t) . For satisfying Eq. (7) at least one of the term of the LHS Eq.(7) should be 1. That is all the terms should not be zeros. Therefore, the probability of satisfying Eq. (7) is, same as, the probability that all terms are not zeros. Under this condition, the link(i,k) is connected. Now, the probability that link(i,k) is not connected at all is given by,
\[
\text{Prob}(\text{link}(i,k,t) = \text{OFF}) = \prod_{x=1}^{Q(i,k)} (1 - p(B(i,k,x),t))
\]                                                             (9)
Now, we define the link connectivity of link(i, k, t) designated by L(i, k, t), as the probability that link(i, k, t) is connected (ON). Link connectivity is somewhat similar to link reliability.
Therefore, using Eq. (9), \( L(i, k, t) \) is given by [ref],

\[
L(i, k, t) = \Prob(link(i, k, t) = \text{connected} (\text{ON}) = \\
1 - \prod_{x=1}^{Q(i,k)} (1 - p(B(i, k, x), t)) = \prod_{x=1}^{Q(i,k)} (1 - p(B(i, k, x), t))
\]  

(10)

Eq. (10) corresponds to parallel connectivity, similar to parallel reliability. Thus \( L(i,k,t) \) gives the reliability of link \((i,k)\). Considering the configuration shown in Fig. 1. At time slot \( t \), let the channel availability probabilities for \( \text{ch}(j) \) be \( p(j, t) \) as given by Eq. (8) for \( j = 1 \) to 4. Now consider link\((1, 2)\). The covering set for link\((1, 2)\) is \( C(1, 2) = [\text{ch}(1), \text{ch}(2), \text{ch}(4)] \). The channel indices \( B(1,2,x) \)'s are \([1, 2, 4]\). Therefore, from Eq. (10), \( L(1, 2, t) \) is given by,

\[
L(1, 2, t) = 1 - \left( (1 - p(1, t)) \times (1 - p(2, t)) \times (1 - p(4, t)) \right)
\]  

(11)

Similarly, it can be shown that,

\[
L(3, 6, t) = 1 - \left( (1 - p(3, t)) \right) = p(3, t)
\]

\[
L(3, 4, t) = 0 \text{ because there is no connectivity at all.}
\]

\[
L(2, 5, t) = 1 - \left( (1 - p(2, t)) \times (1 - p(4, t)) \right) \text{ because link}(2,5,t) \text{ is covered by ch}(2) \text{ and ch}(4). \text{ Similarly other link connectivity's can be calculated.}
\]

**CRN as a Random Graph**

The given CRN is represented by an undirected graph \( G(V, E, t) \) where \( V \) is a set of \( N \) vertices which are the Secondary Users (nodes) of the network whose identities are \( 1, 2, \ldots, N \). The vertices are static and do not change with \( t \). The edge set is \( E \). The elements of the edge set \( E \) are the edges or links represented by link \((i, k, t)\)'s between nodes \( i \) and \( k \) for \( i, k = 1 \) to \( N \) with \( i \neq k \). Each communication link is bidirectional and the edge weights are the link connectivity values \( L(i, k, t) \) with \( 0 \leq L(i, k, t) \leq 1 \). The adjacency matrix of the graph is designated as \( A(t) \). The element of \( A(t) \) at row \( i \) and column \( k \) is the edge weight \( L(i, k, t) \), for \( i, k = 1 \) to \( N \). Thus the size of \( A(t) \) is \( N \times N \). Unconnected links whose \( L(i, k, t) \)'s are zeros, are not shown in the graph. \( L(i, i, t) = 0 \) (no self loop).

**A. Paths in the graph**

A path from a source node \( src \) to a destination node \( dst \) is a sequence of non-repeating adjacent (one hop) nodes starting from \( src \) and ending at \( dst \). Non repetition of nodes assures that the path is free of loops. Adjacency of nodes along the path assures continuous connectivity from the source to the destination. There can be several paths from \( src \) to \( dst \) in a given graph (network). Consider a specific communication path from \( src \) to \( dst \), at a given time slot \( t \), denoted by \( \text{path}(J) \), as shown in Fig. 2.

In Fig. 2, path\((J)\) is made up of \( H \) number of edges (single hop links) as, \( e(1), e(2), \ldots, e(H) \). The respective link connectivity’s (probabilities) are represented by \( r(1), r(2), \ldots r(H) \). That is,

\[
r(u) = \Prob(e(u) = \text{ON})
\]  

(12)

for \( u = 1,2,\ldots,H \). Link reliability \( r(u) \) is same as \( L(i, k) \) where \( i \) and \( k \) are the two ends of link \( u \). Therefore \( r(u) \)'s are calculated using Eq. (10).

**Link Cost**

In Fig. 2, the cost of using a link depends on the cost of acquiring the link, energy cost of transmission, etc. This cost is called the nominal cost. Let the nominal cost of using link \( e(u) \) to send a packet of data be \( c(u) \) in appropriate units, for \( u = 1 \) to \( H \).

1. **Overall probability of the path**: Since the links of path\((J)\) are in series, the overall connectivity of the whole path is,

\[
\Prob(\text{path}(J) \text{ is connected}) = R(J) = r(1) \times r(2) \times \ldots \times r(H)
\]  

(13)

Let the number of packets forwarded from the source towards the destination, be \( n \). Then, because of the probabilistic links, the number of packets received would be \( n \times R(J) \). Thus,

\[
\text{NPR} = n \times R(J)
\]  

(14)

Here, NPR represents the number of packets received by the destination when \( n \) packets are forwarded by the source. Total cost of forwarding \( n \) packets along path\((J)\) of length \( H \), represented by \( TC \) is,

\[
TC = n \times c(1)+n \times c(2)+\ldots+n \times c(H)
\]  

(15)

Therefore, the Cost PerReceived Packet represented by CPRP is calculated using Eqs. (14) and (15). The result is,

\[
\text{CPRP} = \frac{TC}{\text{NPR}} = \frac{n \times c(1)+n \times c(2)+\ldots+n \times c(H)}{n \times R(J)}
\]  

On simplifying the RHS of this equation, we get,

\[
\text{CPRP} = \frac{c(1)+c(2)+\ldots+c(H)}{R(J)}
\]  

(16)

From Eqs. (16) and (13),

\[
\text{CPRP} = \frac{c(1)+c(2)+\ldots+c(H)}{r(1) \times r(2) \times \ldots \times r(H)}
\]  

(17)

Our objective is to find that path which minimizes CPRP.
B. Minimization of CPRP

The direct method is to enumerate all possible paths and then calculate all possible CPRP's and then to find the minimum. But this method is NP-hard [15], because the number of possible paths from the source to the destination, exponentially increases with the number of vertices. The second approach is to use Dijkstra's shortest path method. In Dijkstra method, sum of the distances or weights of the links is minimized. Therefore, CPRP given by Eq. (17) can be minimized, if the RHS of Eq. (17) can be expressed in the form of sum of weights. Since the RHS of Eq. (17) cannot be directly expressed as the sum of weights, we use modified CPRP. We represent the modified CPRP as MCPRP. The modification is done such that minimization of MCPRP implies the minimization of CPRP.

1. Modification of CPRP: We know that $e^x$ is minimum when $x$ is minimum for real values of $x$. That is if $x_1 < x_2$, then $e^{x_1} < e^{x_2}$. Therefore, the numerator of the RHS of Eq. (17) is replaced by, $e^{c(1)+c(2)+...+c(H)}$. Thus the first modification represented by FCPRP is,

\[
FCPRP = \frac{e^{c(1)+c(2)+...+c(H)}}{r(1) * r(2) * ... * r(H)}
\]  

(18)

Therefore, minimization of FCPRP implies the minimization of CPRP. The numerator of the RHS of Eq. (18) is further rewritten as,

\[
e^{c(1)+c(2)+...+c(H)} = e^{c(1)} * e^{c(2)} * ... * e^{c(H)}
\]

Using this identity in Eq. (18) we get,

\[
FCPRP = \frac{e^{c(1)} * e^{c(2)} * ... * e^{c(H)}}{r(1) * r(2) * ... * r(H)}
\]  

(19)

Now, modified CPRP is expressed as,

\[
MCPRP = \log(FCPRP)
\]  

(20)

Note that, minimizing FCPRP automatically minimizes its logarithm for FCPRP > 0. Therefore minimization of MCPRP also minimizes FCPRP which in turn minimizes CPRP. Now, from Eqs. (19) and (20),

\[
MCPRP = \log\left(\frac{e^{c(1)} * e^{c(2)} * ... * e^{c(H)}}{r(1) * r(2) * ... * r(H)}\right)
\]  

(21)

Eq. (21) can be expressed as,

\[
MCPRP = \log\left(\frac{e^{c(1)}}{r(1)}\right) + \log\left(\frac{e^{c(2)}}{r(2)}\right) + ... + \log\left(\frac{e^{c(H)}}{r(H)}\right)
\]  

(22)

Thus, \[MCPRP = w(1)+w(2)+...+w(H)\] (23)

where, $w(1)$, $w(2)$, ..., $w(H)$ are the equivalent weights of the corresponding links (or edges). That is, for edge $e(u)$,

\[
w(u) = \log\left(\frac{e^{c(u)}}{r(u)}\right) = c(u) - \log(r(u))
\]  

(24)

for $u = 1, 2, ..., H$. Here, $c(u)$ and $r(u)$ are the cost and link reliability of edge $e(u)$ respectively. In Eq. (24), $-\log(r(u))$ is a non-negative number, because $r(u)$ is ≤ 1. From Eqs. (23) and (24), we see that, the CPRP minimization problem is reduced to that of minimizing the sum of weights or the shortest path problem. That path which minimizes MCPRP also minimizes CPRP. MCPRP is minimized using Dijkstra algorithm whose time complexity is $[ref]O(E*\log(N))$ where $E$ is the total number of edges in the graph and $N$ is the number of vertices.

In conventional graphics, an edge is identified by its end points (vertices) as shown in Fig. 3. An edge $e(u)$ is represented by $e(i, k)$ where $i$ and $k$ are the two ends of $u$. That is, edge $u$ joins node $i$ and $k$. Cost $c(u)$ is represented by $c(i, k)$, $r(u)$ by $r(i, k)$ and $w(u)$ by $w(i, k)$. This is called the two index notation of an edge. In graph shortest path algorithms, two index edge notations are very much used.

\[i\quad e(u) = e(i, k) \quad k\]

\[c(u) = c(i, k)\]

\[r(u) = r(i, k)\]

\[w(u) = w(i, k)\]

**Figure 3.** Two index notation of an edge

Note that at a specified $t$, $r(i, k) = L(i, k, t)$ for $i, k = 1$ to $N$ with $i \neq k$. Since $w(i, k) = w(u)$, $c(i, k) = c(u)$ and $r(i,k) = r(u)$, from Eq. (24),

\[
w(i, k) = \log\left(\frac{e^{c(u)}}{r(u)}\right) = c(i, k) - \log(r(i, k))
\]  

(25)

**PROPOSED ALGORITHM**

Various steps involved in determining the best path at a given time slot $t$ is given below.

**Algorithm 1.**

Inputs: CRN network configuration and details. Number of host channels $M$, number of SU’s $N$ and Channel probabilities $p(j)$ at $t$ for $j = 1$ to $M$. Cost of every link $c(i, k)$ for $i, k = 1$ to $N$ with $i \neq k$. Cost of non-existing links are taken as $+\infty$. Source and destination nodes for the given data transmission.

Output: The optimal path.

1. From the topology and the configuration of the CRN, get the channel Accessibility matrix $CA$. Here, we assume that $CA$ is independent of time because of static SUs. The size of $CA$ is $N \times M$.
2. From $CA$, get all the HCN($i$)’s for $i = 1$ to $N$. 
3. For every $i \neq k$, $L(i, k, t) = \log\left(\frac{e^{c(i, k)}}{r(i, k)}\right) = c(i, k) - \log(r(i, k))$. 
4. The two index edge notations are used.

Indicates that the input to the algorithm is the topological configuration and details of the CRN network, including the number of host channels, the number of SU’s, and the channel probabilities at a given time $t$. The algorithm then proceeds to determine the optimal path for data transmission by considering the channel accessibility matrix, which provides information about the available channels between any two nodes. The channel reliability and cost are also considered to calculate the channel accessibility for each edge. The algorithm uses Dijkstra’s shortest path algorithm to find the path with the minimum sum of weights (MCPRP), which is equivalent to minimizing the CPRP. The time complexity of this algorithm is $O(E \cdot \log(N))$, where $E$ is the total number of edges and $N$ is the number of vertices in the graph. This complexity is achieved by using the priority queue in Dijkstra’s algorithm, which allows efficient selection of the next vertex to visit based on the minimum distance. The algorithm then identifies the best path for data transmission by considering both the channel accessibility and the associated channel reliability and cost.
3. Get all the links (edges) and their weights as follows.

3.1. For i = 1 to N−1
3.2. For k = i+1 to N
3.3. Calculate set HCL(i, k) as,
HCL(i, k) = HCN(i) ∩ HCN(k)
3.4. If HCL(i, k) is empty,
take w(i, k) = +∞.
Continue (go to 3.2)
Endif
3.5. If HCL(i, k) is not empty,
1. Get Q(i, k) as,
Q(i, k) = |HCL(i, k)|
2. Enumerate all the members of HCL(i, k) as
   Given by Eq. (4). That is,
   Get B(i, k, x)'s for x = 1 to Q(i, k).
   3. Get L(i, k, t) using Eq. (10).
   4. Set r(i, k) = L(i, k, t).
   5. Get w(i, k) using Eq. (25).
// All w(i, k)'s are +ve.
Endif
Endfor k
Endfor i
// Now w(i,k)'s are available
   // for i, k = 1 to N with i ≠ k.

4. Apply Dijkstra shortest path algorithm using w(i, k)'s
   as the weights, to get the best path from the source to
   the destination.

C. A simple Example

Example 1.

The CRN configuration is as shown in Fig.1. The Channel State probabilities at t, are taken as, p(1, t) = 0.8, p(2, t) = 0.7, p(3, t) = 0.4 and p(4, t) = 0.6. The corresponding connectivity
   graph is shown in Fig. 4. Primary Users and other details are
   not shown in Fig. 4.

Figure 4. A simple CRN with 6 SUs

The assumed cost adjacency matrix is shown in Table 4.

<table>
<thead>
<tr>
<th>i, k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2.2</td>
<td>0.3</td>
<td>1.2</td>
<td>2.1</td>
<td>3.9</td>
</tr>
<tr>
<td>2</td>
<td>2.2</td>
<td>0</td>
<td>0.7</td>
<td>3.6</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>0.7</td>
<td>0</td>
<td>∞</td>
<td>∞</td>
<td>3.0</td>
</tr>
<tr>
<td>4</td>
<td>1.2</td>
<td>3.6</td>
<td>∞</td>
<td>0</td>
<td>3.8</td>
<td>∞</td>
</tr>
<tr>
<td>5</td>
<td>2.1</td>
<td>0.8</td>
<td>∞</td>
<td>3.8</td>
<td>0</td>
<td>0.75</td>
</tr>
<tr>
<td>6</td>
<td>3.9</td>
<td>0.9</td>
<td>3.0</td>
<td>∞</td>
<td>0.75</td>
<td>0</td>
</tr>
</tbody>
</table>

The link reliability values r(i, k)'s are calculated using Eq.
(10) and the these are shown in Table 5.

<table>
<thead>
<tr>
<th>i, k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.976</td>
<td>0.800</td>
<td>0.700</td>
<td>0.880</td>
<td>0.600</td>
</tr>
<tr>
<td>2</td>
<td>0.976</td>
<td>0</td>
<td>0.800</td>
<td>0.700</td>
<td>0.880</td>
<td>0.600</td>
</tr>
<tr>
<td>3</td>
<td>0.800</td>
<td>0.800</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.400</td>
</tr>
<tr>
<td>4</td>
<td>0.700</td>
<td>0.700</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.700</td>
</tr>
<tr>
<td>5</td>
<td>0.880</td>
<td>0.880</td>
<td>0</td>
<td>0</td>
<td>0.700</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.600</td>
<td>0.600</td>
<td>0.400</td>
<td>0</td>
<td>0.600</td>
<td>0</td>
</tr>
</tbody>
</table>

The weight adjacency matrix given by Eq. (25) is calculated
and shown in Table 6.
Table 6. $w(i, k)$ values for $i, k = 1$ to $N$

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2.2243</td>
<td>0.5231</td>
<td>1.7567</td>
<td>2.2278</td>
<td>4.4108</td>
</tr>
<tr>
<td>2</td>
<td>2.2243</td>
<td>0</td>
<td>0.9231</td>
<td>3.9567</td>
<td>0.9278</td>
<td>1.4108</td>
</tr>
<tr>
<td>3</td>
<td>0.5231</td>
<td>0.9231</td>
<td>0</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>3.9163</td>
</tr>
<tr>
<td>4</td>
<td>1.7567</td>
<td>3.9567</td>
<td>$\infty$</td>
<td>0</td>
<td>4.1567</td>
<td>$\infty$</td>
</tr>
<tr>
<td>5</td>
<td>2.2278</td>
<td>0.9278</td>
<td>$\infty$</td>
<td>4.1567</td>
<td>0</td>
<td>1.2608</td>
</tr>
<tr>
<td>6</td>
<td>4.4108</td>
<td>1.1408</td>
<td>3.9163</td>
<td>$\infty$</td>
<td>1.2608</td>
<td>0</td>
</tr>
</tbody>
</table>

The result of single source shortest path Dijkstra algorithm for this graph with src = 4 is shown in Table 7. The optimal path from src 4 to dst 6 is found to be 4-1-3-2-6.

Table 7. Minimum Weight Shortest paths from src 4

<table>
<thead>
<tr>
<th>Path</th>
<th>Minimum Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 1</td>
<td>1.7567</td>
</tr>
<tr>
<td>4 1 3 2</td>
<td>3.2030</td>
</tr>
<tr>
<td>41 3</td>
<td>2.2798</td>
</tr>
<tr>
<td>4 1 5</td>
<td>3.9845</td>
</tr>
<tr>
<td>4 1 3 2 6</td>
<td>4.6138</td>
</tr>
</tbody>
</table>

COMPARITIVE RESULT ANALYSIS

A. CRPP versus Number of primary channels

Here, we compare the objective function CRPP values obtained in three cases. In the first case we use our CEHRR method. In the second and third cases we use “Cost only” and “Reliability only” methods respectively. In CEHRR both the link cost and the link reliability are combined to form a single link attribute to calculate the optimal path. In this case $w(i, k)$ is given by Eq. (25). In the “Cost only” method, the link cost alone is taken into consideration. Here, $w(i, k) = c(i, k)$ and $r(i, k)$ is ignored. In the “Reliability only” method, $w(i, k) = -\log(r(i, k))$. In this case, only the link reliability is taken into account and $c(i, k)$ is ignored. In all the cases, the channel accessibility matrix CA is chosen randomly.

Experiment 1. To demonstrate the relative performance, we take a CRN having 25 SUs (nodes). The number of primary Host Channels is varied from 7 to 12 in steps of one. The cost adjacency matrix, the channel accessibility matrix and the link reliability values are generated randomly within the appropriate ranges. The link cost depends on the cost of acquiring, running and maintaining the link. The length of the link, the bandwidth of the link, the ambient noise level and the location of the link etc., contribute to the overall cost of the link. The calculated CRPP values versus M are shown in Table 8. The corresponding graph is shown in Fig. 5.

Table 8. CPRP values in three cases. N = 25

<table>
<thead>
<tr>
<th>M values ↓</th>
<th>CEHRR</th>
<th>Cost only</th>
<th>Reliability only</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.8867</td>
<td>3.3657</td>
<td>6.3186</td>
</tr>
<tr>
<td>6</td>
<td>2.8867</td>
<td>3.3657</td>
<td>4.1941</td>
</tr>
<tr>
<td>7</td>
<td>2.8867</td>
<td>3.3657</td>
<td>4.0377</td>
</tr>
<tr>
<td>8</td>
<td>2.4840</td>
<td>3.3657</td>
<td>3.1751</td>
</tr>
<tr>
<td>9</td>
<td>2.3368</td>
<td>2.5544</td>
<td>3.0513</td>
</tr>
<tr>
<td>10</td>
<td>2.1226</td>
<td>2.5544</td>
<td>2.8555</td>
</tr>
<tr>
<td>11</td>
<td>1.6932</td>
<td>1.6932</td>
<td>3.5767</td>
</tr>
<tr>
<td>12</td>
<td>1.4294</td>
<td>1.6932</td>
<td>6.4078</td>
</tr>
<tr>
<td>13</td>
<td>1.4294</td>
<td>1.6932</td>
<td>5.8201</td>
</tr>
<tr>
<td>14</td>
<td>1.4294</td>
<td>1.6932</td>
<td>6.0292</td>
</tr>
<tr>
<td>15</td>
<td>1.4294</td>
<td>1.6932</td>
<td>5.8856</td>
</tr>
<tr>
<td>16</td>
<td>1.4294</td>
<td>1.6932</td>
<td>4.1366</td>
</tr>
<tr>
<td>17</td>
<td>1.3065</td>
<td>1.4430</td>
<td>2.7657</td>
</tr>
<tr>
<td>18</td>
<td>1.1272</td>
<td>1.1272</td>
<td>2.6288</td>
</tr>
</tbody>
</table>

From Fig. 5, we can see that, in general, CPRP decreases as the number of host channels increases.

B. Performance with respect to the Channel Accessibility

From Eq. (14), we see that the Packet Delivery Ratio (PDR) for path $J$, is given by,
Experiment 2. In this experiment, the Channel Accessibility Indicator (CAI) as given by Eq. (2) is uniformly increased in fixed steps and the corresponding values of R(J) are calculated for the optimal (shortest) path taking only reliability \((-\log(r))\) into account. Because, \(R(j)\) depends on the reliability of the individual links forming the path as given by Eq. (13). For the purpose of comparison, we have found the PDR’s for not only the first shortest path, but also for the second, the third and the fourth shortest paths. The results are shown in Table 9. The corresponding bar graph is shown in Fig. 6.

\[
PDR = \frac{\text{NPR}}{n} = R(j) \quad (25)
\]

Table 9. Variation of PDR with respect to CAI

<table>
<thead>
<tr>
<th>CAI</th>
<th>Path 1 Shortest</th>
<th>Path 2 2nd shortest</th>
<th>Path 3 3rd shortest</th>
<th>Path 4 4th shortest</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/25</td>
<td>0.4773</td>
<td>0.4773</td>
<td>0.4496</td>
<td>0.4288</td>
</tr>
<tr>
<td>13/25</td>
<td>0.6195</td>
<td>0.5865</td>
<td>0.5862</td>
<td>0.5710</td>
</tr>
<tr>
<td>14/25</td>
<td>0.6414</td>
<td>0.6265</td>
<td>0.6043</td>
<td>0.5957</td>
</tr>
<tr>
<td>15/25</td>
<td>0.8224</td>
<td>0.8034</td>
<td>0.7874</td>
<td>0.7792</td>
</tr>
<tr>
<td>16/25</td>
<td>0.8378</td>
<td>0.8378</td>
<td>0.8264</td>
<td>0.8264</td>
</tr>
<tr>
<td>17/25</td>
<td>0.8583</td>
<td>0.8583</td>
<td>0.8466</td>
<td>0.8466</td>
</tr>
<tr>
<td>18/25</td>
<td>0.8583</td>
<td>0.8583</td>
<td>0.8466</td>
<td>0.8466</td>
</tr>
<tr>
<td>19/25</td>
<td>0.8583</td>
<td>0.8583</td>
<td>0.8481</td>
<td>0.8466</td>
</tr>
<tr>
<td>20/25</td>
<td>0.8583</td>
<td>0.8583</td>
<td>0.8583</td>
<td>0.8481</td>
</tr>
</tbody>
</table>

From Fig. 6, we see that as CAI increases, PDR also increases. But, for higher values of CAI, the changes in PDR are insignificant.

Experiment 3. Here the variation of CPRP with respect to the Channel Accessibility Indicator is determined. In this case, \(N = 25\) and \(M = 9\). The CAI is varied from 12/25 to 20/25 as in Experiment 2. For determining CPRP, the link weights used are \(w(u)’s\) as given by Eq. (24). The numerical results are shown in Table 10 and the corresponding bar graph is shown in Fig. 7.

Table 10. Variation of CPRP with respect to CAI

<table>
<thead>
<tr>
<th>CAI</th>
<th>Path 1 Shortest</th>
<th>Path 2 2nd shortest</th>
<th>Path 3 3rd shortest</th>
<th>Path 4 4th shortest</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/25</td>
<td>1.6754</td>
<td>2.7715</td>
<td>3.5342</td>
<td>3.6829</td>
</tr>
<tr>
<td>13/25</td>
<td>1.6754</td>
<td>1.9675</td>
<td>2.7715</td>
<td>3.5342</td>
</tr>
<tr>
<td>14/25</td>
<td>1.6754</td>
<td>1.9675</td>
<td>2.7715</td>
<td>3.1213</td>
</tr>
<tr>
<td>15/25</td>
<td>1.0027</td>
<td>1.0660</td>
<td>1.2413</td>
<td>1.8192</td>
</tr>
<tr>
<td>16/25</td>
<td>0.8865</td>
<td>1.0236</td>
<td>1.1777</td>
<td>1.5037</td>
</tr>
<tr>
<td>17/25</td>
<td>0.7914</td>
<td>0.9835</td>
<td>1.1077</td>
<td>1.2514</td>
</tr>
<tr>
<td>18/25</td>
<td>0.7509</td>
<td>0.9835</td>
<td>1.0068</td>
<td>1.1347</td>
</tr>
<tr>
<td>19/25</td>
<td>0.7509</td>
<td>0.9631</td>
<td>0.9835</td>
<td>1.1347</td>
</tr>
<tr>
<td>20/25</td>
<td>0.7509</td>
<td>0.9631</td>
<td>0.9835</td>
<td>1.1347</td>
</tr>
</tbody>
</table>

From Fig. 7, we see that, at lower CAI’s, the differences in CPRP’s for successive shortest paths differ over wide range than at higher CAI’s.

Our method can be modified to take care of other parameters like energy levels, delay due to congestion, throughput, and bandwidth utilisation.
CONCLUSION

Using modified CPRP, the optimal path can be explored from source to destination. In MCPRP, cost reduction and reliability increase is achieved. We use principle of parallel and series reliability. The technique used in this paper can be modified into an equivalent distributed algorithm, implemented at individual nodes. This method can be easily adopted in wireless mesh network.

REFERENCES


