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On optimization of a Multi-Product EPQ Model with Scrap and an Improved Multi-Delivery Policy

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Abstract

This study presents an alternative approach to optimize a multi-product economic production quantity (EPQ) model with scrap and an improved multi-delivery policy. Traditional method for deriving the optimal common production cycle time is to apply differential calculus to the system cost function, whereas the present study proposes an algebraic approach without using differential calculus. Such a simplification may assist practitioners in resolving the problem more effectively.

Keywords: Optimization, multi-item EPQ, common production cycle time, multi-delivery plan, scrap

INTRODUCTION

With the aim of maximizing machine utilization, managers in manufacturing firms often plan to fabricate multiple products in turn on a single machine. Unsurprisingly, various aspects of multi-product systems have received extensive attentions by both researchers and practitioners for several decades [1-17]. Zahorik *et al*. [1] explored multi-product, multiple levels production planning problems with linear costs and production/inventory constraints at a key machine. Shipping capability constraint was assumed in their first studied model, and the bottleneck machine in the final stage was assumed in their second studied model. The problem of multi-item facilities-in-series was formulated as a linear program. A rolling heuristic for T-period problems were also examined, and computational comparisons to standard linear programming were provided. Clausen and Ju [2] investigated an economic lot and delivery scheduling problem (ELDSP). They assumed a vendor fabricates and distributes several kinds of components/parts in batches to a buyer. The objective is to determine fabrication sequence and the cycle time that minimize the overall system cost per unit time. They investigated the computational behavior of two algorithms – a heuristic and an optimal algorithm were. With large number of component types, the optimal algorithm has long running times, hence, a hybrid algorithm was developed with the aim of efficiently finding optimal solution. Wu et al. [3] determined the optimal common production cycle time policy for a multi-product EPQ model with scrap and an improved multi-delivery policy. They used the conventional differential calculus method to first prove convexity of the system cost function, and then, determine the optimal common cycle time from the first derivative of system cost function.

Grubbstrom and Erdem [18] proposed an algebraic method to economic order quantity model with backlogging without using derivatives. A couple of studies adopted such an approach to resolve different aspects of vendor-buyer inventory systems and manufacturing batch size issues [19-21]. This study extends such an approach to the problem of Wu et al. [3] and demonstrates that the optimal common production cycle time can be obtained without using the differential calculus.

PROBLEM AND THE PROPOSED APPROACH

Reconsider the specific multi-product EPQ model examined by Wu et al. [3] as follows. There are L products to be fabricated in sequence on a machine with unit production cost C_i, where $i = 1, 2, \ldots, L$. During the production process, a x_i portion of scrap items may be randomly produced at a rate of d_i. Without permitting shortages, the production rate P_i must satisfy $(P_i - d_i - \lambda_i) > 0$, where λ_i is the demand rate for product i, and $d_i = x_iP_i$. A $n + 1$ multi-delivery policy is used to distribute all perfect items to buyer, where the first delivery occurs within uptime and followed by n equal-size distribution in the end of uptime [3] for each product i. (see Figure 1).

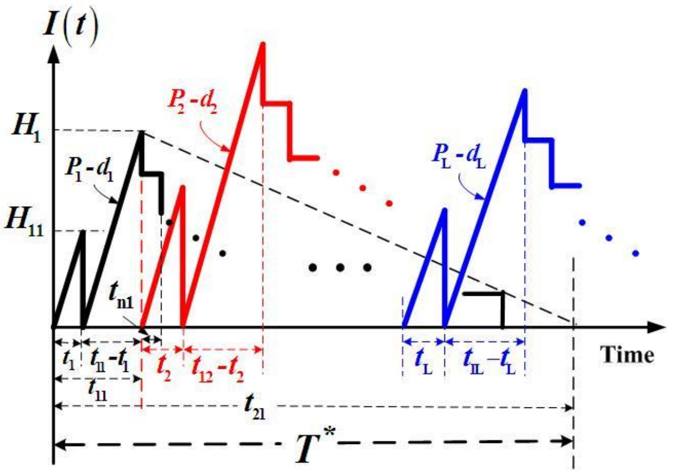

Figure 1. On-hand inventory of perfect quality items in the proposed multi-item EPQ model under common cycle time policy [3]

To ease readers' comparison efforts, this study uses the same notation as in [3] where $i = 1, 2, \ldots, L$, as follows:

T = rotation cycle time - the decision variable,

K_i = setup cost, where $i = 1, 2, \ldots, L$

t_i = time needed to produce enough items of product i to meet customer's demand during uptime t_{1i},

t_{1i} = uptime for product i,

t_{2i} = delivery time for product i,

C_{Si} = unit disposal cost,

K_{1i} = fixed delivery cost,

C_{Ti} = unit delivery cost,

H_{1i} = level of on-hand inventory to meet customer's demand during uptime t_{1i},

H_i = maximum level of on-hand inventory of product i when the regular production ends,

h_i = setup cost,

n = number of fixed quantity installments of the finished lot to be delivered to customers in each cycle,

Q_i = batch size per cycle for product i,

t_{ni} = fixed interval of time between each installment of finished product i being delivered during t_{2i},

$TC(Q_i)$ = per cycle production-inventory-delivery cost,

$E[TCU(T)]$ = total expected production-inventory-delivery cost per unit time for L products in the proposed system using the common cycle time T as the decision variable.

From the proposed model, the following production setup cost, variable fabrication cost, and disposal cost can be obtained for product $i = 1, 2, \ldots, L$, as follows:

$$\sum_{i=1}^{L}\left[K_i + C_iQ_i + C_{Si}\left(x_iQ_i\right)\right] \tag{1}$$

Total fixed and variable transportation costs for product $i = 1, 2, \ldots, L$ in a cycle are

$$\sum_{i=1}^{L}\left[\left(n+1\right)K_{1i} + C_{Ti}Q_i\right] \tag{2}$$

Total inventory holding cost in a production cycle for product $i = 1, 2, \ldots, L$ is

$$\sum_{i=1}^{L} h_i\left[\frac{H_{1i}}{2}\left(t_i\right) + \frac{H_i}{2}\left(t_{1i} - t_i\right) + \frac{d_it_{1i}}{2}\left(t_{1i}\right) + \left(\frac{n-1}{2n}\right)H_it_{2i}\right] \tag{3}$$

Therefore, $TC(Q_i)$ for L products in a cycle is

$$\sum_{i=1}^{L} TC\left(Q_i\right) = \sum_{i=1}^{L}\left\{\begin{array}{l} K_i + C_iQ_i + C_{Si}\left(x_iQ_i\right) + \left(n+1\right)K_{1i} + C_{Ti}Q_i \\ + h_i\left[\frac{H_{1i}}{2}\left(t_i\right) + \frac{H_i}{2}\left(t_{1i} - t_i\right) + \frac{d_it_{1i}}{2}\left(t_{1i}\right) + \left(\frac{n-1}{2n}\right)H_it_{2i}\right] \end{array}\right\} \tag{4}$$

Taking into consideration of random scrap rate and with further derivations the expected $E[TCU(T)]$ can be obtained [3] as

$$E\left[TCU\left(T\right)\right] = \sum_{i=1}^{L}\left\{\begin{array}{l} C_i\lambda_i + \frac{K_i}{T} + C_{Si}\lambda_iE\left[x_i\right] + C_{Ti}\lambda_i + \frac{\left(n+1\right)K_{1i}}{T} \\ + \frac{h_iT\lambda_i^2}{2}\left\{\lambda_i\left(\frac{1}{P_i}\right)^2\left[\frac{2\lambda_i}{P_i\left[1-E\left[x_i\right]\right]}\right] + \frac{1}{P_i} + \left(1-\frac{1}{n}\right)\left[\left[\frac{1}{\lambda_i} - \frac{2}{P_i}\right]\right]\right\} \end{array}\right\} \tag{5}$$

This paper proposes a simplified algebraic approach. Let γ_0, γ_1, and γ_2 be the following:

$$\gamma_0 = \sum_{i=1}^{L}\left\{C_i\lambda_i + C_{Si}\lambda_iE\left[x_i\right] + C_{Ti}\lambda_i\right\} \tag{6}$$

$$\gamma_1=\sum_{i=1}^{L}\left\{\frac{h_i\lambda_i^2}{2}\left\{\lambda_i\left(\frac{1}{P_i}\right)^2\left[\frac{2\lambda_i}{P_i\left[1-E\left[x_i\right]\right]}\right]+\frac{1}{P_i}+\left(1-\frac{1}{n}\right)\left[\left[\frac{1}{\lambda_i}-\frac{2}{P_i}\right]\right]\right\}\right\} \quad (7)$$

$$\gamma_2=\sum_{i=1}^{L}\left[K_i+(n+1)K_{1i}\right] \quad (8)$$

Then, Eq. (5) becomes

$$E\left[TCU(T)\right]=\gamma_0+\gamma_1(T)+\gamma_2\left(T^{-1}\right) \quad (9)$$

With further rearrangement, Eq. (9) becomes

$$E\left[TCU(T)\right]=\gamma_0+T\left(\sqrt{\gamma_1}-\sqrt{\gamma_2}T^{-1}\right)^2+2\sqrt{\gamma_1\gamma_2} \quad (10)$$

It is noted that $E[TCU(T)]$ can be minimized if the second term of Eq. (10) equals zero. So,

$$\sqrt{\gamma_1}=\sqrt{\gamma_2}T^{-1}, \text{ or } T=\sqrt{\frac{\gamma_2}{\gamma_1}} \quad (11)$$

By substituting γ_1 and γ_2 in Eq. (11), the optimal common production cycle length T^* can be derived as follows:

$$T^*=\sqrt{\frac{\sum_{i=1}^{L}\left[K_i+(n+1)K_{1i}\right]}{\sum_{i=1}^{L}\frac{h_i\lambda_i^2}{2}\cdot\left\{\lambda_i\left(\frac{1}{P_i}\right)^2\left[\frac{2\lambda_i}{P_i(1-E\left[x_i\right])}\right]+\frac{1}{P_i}+\left(1-\frac{1}{n}\right)\left[\left[\frac{1}{\lambda_i}-\frac{2}{P_i}\right]\right]\right\}}} \quad (12)$$

It is noted that Eq. (12) is identical to that in [3], which was derived by the conventional differential calculus. Applying T^* in Eq. (10), one obtains the expected system cost as follows:

$$E\left[TCU(T^*)\right]=\gamma_0+2\sqrt{\gamma_1\gamma_2} \quad (13)$$

CONCLUSIONS

In this paper, a straightforward algebraic approach is proposed to resolve the optimization problem of multi-product EPQ model with scrap and an improved multi-delivery policy as studied by Wu et al. [3]. This simplified method can assist practitioners in planning their real-life multi-product fabrication problem more effectively.

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